REFERENCES

1. Berge,C. Graphs and Hypergraphs. Amsterdam: North-Halland Publishing Company, 1973.

2. Hall,F. Graph Theory. New York: Addison-Wesley Publishing Company, 1971.

3. Hall,M. The Theory of Groups. New York: The Macmillan Company, 1959.

4. Jeamsak Trisirirat. "Algebraic Hypergraphs." Master's thesis, Department of Mathematics, Graduate School, Chulalongkorn University, 1980.

APPENDIX I

This appendix deals with admissible sets in groups. We describe a method which enable us to determine all the admissible sets in a group. For a given group G and a given positive integer $r \geq 2$, the set of all admissible sets of (r-1)-subsets of G is partially ordered by set inclusion. By minimal admissible sets we mean those admissible sets which are minimal with respect to this partial order.

1 Theorem. Let $(G,*)$ be a group. Then any admissible set of (r-1)-subsets of G is a union of minimal admissible sets of (r-1)-subsets of G.

2 Theorem. Let $(G,*)$ be a group. Then any minimal admissible set of (r-1)-subsets of G is of the form

$$\{\{a_1,a_2,\ldots,a_{r-1}\}\} \cup \{a_i^{-1} * \{a_1,a_2,\ldots,a_{i-1},e,a_{i+1},\ldots,a_{r-1}\} / i = 1,2,\ldots,r-1\},$$

for some distinct a_i, $i = 1,2,\ldots,r-1$, in $G-\{e\}$.

To prove Theorem 1 and Theorem 2 we need the following lemmas.

1 Lemma. Let $(G,*)$ be a group and $\mathcal{A}$ be a set of (r-1)-subsets of $G-\{e\}$. If for each A in $\mathcal{A}$ and each a in A, there exists B_a in $\mathcal{A}$ such that

$$A - \{a\} = (a * B_a) - \{e\} ,$$

then $\mathcal{A}$ is admissible.

Proof. Let $(G, *)$ be a group and $\mathcal{A}$ be a set of $(r-1)$-subsets of $G-\{e\}$. Suppose that for each A in $\mathcal{A}$ and each a in A, there exists B_a in $\mathcal{A}$ such that

$$A - \{a\} = (a * B_a) - \{e\} .$$

Let A be any set in $\mathcal{A}$. Let a be any element in A and g be any element in G. By supposition, there exists B_a in $\mathcal{A}$ such that

$$A - \{a\} = (a * B_a) - \{e\} .$$

Therefore

$$\begin{aligned}(\{e\} \cup A) - \{a\} &= \{e\} \cup (A - \{a\}) ,\\ &= \{e\} \cup ((a * B_a) - \{e\}),\\ &= a * B_a .\end{aligned}$$

Hence

$$\begin{aligned}(\{g\} \cup g * A) - \{g * a\} &= g * (\{e\} \cup e * A) - g * \{a\},\\ &= g * [(\{e\} \cup A) - \{a\}],\\ &= g * (a * B_a),\\ &= (g * a) * B_a .\end{aligned}$$

Hence $\mathcal{A}$ is admissible. #

2 Lemma. Let $(G,*)$ be a group and $\mathcal{A}$ be an admissible set of $(r-1)$-subsets of G. Then e does not belong to A for all A in $\mathcal{A}$.

Proof. Let $(G,*)$ be a group and $\mathcal{A}$ be an admissible set of $(r-1)$-subsets of G. Suppose that there exists A in $\mathcal{A}$ such that e belongs to A. Since $\mathcal{A}$ is admissible, hence there exists B in $\mathcal{A}$ such that

$$(\{e\} \cup e * A) - \{e * e\} = (e * e) * B .$$

Note that

$$\begin{aligned} |(\{e\} \cup e * A) - \{e * e\}| &= |(\{e\} \cup A) - \{e\}| , \\ &= |A - \{e\}| , \\ &= r-2 . \end{aligned}$$

But $|(e * e) * B| = |B| = r-1$. Hence we have a contradiction. Therefore e does not belong to A for all A in $\mathcal{A}$. #

