Chapter 2

Theory of Voltage Transformer

A voltage transformer is intended to operate normally with the rated voltage of the network to which it primary winding is connected, and to provide at its secondary terminals a specified rated voltage. The voltage transformer works at the line voltage which in practice is nearly constant and is, in consequence, much less subject to source of error in ratio and phase angle. Let V_{np} be the rated primary voltage and V_{ns} the rated secondary voltage; then the nominal or rated voltage ratio for which the transformer is designed is

$$m = V_{np}/V_{ns}$$

sometime also called the marked ratio .

In practice the actual ratio of a voltage transformer varies with changes in primary voltage, the secondary load and other factors, and may be appreciably different from m. Let V_p be any value of primary voltage and V_s the corresponding value of secondary voltage; then the true or actual voltage ratio will be

$$R = V_p/V_s$$

The voltage V_s will differ from the ideal value V_p/m in magnitude and from exact phase-opposition to V_p by a small angle, γ

called the phase angle, phase error or phase displacement of the voltage transformer. The voltage ratio correction factor is

$$RCF = R/m$$

and the percentage error in a voltage measurement, the percentage voltage error or ratio error is

$$\varepsilon_v = \frac{m\,V_s - R\,V_s}{R\,V_s} \times 100$$

$$= \frac{m\,V_s - V_p}{V_p} \times 100$$

The error in a voltage measurement is positive when, with a given secondary voltage, the primary voltage indicated by the instrument in the secondary circuit exceeds the actual value, i.e. when the instrument read too high; it is understood that this instrument is scaled in nominal primary values. Otherwise, error is positive when, with a given primary voltage, the actual secondary voltage is in excess of its ideal value V_p/m ; or again the nominal ratio exceeds the actual ratio. Further

$$\text{True voltage} = R \times \text{measured voltage}$$

The phase angle γ is generally very small and, though customarily expressed in minutes or arc, would be more rationally stated in centriradians. The phase angle is positive when the reversed secondary voltage vector leads the primary voltage vector.

The external load, consisting of a voltmeter or other instrument with its connecting leads, connected to the secondary terminal of a voltage transformer is its burden and as before, may be specified either in ohms or in volt-amperes with a stated power factor. The rated burden is that with which the voltage transformer can be loaded at the rated secondary voltage without the value of ratio error and phase angle exceeding the limits laid down in the national standard specified.

2.1 Voltage Transformer Equivalent Circuit

In general, the secondary voltage of a constant voltage transformer is proportional to turns ratio, and in phase with or in phase opposition to the primary voltage (depending upon the reference terminal designation) Such would be the case of an ideal transformer, which has no leakage impedance, losses, or exciting current. However, in the actual constant voltage transformer, the energy necessary to magnetize the magnetic circuit of the transformer must be supplied from the primary lines through the leakage impedance of the primary winding. Also the presence of load current in the transformer windings. causes a voltage drop in the leakage impedance of the primary and secondary windings. Load current and exciting current produce an overall voltage drop in the transformer, which results in a ratio error and phase angle other than 180° between primary and secondary terminal voltages.

In the usual analysis of the voltage transformer, the saturation effect may be neglected and assumption of linear, bilateral impedances is valid. Although condition may exist in which the effect of non linearity are not discussed here. Usually the voltage transformer can be analyzed with reasonable accuracy by representing the transformer by the equivalent circuit shown in Fig. 2.1.

