CHAPTER I

INTRODUCTION

Roughtly speaking a graph consists of points and lines joining certain points. If each line are given a direction we have a directed graph or digraph. Fig. a and Fig. b below are examples of a graph and a digraph respectively.

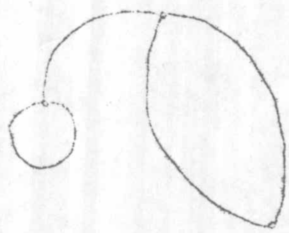

Fig. a

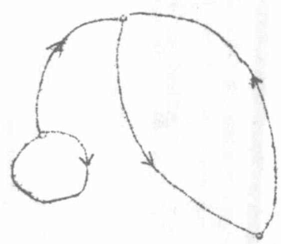

Fig. b

In a graph, points and lines are also known as vertices and edges. An edge in a graph is said to be a loop if it joins a vertex with itself. If there are more than one edge joining the same pair of vertices these edges are called multiple edges. A simple graph is a graph which does not contain any loop nor any nultiple edges. Strictly speaking a simple graph is an ordered pair $(V,\mathcal{E})$, where V is a finite non-empty set and $\mathcal{E}$ is a set of unordered pairs of elements of V. The elements of V are the vertices and the unordered pairs in $\mathcal{E}$ are the edges. In this thesis we consider only simple graphs. The points and directed line in a digraph are also known as vertices and arcs. A simple digraph is a digraph in which for any given two vertices u, v there is at most one arc from u to v. Strictly speaking a simple digraph is an ordered pair (V,A), where

V is finite non-empty set and A is a set of ordered pairs of elements of V. The elements of V are the vertices and the ordered pairs in A are the arcs.

Let $(Q,\circ)$ be a quasi-group and $\mathcal{A}$ be any subset of Q. By means of $\mathcal{A}$, we can make Q into a simple digraph by taking elements of Q to be the vertices and join an arc from q to $q \circ a$ for all q in Q all a in $\mathcal{A}$. Any digraph which can be obtained in this way will be called a quasi-group digraph. If the quasi-group can be chosen to be a group, then the digraph will be called a group digraph. Let $(Q, \circ)$ be a group and $\mathcal{A}$ be any subset of Q. We can make Q into a simple graph by taking elements of Q to be the vertices and join an edge from q to $q \circ a$ for all q in Q and all a in $\mathcal{A}$. From $\{q, q \circ a\} = \{q \circ a, q\}$ we see that we must have $q = (q \circ a) \circ a'$ for some a' in $\mathcal{A}$. This require that for each a in $\mathcal{A}$, a^{-1} must also belong to $\mathcal{A}$. Hence $\mathcal{A}^{-1} \subseteq \mathcal{A}$. Any graph which can be obtained in this way will be called a group graph.. Therefore, in definition group graph only those subset $\mathcal{A}$ such that $\mathcal{A}^{-1} \subseteq \mathcal{A}$ are admissible.

Characterization of quasi-group digraph is given by H.H.Teh [4]. Sabidussi [5] provides a necessary and sufficient condition for a graph to be a group graph. The purpose of this thesis is to define and characterize quasi-group hypergraphs and group hypergraphs.