3 Lemma. Let $(G,*)$ be a group and $a_1, a_2, \ldots, a_{r-1}$ be distinct elements in $G-\{e\}$. Then any minimal admissible set containing $\{a_1, a_2, \ldots, a_{r-1}\}$ is of the form

$$\{\{a_1, a_2, \ldots, a_{r-1}\}\} \cup \{a_i^{-1} * \{a_1, a_2, \ldots, a_{i-1}, e, a_{i+1}, \ldots, a_{r-1}\} /$$
$$i = 1,2,3,\ldots,r-1\}.$$

Proof. Let $(G,*)$ be a group and $a_1, a_2, \ldots, a_{r-1}$ be distinct elements in $G-\{e\}$.

First, we show that the set

$$\mathcal{A} = \{\{a_1,a_2,\ldots,a_{r-1}\}\} \cup \{a_i^{-1} * \{a_1,a_2,\ldots,a_{i-1},e,a_{i+1},\ldots,a_{r-1}\} / i = 1,2,3,\ldots,r-1\}.$$

is admissible. Let A be any element in $\mathcal{A}$.

<u>Case 1.</u> If $A = \{a_1,a_2,\ldots,a_{r-1}\}$, we see that

$$A-\{a_i\} = (a_i * (a_i^{-1} * \{a_1,a_2,\ldots,a_{i-1},e,a_{i+1},\ldots,a_{r-1}\}))-\{e\},$$

for all a_i, $i = 1,2,\ldots,r-1$.

<u>Case 2.</u> If $A = a_i^{-1} * \{a_1,a_2,\ldots,a_{i-1},e,a_{i+1},\ldots,a_{r-1}\}$ for some $i = 1,2,\ldots,r-1$, we see that

$$A-\{a_i^{-1}\} = (a_i^{-1}*\{a_1,a_2,\ldots,a_{r-1}\})-\{e\} ,$$

and for $j = 1,2,\ldots,i-1,i+1,\ldots,r-1$,

$$A-\{a_i^{-1} * a_j\} = ((a_i^{-1} * a_j)*(a_j^{-1} * \{a_1,a_2,\ldots,a_{j-1},e,a_{j+1},\ldots,a_{r-1}\ \ldots,a_{r-1}\}))-\{e\}.$$

Hence for any a in A there exists B_a in $\mathcal{A}$ such that

$$A-\{a\} = (a * B_a)-\{e\} .$$

Hence, by Lemma 1, $\mathcal{A}$ is admissible.

Next, we show that $\mathcal{A}$ is the only minimal admissible set of (r-1)-subsets of G containing $\{a_1,a_2,\ldots,a_{r-1}\}$. Let $\mathcal{B}$ be any minimal admissible set of (r-1)-subsets of G containing $\{a_1,a_2,\ldots,a_{r-1}\}$.

Hence for each a_i, $i = 1,2,\ldots,r-1$, there exists B_{a_i} in $\mathcal{B}$ such that

$$(\{e\} \cup e * \{a_1,a_2,\ldots,a_{r-1}\}) - \{e * a_i\} = (e * a_i) * B_{a_i} .$$

By straightforward verifications, it can be shown that

$$B_{a_i} = a_i^{-1} * \{a_1,a_2,\ldots,a_{i-1},e,a_{i+1},\ldots,a_{r-1}\} ,$$

for all $i = 1,2,\ldots,r-1$. Hence

$$\mathcal{A} \subseteq \mathcal{B}$$

Since $\mathcal{A}$ is admissible and $\{a_1,a_2,\ldots,a_{r-1}\}$ is in $\mathcal{A}$, hence

$$\mathcal{B} \subseteq \mathcal{A}$$

Therefore $\mathcal{A} = \mathcal{B}$. Hence $\mathcal{A}$ is the only minimal admissible set of $(r-1)$-subsets of G containing $\{a_1,a_2,\ldots,a_{r-1}\}$. #