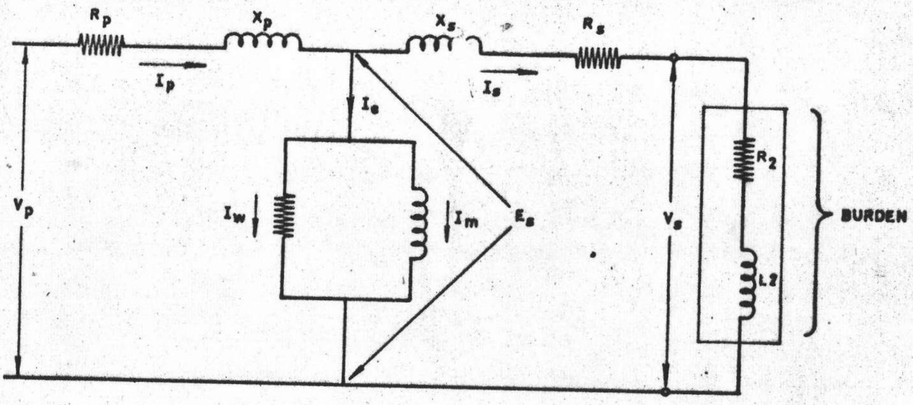

FIG 2-1 VOLTAGE TRANSFORMER EQUIVALENT CIRCUIT

2.2 Actual Ratio & Phase Angle Formulas

The vector diagram resulting from sinusoidal applied voltage is shown in Fig. 2.2 Starting with the main flux ϕ in the core, which links the primary and secondary windings, the induced secondary voltage E_s lags this flux by 90 ° This voltage causes a current I_s to flow in the secondary circuit and a terminal voltage V_s to appear at the secondary burden (the instrument load on transformer). This terminal voltage will be less than E_s because of the voltage drop $I_s Z_s$ in the secondary winding. The primary terminal voltage V_p may be divided into two parts : the primary impedance drop, $I_p Z_p$ and the primary induced voltage, $E_p = -n E_s$ (n being the ratio of primary to secondary turns in the transformer), leading the main flux ϕ by 90 °. The primary current I_p contains two components : I_e the exciting current of the transformer, and $- I_s/n$, the reflection of the secondary current. I_e will itself have two components : I_m, the magnetizing current in phase with the main flux, and I_w, the core-loss current leading the flux by 90 ° and in phase with $E_p = -n E_s$.

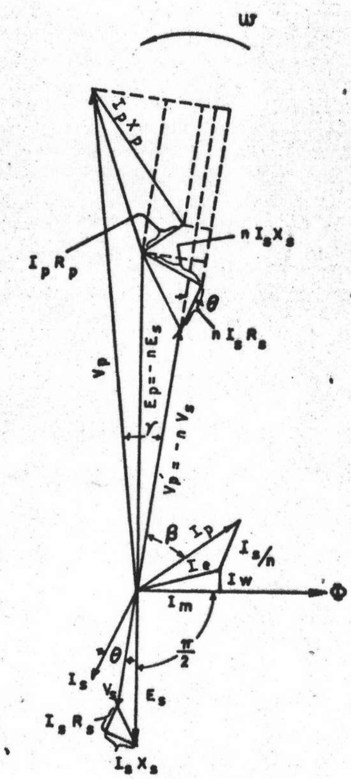

FIG 2-2 VOLTAGE TRANSFORMER VECTER DIAGRAM .

In using such a transformer as an instrument transformer, we are interested in the ratio of primary to secondary terminal voltages and in the phase angle γ between the primary and reversed secondary terminal voltages. R_s, X_s and R_p, X_p are the resistance and leakage reactance of the secondary and primary windings respectively; V_s and V_p are the corresponding terminal voltages; and Θ

is the phase angle of the secondary burden.

If we resolve V_p along V_s reversed and ignore the sign associated in the vector diagram with the reversed secondary vectors, we have :

$$V_p \cos\gamma = nV_s + nI_s(R_s\cos\theta + X_s\sin\theta) + I_p(R_p\cos\beta + X_p\sin\beta)$$

Now, $\gamma < 1°$ for a well-designed voltage transformer at any burden within its rating, and we can say that

$$\cos\gamma = 1 \; ; \; I_p\cos\beta = I_w + \frac{I_s}{n}\cos\theta \; ; \text{ and } I_p\sin\beta = I_m + \frac{I_s}{n}\sin\theta$$

Then.

$$V_p = nV_s + nI_s(R_s\cos\theta + X_s\sin\theta) + R_p(I_w + \frac{I_s}{n}\cos\theta)$$
$$+ X_p(I_m + \frac{I_s}{n}\sin\theta)$$
$$= nV_s + I_s\cos\theta\,(nR_s + \frac{R_p}{n}) + I_s\sin\theta\,(nX_s + \frac{X_p}{n}) + I_wR_p + I_mX_p$$
$$= n\left[V_s + I_s\cos\theta\,(R_s + \frac{R_p}{n^2}) + I_s\sin\theta\,(X_s + \frac{X_p}{n^2}) + \frac{I_wR_p + I_mX_p}{n}\right]$$

we will write

$$R_s + \frac{R_p}{n^2} = R_{ts} \; ; \quad X_s + \frac{X_p}{n^2} = X_{ts}$$

and note that R_{ts}, X_{ts} are the equivalent total resistance and reactance of the transformer referred to its secondary circuit. Substituting these values in the equation above and dividing by V_s, we have

$$\text{actual Ratio} = R = \frac{V_p}{V_s}$$
$$= n\left[1 + \frac{I_s(R_{ts}\cos\theta + X_{ts}\sin\theta) + \frac{I_wR_p + I_mX_p}{n}}{V_s}\right]$$

$$= n + \frac{I_w R_p + I_m X_p}{V_s} + \frac{nI_s}{V_s}\left(R_{ts}\cos\theta + X_{ts}\sin\theta\right) \;....(2.1)$$

