CHAPTER I

EXACT RESULT IN ONE DIMENSION

1.1 Introduction

Many problems of theoretical physics have often had one - demensional analogs which have turned out to be simpler than those in three dimensions. The quantum theory of disordered structures are representative of problems or models which have an exact solution in one dimension, but are understood only approximately in three dimensions.[1] One of the problems which has been studied successfully in one dimension is the calculation of the density of states for a white noise model. The one - dimensional white noise model is a simple problem of disordered systems, and has an exact form for the density of states.[2] Furthermore, an exact asymptotic form of the density of states can be conventionally set up for testing the validity of the general approximation schemes.

1.2 White Noise Model

The white noise model arises when we consider a system as follows : An electron moves in a high density of atoms. The atoms, all of one kind, have randomly fixed positions, and the electron - atom potentials are assumed to be weak, Dirac delta functions. Moreover, the positions of the atoms are considered to be random variables which obey Gaussian statistics.

1.3 One - Dimensional White Noise Model

When the white noise model appears in one dimension, it is ..

usually called " the one - dimensional white noise model." In the case of the one - dimensional system, the high density of the atoms implies a one - dimensional array of the atoms on a line segment. Furthermore, Gaussian distribution formally requires that the line must be very large. Since the potential of the atoms is weak, and the atoms density is high, the fluctuation about the average potential of the system, denoted by ξ , tends to zero. This means that the asymptotic form of the density of states can be studied by taking $\xi \to 0$.

1.4 Density of States

If we have a function N(E) which is the total density of states at a given energy E of an electron - atoms system, then a function which is called the density of states is defined by

$$\rho(E) \triangleq \frac{d}{dE} N(E) \qquad 1.4.1$$

or, equivalently,

$$\rho(E) = \frac{1}{\Omega} \sum_{k=1} \delta(E - E_k), \qquad 1.4.2$$

where E_k is the energy of the k th eigenstate, Ω is the volume of the system. If the system is disordered, we must average (1.4.2) over the statistical ensemble for the random positions.

1.5 Exact Asymptotic Result

The one - dimensional disordered systems was first studied by Frisch and Lloyd[3] using the method of phase process. Halperin[2] has studied in detail the case of the one - dimensional white noise model using the one-electron Green's function method. He found the exact form of the density of states, and established the exact asymptotic form of the density of states, as $\xi \to 0$, in the form :

$$\rho_{as}(E) = \frac{4}{\pi} \cdot \frac{(-E)}{\xi} \exp \left\{ - \frac{4\sqrt{2}}{3} \cdot \frac{\hbar}{\sqrt{m}} \cdot \left(\frac{(-E)}{\xi} \right)^{3/2} \right\}, \quad 1.5.1$$

where m is an electron mass, $\hbar$ is Planck's constant divided by 2π. Furthermore, the exact asymptotic form (1.5.1) has been reproduced by Zittartz and Langer[4] using the method of functional integration.

1.6 Approximate Density of States

The methods we mentioned in the preceding section give us the exact asymptotic form of the density of states. However, it is not useful for handling the disordered phenomena in three dimensions such as a heavily doped semiconductor. For the three dimensional systems, other approaches have been introduced. One should be to check the validity of the new theories by comparing the new expressions against the exact expression of the density of states for the one - dimensional white noise model. A few of the new methods developed for the three dimensional systems are those of Halperin and Lax[5], Edwards[6], Sa-yakanit[7], and Gross[8]. All of them obtain their results by different approaches, and have all tested their theories with the exact expression (1.5.1). However, for comparison, the two of which gave the good approximate density of states for a screened Coulomb potential in three dimensions will be discussed in terms of the one - dimensional white noise model.

Firstly we consider the Halperin and Lax theory in Chapter II. The density of states is given for their first order approximation[5] by

$$\rho_1(E) = \frac{1}{\sqrt{5}} \cdot \frac{4}{\pi} \cdot \frac{(-E)}{\xi} \cdot \exp\{-\frac{4\sqrt{2}}{3} \cdot \frac{\hbar}{\sqrt{m}} \cdot \frac{(-E)}{\xi}^{3/2}\}, \tag{1.6.1}$$

and for their second order appoximation[9] by

$$\rho_2(E) = \frac{e^{13/18}}{\sqrt{5}} \cdot \frac{4}{\pi} \cdot \frac{(-E)}{\xi} \cdot \exp\{-\frac{4\sqrt{2}}{3} \cdot \frac{\hbar}{\sqrt{m}} \cdot \frac{(-E)}{\xi}^{3/2}\}. \tag{1.6.2}$$

In Chapter III, Sa-yakanit's theory[10] which is more practical than Halperin and Lax's theory is discussed. The density of states for the work of Sa-yakanit in the first cumulant approximation[7] is given by

$$\rho_\ell(E) = \frac{\sqrt{2\pi}}{6} \cdot \frac{4}{\pi} \cdot \frac{(-E)}{\xi} \cdot \exp\{-(\frac{\pi}{3})^{1/2} \cdot \frac{4\sqrt{2}}{3} \cdot \frac{\hbar}{\sqrt{m}} \cdot \frac{(-E)}{\xi}^{3/2}\}. \tag{1.6.3}$$

Next we present our work in Chapter IV which is the extension of the Sa-yakanit calculation in Chapter III. The calculation used the complete first cumulant, and is then extended to include the second cumulant. With the complete first cumulant, the density of states is given by

$$\rho_1(E) = e^{-1/2} \cdot \frac{4\sqrt{2\pi}}{6} \cdot \frac{4}{\pi} \cdot \frac{(-E)}{\xi} \cdot \exp\{-(\frac{\pi}{3})^{1/2} \cdot \frac{4\sqrt{2}}{3} \cdot \frac{\hbar}{\sqrt{m}} \cdot \frac{(-E)}{\xi}^{3/2}\}. \tag{1.6.4}$$

Including the second cumulant correction, the density of states becomes

$$\rho_2(E) = e^{-1/2} \cdot \frac{4\sqrt{2\pi}}{6} \cdot \frac{4}{\pi} \cdot \frac{(-E)}{\xi} \exp\{-(\frac{3031}{3072})(\frac{\pi}{3})^{1/2} \cdot \frac{4\sqrt{2}}{3} \cdot \frac{\hbar}{\sqrt{m}} \cdot \frac{(E)}{\xi}^{3/2}\}. \quad 1.6.5$$

Finally in Chapter V, we compare our results with the Halperin and Lax results as well as with the exact result.