<u>Proof of Theorem 2.</u> Let $(G,*)$ be a group. Let $\mathcal{A}$ be any minimal admissible set of $(r-1)$-subsets of G. Since $\mathcal{A}$ is non-empty, hence there exists $(r-1)$-subset $\{a_1,a_2,\ldots,a_{r-1}\}$ of G in $\mathcal{A}$. Hence, by Lemma 2, $a_1,a_2,\ldots,a_{r-1}$ are distinct elements in $G-\{e\}$. Hence, by Lemma 3,

$$\mathcal{A} = \{\{a_1,a_2,\ldots,a_{r-1}\}\} \cup \{a_i^{-1} * \{a_1,a_2,\ldots,a_{i-1},e,a_{i+1},\ldots,a_{r-1}\} / i = 1,2,\ldots,r-1\}.$$

Hence we have Theorem 2.

4 <u>Proposition.</u> Let $(G,*)$ be a group. Then distinct minimal admissible sets of $(r-1)$-subsets of G are disjoint.

<u>Proof.</u> Let $(G,*)$ be a group. Let $\mathcal{A}$, $\mathcal{B}$ be distinct minimal admissible sets of $(r-1)$-subsets of G. Suppose that $\mathcal{A} \cap \mathcal{B} \neq \emptyset$. Hence there exists $(r-1)$-subset $\{a_1,a_2,\ldots,a_{r-1}\}$ of G in $\mathcal{A} \cap \mathcal{B}$. Hence, by Lemma 2, $a_1,a_2,\ldots,a_{r-1}$ are distinct elements in $G-\{e\}$. Therefore $\mathcal{A}$ and $\mathcal{B}$ are minimal admissible sets of $(r-1)$-subsets of G containing $\{a_1,a_2,\ldots,a_{r-1}\}$. Hence, by Lemma 3,

$$\mathcal{A} = \{\{a_1,a_2,\ldots,a_{r-1}\}\} \cup \{a_i^{-1} * \{a_1,a_2,\ldots,a_{i-1},e,a_{i+1},\ldots,a_{r-1}\} / i = 1,2,3,\ldots,r-1\},$$

$$= \mathcal{B} .$$

Therefore we have a contradiction. Hence distinct minimal admissible sets of $(r-1)$-subsets of G are disjoint. #

5 <u>Proposition.</u> Let $(G.*)$ be a group. The set of all minimal admissible sets of $(r-1)$-subsets of G forms a partition of $P_{r-1}(G-\{e\})$, the set of all $(r-1)$-subsets of $G-\{e\}$.

<u>Proof.</u> Let $(G,*)$ be a group. Let Ω be the set of all minimal admissible sets of $(r-1)$-subsets of G. Hence

$$\cup \Omega \subseteq P_{r-1}(G-\{e\}). \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Let $\{a_1,a_2,\ldots,a_{r-1}\}$ be any set in $P_{r-1}(G-\{e\})$. Hence $a_1,a_2,\ldots$ $\ldots,a_{r-1}$ are distinct elements in $G-\{e\}$. Therefore, by Lemma 3,

$\mathcal{A} = \{\{a_1,a_2,\ldots,a_{r-1}\}\} \cup \{a_i^{-1} * \{a_i,a_2,\ldots,a_{i-1},e,a_{i+1},\ldots,a_{r-1}\} /$
$i = 1,2,3,\ldots,r-1\}$

is a minimal admissible set of (r-1)-subsets of G. Hence $\{a_1,a_2,\ldots,a_{r-1}\}$ belongs to $\cup\Omega$. Hence

$$P_{r-1}(G-\{e\}) \subseteq \cup\Omega \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From (1) and (2), we have

$$\cup\Omega = P_{r-1}(G-\{e\}).$$

By Proposition 3, any elements $\mathcal{A}$, $\mathcal{B}$ in Ω, $\mathcal{A} = \mathcal{B}$ or $\mathcal{A} \cap \mathcal{B} = \emptyset$. Hence Ω is a partition of $P_{r-1}(G-\{e\})$. #

By Theorem 1 and Proposition 5 we can find all minimal admissible sets of (r-1)-subsets of G.