Going back to the vector diagram of Fig 2.2 if we write the component of V_p perpendicular to V_s we have

$$V_p \sin\gamma = I_p X_p \cos\beta - I_p R_p \sin\beta + nI_s X_s \cos\theta - nI_s R_s \sin\theta$$

But under the condition of our approximation $\sin\gamma = \gamma$ and $V_p = nV_s$. Also the accepted sign convention on phase angles of instrument transformer required that the angle shall be considered positive when the reversed secondary vector leads the primary vector. This convention requires that the sign of γ in our vector diagram and equation be negative. Then, on making the indicated substitutions, we may write

$$-\gamma = \frac{X_p\left(I_w + \frac{I_s}{n}\cos\theta\right) - R_p\left(I_m + \frac{I_s}{n}\sin\theta\right) + nI_s\left(X_s\cos\theta - R_s\sin\theta\right)}{nV_s}$$

or

$$\gamma = \frac{R_p I_m - X_p I_w}{nV_s} - \frac{I_s}{V_s}\left[\left(\frac{X_p}{n^2} + X_s\right)\cos\theta - \left(\frac{R_p}{n^2} + R_s\right)\sin\theta\right]$$

$$= \frac{R_p I_m - X_p I_w}{nV_s} - \frac{I_s}{V_s}\left(X_{ts}\cos\theta - R_{ts}\sin\theta\right) \;---- (2.2)$$

At no load (secondary circuit open so that $I_s = 0$) we have for the ratio and phase angle

$$R_o = n\left(1 + \frac{I_w R_p + I_m X_p}{nV_s}\right) \;-----------(2.3)$$

and

$$\gamma_o = \frac{R_p I_m - X_p I_w}{nV_s} \;------------ (2.4)$$

Then

$$R = R_o + \frac{nI_s(R_{ts}\cos\theta + X_{ts}\sin\theta)}{V_s} \quad \text{-------- (2.5)}$$

and

$$\gamma \text{ (radian)} = \gamma_o - \frac{I_s}{V_s}(X_{ts}\cos\theta - R_{ts}\sin\theta) \quad \text{-------- (2.6)}$$

$$\gamma \text{ (minutes)} = 3438\left[\gamma_o - \frac{I_s}{V_s}(X_{ts}\cos\theta - R_{ts}\sin\theta)\right] \quad \text{---- (2.7)}$$

(1 radian = 3438 minutes)

2.3 Method for Reducing Ratio Error and Phase Angle Error

The equation for voltage ratio and phase angle may be written

$$R = n + \frac{I_wR_p + I_mX_p}{V_s} + \frac{nI_s}{V_s}(R_{ts}\cos\theta + X_{ts}\sin\theta)$$

$$\gamma = \frac{I_mR_p - I_wX_p}{nV_s} - \frac{I_s}{V_s}(X_{ts}\cos\theta - R_{ts}\sin\theta)$$

the deviation of the actual ratio from the turns ratio, R-n and the phase angle γ each consist of 2 parts. The first depends on the drop of voltage due to the reactance of the primary winding only; while the second depends on the load current I_s determined by the secondary burden, flowing in the equivalent impedance of the whole transformer, looked from the primary side.

At no load the actual ratio exceeds the turns ratio by an amount $(I_wR_p + I_mX_p)/V_s$, i.e. because of the drop in voltage resulting from the exciting current in the resistance and leakage reactance of the primary winding. Hence the actual ratio can be brought to nominal if we reduce the turn ratio by this amount. The manner in which the leakage reactance is divided between the primary and secondary circuits cannot be precisely evaluated. For most purposes it is satisfactory to assign half of the total equivalent reactance to the primary circuit. With any inductive or resistive load there is a further increase of ratio because of the voltage drop resulting from the load current in the equivalent total impedance of the primary and secondary transformer windings. This would require a further reduction in turn ratio to bring the actual ratio to nominal. Thus the ratio factor of a voltage transformer can be brought to unity for some particular combination of load and voltage by adjusting the turn ratio. This may be done either by adding secondary turns or by removing an equivalent percentage of primary turns. It will be seen from the equations developed above that the no-load phase angle, γ_0 , could be made zero if $I_m R_p = I_w X_p$. However, it is not generally practical to design for this conditions, and the no - load phase angle is generally positive because $I_m R_p > I_w X_p$. The means which can be taken to reduce R_p will increase I_m, and a compromise must be made. Low values of R and X are of importance in decreasing the errors under load, and accordingly the best design is usually one with a minimum number of turns and as high a flux density in the core as is feasible without approaching too closely to saturation. Designers are not altogether in agreement as to the proper value of flux density to use, but high permeability (low I_m) is more important than low loss (low I_w).

