CHAPTER I

INTRODUCTION

A hypergraph H is an ordered pair $(V,\mathcal{E})$, where V is a finite set and $\mathcal{E}$ is a set of non-empty subsets of V such that $\cup\mathcal{E} = V$. Any element v in V is called a vertex and any element E in $\mathcal{E}$ is called an edge. For example, let

$$V = \{1,2,3,4,5\}$$

and

$$\mathcal{E} = \{\{4\},\{3,5\},\{1,2,3\},\{1,4,5\}\}.$$

We see that $\cup\mathcal{E} = V$. Hence $H = (V,\mathcal{E})$ is a hypergraph. To represent a hypergraph $(V,\mathcal{E})$ by a diagram, we represent each vertex v by a point and each edge E is drawn as a curve encircling all the points respresenting the vertices that belong to E. The hypergraph in the above example can be represented by the diagram in Fig 1 :

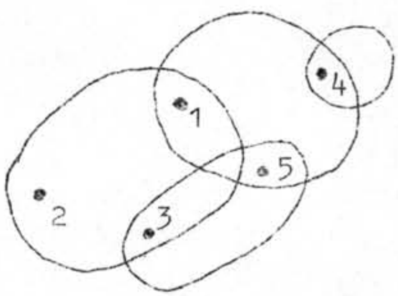

Fig 1.

To each vertex v of a hypergraph $H = (V, \mathcal{E})$ we associate a hypergraph, called the neighbourhood hypergraph of H at v and will be denoted by $vH = (vV, v\mathcal{E})$, as follows. First we delete all the edges not containing the vertex v. If $\{v\}$ is an edge, it is also deleted. Finally we delete the vertex v. In the above example, for v = 1 its neighbourhood hypergraph can be represented by the diagram in Fig 2.:

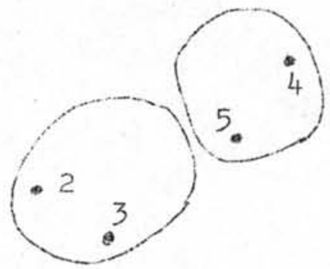

Fig 2.

In another word, for any vertex v we can write

$$v\mathcal{E} = \{E-\{v\}/E \in \mathcal{E},\ v \in E \text{ and } E-\{v\} \neq \emptyset\},$$

$$vV = \cup v\mathcal{E}.$$

Hence for the vertices 2,3,4,5, in the above example, we have

(1) $2H = (\{1,3\}, \{\{1,3\}\})$, see Fig 3.

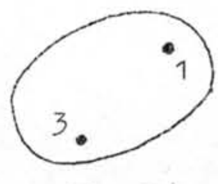

Fig 3.

(2) $3H = (\{1,2,5\},\{\{5\},\{1,2\}\})$, see Fig 4.

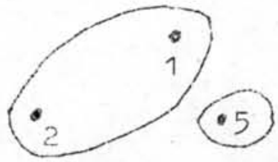

Fig 4.

(3) $4H = (\{1,5\},\{\{1,5\}\})$, see Fig 5.

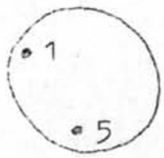

Fig 5.

(4) $5H = (\{1,3,4\},\{\{3\},\{1,4\}\})$, see Fig 6.

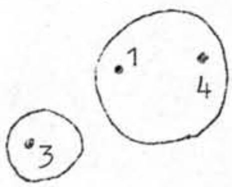

Fig 6.

We see that given a hypergraph $H = (V,\mathcal{E})$, we can associate a family $(vH)_{v \in V}$ of its neighbourhood hypergraphs. In this study we are interested in the opposite situation. Namely, give a family of hypergraphs $(K_v)_{v \in V}$, we want to determine whether there exists a hypergraph H whose neighbourhood hypergraphs are prescribed the hypergraphs of the given family, and to find them if any exists.