<u>Proof of Theorem 1.</u> Let $(G,*)$ be a group. Let $\mathcal{A}$ be any admissible set of (r-1)-subsets of G. For each A in $\mathcal{A}$, there exist distinct elements $a_1,a_2,\ldots,a_{r-1}$ in $G-\{e\}$ such that

$$A = \{a_1,a_2,\ldots,a_{r-1}\}.$$

Let

$M(A) = \{\{a_1,a_2,\ldots,a_{r-1}\}\} \cup \{a_i^{-1} * \{a_1,a_2,\ldots,a_{i-1},e,a_{i+1},\ldots,a_{r-1}\} /$
$i = 1,2,3,\ldots,r-1\}$.

Hence M(A) is a minimal admissible set of (r-1)-subsets of G.

Hence

$$M(A) \subseteq \mathcal{A}$$

for all A in $\mathcal{A}$. Therefore

$$\mathcal{A} \subseteq \bigcup_{A \in \mathcal{A}} M(A),$$

$$\subseteq \mathcal{A} .$$

Hence

$$\mathcal{A} = \bigcup_{A \in \mathcal{A}} M(A).$$

Therefore any admissible set of (r-1)-subsets of G is a union of minimal admissible sets of (r-1)-subsets of G. #

APPENDIX II

This appendix is a supplement of Example 4.3.1. In this appendix we prove the followings;

(1) There does not exist a compatible full family of Γ-injections of type 1: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^1, \alpha_5^4)$;

(2) There does not exist a compatible full family of Γ-injections of type 7: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^1, \alpha_5^1)$;

(3) $A_{10} = (\alpha_1^4, \alpha_2^5, \alpha_3^1, \alpha_4^1, \alpha_5^2)$ is the unique compatible full family of Γ-injections of type 10: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^1, \alpha_5^2)$;

(4) $A_{11} = (\alpha_1^2, \alpha_2^1, \alpha_3^5, \alpha_4^1, \alpha_5^3)$ is the unique compatible full family of Γ-injections of type 11: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^1, \alpha_5^3)$;

(5) $A_{12} = (\alpha_1^5, \alpha_2^4, \alpha_3^1, \alpha_4^2, \alpha_5^1)$ is the unique compatible full family of Γ-injections of type 12: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^2, \alpha_5^1)$;

(6) $A_{13} = (\alpha_1^1, \alpha_2^2, \alpha_3^3, \alpha_4^2, \alpha_5^3)$ is the unique compatible full family of Γ-injections of type 13: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^2, \alpha_5^3)$;

(7) $A_{14} = (\alpha_1^3, \alpha_2^1, \alpha_3^4, \alpha_4^3, \alpha_5^1)$ is the unique compatible full family of Γ-injections of type 14: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^3, \alpha_5^1)$;

(8) $A_{15} = (\alpha_1^1, \alpha_2^3, \alpha_3^2, \alpha_4^3, \alpha_5^2)$ is the unique compatible full family of Γ-injections of type 15: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^3, \alpha_5^2)$;

(9) $A_{16} = (\alpha_1^1, \alpha_2^1, \alpha_3^1, \alpha_4^4, \alpha_5^4)$ is the unique compatible full family of Γ-injections of type 16: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^4, \alpha_5^4)$.

Proof of (1). Observe that

$$\mathcal{S}(\alpha_4^1, 5) = \emptyset$$

and

$$\mathcal{S}(\alpha_5^4, 4) = \{\{4,5\}\},$$

hence α_4^1 and α_5^4 are not compatible. Hence there does not exist any compatible full family of Γ-injections of type 1: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^1, \alpha_5^4)$.