2.4 Correction for Ratio and Phase Angle Errors.

The ratio correction factor (RCF) of a voltage transformer is that factor by which the marked ratio (ratio as indicated on the nameplate) must be multiplied to obtain the true ratio, V_p/V_s and is given by

$$RCF = \frac{V_p/V_s}{\text{Marked Ratio}} \quad \text{----------} \quad (2.8)$$

The phase angle correction factor (PACF) of a voltage transformer is that factor by which the apparent power factor must be multiplied to obtain the true (system) power factor. For positive transformer phase angle, reversed V_s leads V_p; therefore for a lagging power factor load, the true power factor angle θp, is less than the indicated power factor angle θ. Assuming no ratio error exists, the phase angle correction factor for a voltage transformer is given by

$$PACF = \frac{\cos \theta p}{\cos \theta} = \frac{\cos(\theta - \gamma)}{\cos \theta} \quad \text{--------} \quad (2.9)$$

The PACF can be expressed in terms of the apparent system power factor angle θ (as indicated on the secondary side of the transformer), as is given by equation (2.9) In regard to the standard accuracy classifications for metering service which are given in the standards on instrument transformers, the accuracy standards are basedon the system power factor.

Since γ is usually very small of the order of minutes, PACF can be given in terms of θ with sufficient accuracy by

$$PACF = 1 + \gamma \frac{\tan \theta}{3438} \quad \text{-------------} \quad (2.10)$$

where γ is expressed in minutes.

The transformer correction factor (TCF) is the factor by which the reading on a wattmer or registration of a watthour meter must be multiplied to correct for the effect of the error in ratio and phase angle of the voltage transformer. It is numerically equal to the product of RCF and PACF. The ratio of the true system power to the indicated system power (as indicated by a wattmeter, including the marked turns ratio) is given approximately by

$$TCF = \frac{P_t}{P_m} = \frac{V_p I_p \cos(\theta - \gamma)}{(\text{Marked Ratio})\, V_s I_p \cos\theta}$$

$$\text{Marked Ratio} = \frac{V_p}{V_s \times (RCF)}$$

$$\therefore\ TCF = (RCF) \times (PACF)$$

$$= RCF\left(1 + \frac{\gamma \tan\theta}{3438}\right) \qquad \text{-----------} \qquad (2.11)$$

In measurement of voltage only, the ratio error is the only error of importance. However, in the measurement of a quantity which is a function of the product of voltage and current (Watt meters and watthour meters), the ratio error and phase angle are both involved.

An RCF greater than the unity indicates that the true turns ratio is greater than marked ratio, and results in a low meter reading, based on the marked ratio.

Positive phase angle results in low and high meter readings and PACF greater and less than unity for lagging and leading power factor loads, respectively.

2.5 Service Conditions Affecting Voltage Transformer Errors

2.5.1 Burden

The ratio is approximately linear with burden at constant burden power factor and voltage. Characteristics at various burden power factors meet at zero burden. This factor makes it possible to determine the characteristics of any burden power factor, if the characteristics are known for one power factor.

At a particular burden power factor, the angle of which is equal to the through impedance angle of the transformer, the phase angle is equal to the phase angle at zero burden for the usual range of burdens. However, for the same burden power factor, the rate of change of RCF with burden is maximum. At extremely low and high burden power factors, the effects of ratio and phase angle errors tend to compensate for one another in the measurement of power.

2.5.2 Wave - form

The effect of wave - form is relatively important in practice since the deviation of practical voltage wave - form the sine shape is usually slight. With a third harmonic as large as 30 percent of the fundamental, the ratio changes by less than 0.1 percent from the values found with a sine wave; a peaked wave lowers the ratio and a dimpled wave increases it. Since such a harmonic content would be quite exceptional in practice, it may safely be assumed that the characteristics of a voltage transformer are not appreciably changed by altering the shape of the primary voltage wave. Oscillograph tests also show that the secondary voltage wave is an exact copy of the primary wave, indicating a negligible distortion.

2.5.3 Frequency

Increasing frequency at a fixed voltage has little effect in the voltage transformer characteristics, since it increases the voltage drop due to leakage flux and decrease the flux density and the required exciting current. Unless the frequency is more than doubled, the effect on performance is usually small.

Decreasing the frequency results in an increase in flux density and a corresponding increase in exciting current. A voltage transformer should not be operated at less than 95 percent of rated frequency.

2.5.4 Voltage

The accuracy of a voltage transformer at other than rated voltage is influenced by the degree by which the exciting current deviates from the normal value. The accuracy characteristics are unchanged at reduced voltage. At abnormally high voltage, the exciting current is excessive and may result in serious error. Normal voltage variations do not result in serious errors.