Proof of (2). Observe that

$$\mathcal{S}(\alpha_4^1, 1) = \{\{1,4\}\}$$

and

$$\mathcal{S}(\alpha_1^1, 4) = \mathcal{S}(\alpha_1^3, 4) = \mathcal{S}(\alpha_1^5, 4) = \emptyset ,$$

hence α_1^1, α_1^3 and α_1^5 are not compatible to α_4^1. Observe that

$$\mathcal{S}(\alpha_5^1, 1) = \{\{1,5\}\}$$

and

$$\mathcal{S}(\alpha_1^2, 5) = \mathcal{S}(\alpha_1^4, 5) = \emptyset ,$$

hence α_1^2 and α_1^4 are not compatible to α_5^1. Observe that

$$\mathcal{S}(\alpha_1^6,5) = \{\{1,5\}\} = \mathcal{S}(\alpha_5^1,1)$$

and

$$\mathcal{S}(\alpha_1^6,4) = \{\{1,4\}\} = \mathcal{S}(\alpha_4^1,1).$$

Hence α_1^6 is the only $(\Gamma,1)$-injection which is compatible to both α_4^1 and α_5^1. Therefore, if $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^1, \alpha_5^1)$ is compatible, then α_1^i must be α_1^6. Observe that

$$\mathcal{S}(\alpha_1^6,2) = \emptyset$$

and

$$\mathcal{S}(\alpha_2^1,1) = \mathcal{S}(\alpha_2^2,1) = \mathcal{S}(\alpha_2^3,1) = \{\{1,2\}\},$$

hence α_2^1, α_2^2 and α_2^3 are not compatible to α_1^6. Observe that

$$\mathcal{S}(\alpha_2^4,4) = \{\{2,4\}\}$$

and

$$\mathcal{S}(\alpha_4^1,2) = \emptyset.$$

hence α_2^4 is not compatible to α_4^1. Observe that

$$\mathcal{S}(\alpha_5^1,2) = \emptyset$$

and

$$\mathcal{S}(\alpha_2^5,5) = \mathcal{S}(\alpha_2^6,5) = \{\{2,5\}\},$$

hence α_2^5 and α_2^6 are not compatible to α_5^1. Hence there does not exist any α_2^j which is compatible to α_1^6, α_4^1, α_5^1. Therefore there does not exist a compatible full family of Γ-injections of type 7: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^1, \alpha_5^1)$.

Proof of (3). First, we find all the (Γ,3)-injections which are compatible to α_4^1 and α_5^2. Observe that

$$\mathcal{S}(\alpha_4^1,3) = \emptyset = \mathcal{S}(\alpha_3^1,4)$$

and

$$\mathcal{S}(\alpha_5^2,3) = \emptyset = \mathcal{S}(\alpha_3^1,5),$$

hence α_3^1 is compatible to α_4^1 and α_5^2. Observe that

$$\mathcal{S}(\alpha_4^1,3) = \emptyset$$

and

$$\mathcal{S}(\alpha_3^2,4) = \mathcal{S}(\alpha_3^4,4) = \mathcal{S}(\alpha_3^6,4) = \{\{3,4\}\},$$

hence α_3^2, α_3^4 and α_3^6 are not compatible to α_4^1. Observe that

$$\mathcal{S}(\alpha_5^2,3) = \emptyset$$

and

$$\mathcal{S}(\alpha_3^3,5) = \mathcal{S}(\alpha_3^5,5) = \{\{3,5\}\},$$

hence α_3^3 and α_3^5 are not compatible to α_5^2. Hence α_3^1 is the only (Γ,3)-injection which is compatible to α_4^1 and α_5^2.

Next, we find all the $(\Gamma,2)$-injections which are compatible to α_3^1, α_4^1 and α_5^2 . Observe that

$$\mathcal{S}(\alpha_5^2,2) = \{\{2,5\}\}$$

and

$$\mathcal{S}(\alpha_2^1,5) = \mathcal{S}(\alpha_2^2,5) = \mathcal{S}(\alpha_2^4,5) = \emptyset ,$$

hence α_2^1, α_2^2 and α_2^4 are not compatible to α_5^2. Observe that

$$\mathcal{S}(\alpha_3^1,2) = \{\{2,3\}\}$$

and

$$\mathcal{S}(\alpha_2^3,3) = \mathcal{S}(\alpha_2^6,3) = \emptyset ,$$

hence α_2^3 and α_2^6 are not compatible to α_3^1. Observe that

$$\mathcal{S}(\alpha_2^5,3) = \{\{2,3\}\} = \mathcal{S}(\alpha_3^1,2) ,$$

$$\mathcal{S}(\alpha_2^5,4) = \emptyset = \mathcal{S}(\alpha_4^1,2)$$

and

$$\mathcal{S}(\alpha_2^5,5) = \{\{2,5\}\} = \mathcal{S}(\alpha_5^2,2) ,$$

hence α_2^5 is compatible to α_3^1, α_4^1 and α_5^2. Hence α_2^5 is the only $(\Gamma,2)$-injection which is compatible to α_3^1, α_4^1 and α_5^2.

Finally, we find all the $(\Gamma,1)$-injections which are compatible to α_2^5, α_3^1, α_4^1 and α_5^2 . Observe that

$$\mathcal{S}(\alpha_4^1, 1) = \{\{1,4\}\}$$

and

$$\mathcal{S}(\alpha_1^1, 4) = \mathcal{S}(\alpha_1^3, 4) = \mathcal{S}(\alpha_1^5, 4) = \emptyset ,$$

hence α_1^1, α_1^3 and α_1^5 are not compatible to α_4^1. Observe that

$$\mathcal{S}(\alpha_1^2, 2) = \{\{1,2\}\}$$

and

$$\mathcal{S}(\alpha_2^5, 1) = \emptyset ,$$

hence α_1^2 is not compatible to α_2^5. Observe that

$$\mathcal{S}(\alpha_1^6, 5) = \{\{1,5\}\}$$

and

$$\mathcal{S}(\alpha_5^2, 1) = \emptyset ,$$

hence α_1^6 is not compatible to α_5^2. Observe that

$$\mathcal{S}(\alpha_1^4, 2) = \emptyset = \mathcal{S}(\alpha_2^5, 1) ,$$

$$\mathcal{S}(\alpha_1^4, 3) = \{\{1,3\}\} = \mathcal{S}(\alpha_3^1, 1),$$

$$\mathcal{S}(\alpha_1^4, 4) = \{\{1,4\}\} = \mathcal{S}(\alpha_4^1, 1).$$

and

$$\mathcal{S}(\alpha_1^4, 5) = \emptyset = \mathcal{S}(\alpha_5^2, 1) ,$$

hence α_1^4 is compatible to α_2^5, α_3^1, α_4^1 and α_5^2. Hence α_1^4 is the only $(\Gamma,1)$-injection which is compatible to α_2^5, α_3^1, α_4^1 and α_5^2. Hence

$A_{10} = (\alpha_1^4, \alpha_2^5, \alpha_3^1, \alpha_4^1, \alpha_5^2)$ is the unique compatible full family of Γ-injections of type 10: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^1, \alpha_5^2)$.

<u>Proof of (4).</u> First, we determine all the $(\Gamma,2)$-injections which are compatible to α_4^1 and α_5^3. It turns out that α_2^1 is the only $(\Gamma,2)$-injection which is compatible to α_4^1 and α_5^3. Next, we determine all the $(\Gamma,3)$-injections which are compatible to α_2^1, α_4^1 and α_5^3. We find that α_3^5 is the only $(\Gamma,3)$-injection which is compatible to α_2^1, α_4^1 and α_5^3. Finally, we determine all the $(\Gamma,1)$-injections which are compatible to α_2^1, α_3^5, α_4^1 and α_5^3. α_1^2 is the only $(\Gamma,1)$-injection which is compatible to α_2^1, α_3^5, α_4^1 and α_5^3. Hence $A_{11} = (\alpha_1^2, \alpha_2^1, \alpha_3^5, \alpha_4^1, \alpha_5^3)$ is the unique compatible full family of Γ-injection of type 11: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^1, \alpha_5^3)$.

<u>Proof of (5).</u> To prove (5), we do similarly as we prove (3)

It can be shown that α_3^1 is the only $(\Gamma,3)$-injection which is compatible to α_4^2 and α_5^1, α_2^4 is the only $(\Gamma,2)$-injection which is compatible to α_3^1, α_4^2 and α_5^1, α_1^5 is the only $(\Gamma,1)$-injection which is compatible to α_2^4, α_3^1, α_4^2 and α_5^1. Hence $A_{12} = (\alpha_1^5, \alpha_2^4, \alpha_3^1, \alpha_4^2, \alpha_5^1)$ is the unique compatible full family of Γ-injections of type 12: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^2, \alpha_5^1)$.

<u>Proof of (6).</u> First, we determine all the $(\Gamma,1)$-injections which are compatible to α_4^2 and α_5^3. It turns out that α_1^1 is the only $(\Gamma,1)$-injection which is compatible to α_4^2 and α_5^3. Next, we determine all the $(\Gamma,2)$-injections which are compatible to α_1^1, α_4^2 and α_5^3. We

find that α_2^2 is the only $(\Gamma,2)$-injection which is compatible to α_1^1, α_4^2 and α_5^3. Finally, we determine all the $(\Gamma,3)$-injections which are compatible to α_1^1, α_2^2, α_4^2 and α_5^3. α_3^3 is the only $(\Gamma,3)$-injection which is compatible to α_1^1, α_2^2, α_4^2 and α_5^3. Hence $A_{13} = (\alpha_1^1, \alpha_2^2, \alpha_3^3, \alpha_4^2, \alpha_5^3)$ is the unique compatible full family of Γ-injections of type 13: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^2, \alpha_5^3)$.

<u>Proof of (7).</u> To prove (7), we do similarly as we prove (4). It can be shown that, α_2^1 is the only $(\Gamma,2)$-injection which is compatible to α_4^3 and α_5^1, α_3^4 is the only $(\Gamma,3)$-injection which is compatible to α_2^1, α_4^3 and α_5^1, α_1^3 is the only $(\Gamma,1)$-injection which is compatible to α_2^1, α_3^4, α_4^3 and α_5^1. Hence $A_{14} = (\alpha_1^3, \alpha_2^1, \alpha_3^4, \alpha_4^3, \alpha_5^1)$ is the unique compatible full family of Γ-injections of type 14: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^3, \alpha_5^1)$.

<u>Proof of (8).</u> To prove (8), we do similarly as we prove (6). It can be shown that, α_1^1 is the only $(\Gamma,1)$-injection which is compatible to α_4^3 and α_5^2, α_2^3 is the only $(\Gamma,2)$-injection which is compatible to α_1^1, α_4^3 and α_5^2, α_3^2 is the only $(\Gamma,3)$-injection which is compatible to α_1^1, α_2^3, α_4^3 and α_5^2. Hence $A_{15} = (\alpha_1^1, \alpha_2^3, \alpha_3^2, \alpha_4^3, \alpha_5^2)$ is the unique compatible full family of Γ-injections of type 15: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^3, \alpha_5^2)$.

Proof of (9). To prove (9), we do similarly as we prove (4). It can be shown that, α_3^1 is the only $(\Gamma,3)$-injection which is compatible to α_4^4 and α_5^4, α_2^1 is the only $(\Gamma,2)$-injection which is compatible to α_3^1, α_4^4 and α_5^4, α_1^1 is the only $(\Gamma,1)$-injection which is compatible to α_2^1, α_3^1, α_4^4 and α_5^4. Hence $A_{16} = (\alpha_1^1, \alpha_2^1, \alpha_3^1, \alpha_4^4, \alpha_5^4)$ is the unique compatible full family of Γ-injections of type 16: $(\alpha_1^i, \alpha_2^j, \alpha_3^k, \alpha_4^4, \alpha_5^4)$.

VITA

Name : Mr. Dumrong Tipyotha

Degree : B.Sc., 1979.

Chulalongkorn University, Bangkok, Thailand.

Scholarship : University Development Commission (U.D.C),

Thai Government.