Chapter I

INTRODUCTION

1.1 General and literature reviews

In present progress in designing heat transfer equipments, there is a need to search for some new surfaces which will give a high ratio of heat transfer area to core volume. These more compact surfaces will, of course, give higher heat transfer coefficients, lower capital cost, smaller size, and lighter weight equipment. The flow passages will also have a very small hydraulic diameter, therefore, the flow, of air for example, can be laminar. Hence, the theory concerning laminar flow solutions for flow characteristics and heat transfer in ducts of various geometries become important. These had been done by many investigators and their works were fully compiled by Shah and London.[1] In addition to circular ducts as well as parallel plates, interests on non-circular ducts have been greatly increased. On account of the fact that most of heat exchange equipments such as air conditioning units, rocket power

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[1]Shah R.K., and London A.L., "Laminar Flow Forced Convection Heat Transfer and Flow Friction in Straight and Curved Ducts - A Summary of Analytical Solutions." Technical Report No. 75, (November 1971).

plants, gas turbine regenerators,[2] radiators etc., the heat transfer ducts usually have cross sections that are non-circulars.

In order to design such an equipment, value of the heat transfer coefficient, h, of each duct to be employed must be known in order that heat transfer rate, $\dot{q}$, can be estimated from Newton's law of cooling,[3] that is

$$\dot{q} = hA(t_w - t_b)$$

where A = surface area of the duct wall in contact with the fluid,

t_w = wall temperature,

t_b = bulk or mean temperature of the fluid.

The heat transfer coefficient at any section of a duct depends upon the shape of the velocity and temperature profiles of the fluid at that section, and fluid properties.

When fluid flows into a closed duct with an initially uniform velocity profile shear forces retard local velocities near the wall. The retarded fluid layer is called the

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[2]Ibid. pp. 235-242.

[3]Bird R.B., Stewart W.E., and Lightfoot E.N., "Transport Phenomena," (New York: John Wiley & Sons, Inc., 1960), p. 391.

boundary layer.[4] The central region is called the potential or inviscid core.[5] While the boundary layer grows towards the centre of the duct, the velocity profile is called a developing profile. The temperature profile also exhibits a similar boundary layer growth, the rate of which depends upon the method of heat transfer across the duct wall. The associated thermal boundary conditions[6] commonly are constant wall temperature, constant heat flux and constant wall temperature gradient.

As already mentioned, in many cases where low flow rates occur in small ducts the flows of gas tend to be laminar. This is true for various duct geometries. And in many practical cases, heat transfer begins as soon as fluid enters the duct. It follows then that the theoretically[7] derived laminar boundary layer solutions for the two developing profiles may be applicable. However, the

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[4]The boundary layer is a thin region, very close to the duct wall, in which the velocity gradients are large enough so that the influence of viscosity cannot be neglected.

[5]The potential core is defined as the region in which the influence of the pressure of the duct wall has died out enough so that the velocity gradients are so small that the fluid viscosity can be ignored.

[6]Shah and London, op. cit. pp. 7-22

[7]Kays W.M., "Convective Heat and Mass Transfer," (New York: McGraw-Hill, Inc., 1968), p. 25, p. 34

Navier-Stokes equations[8] and the equation of energy[9] for a laminar flow in a duct may also be employed.

Laminar forced convection heat transfer solutions in triangular ducts of various cross sections, by theoretical analysis with some experiments,[10] have been obtained for

(1) Fully developed velocity and temperature profiles.

(2) Fully developed velocity profiles and developing temperature profiles.

(3) Simultaneously developing velocity and temperature profiles.

For the former one case, Sparrow and Haji-Sheikh[11] solved the momentum and energy equations numerically by the finite difference method and obtained fully developed Nusselt number for constant heat flux, Nu_{H1}, for the isosceles triangular and right triangular ducts with opening angle, 2ϕ, varying from 0° to 180°. Results were reported graphically. They show that the equilateral

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[8]Schlichting Hermann, "Boundary-Layer Theory," (New York: McGraw-Hill, Inc., 1968), p. 61.

[9]Bird et al., op. cit. p. 315.

[10]Shah and London, op. cit.

[11]Sparrow E.M., and Haji-Sheikh A., "Laminar Heat Transfer and Pressure Drop in Isosceles Triangular, Right Triangular, and Circular Sector Ducts," Tran. ASME, J. Heat Transfer, 87, (1965), 426-27.

triangular duct gives the highest Nu_{H1}. The respective results for $\phi = 0$ are the same as those for $\phi = 90^{\circ}$. In addition the results for the right triangular duct are symmetric about $\phi = 45^{\circ}$. Kutateladze and Borishanskii[12] obtained fully developed Nusselt number for constant wall temperature, Nu_T, for isosceles triangular duct with opening angle, $2\phi = 20^{\circ}, 40^{\circ}, 60^{\circ}, 80^{\circ}, 90^{\circ}$ and 100°. Results show that the greater the opening angle, the higher the Nusselt number.

For the latter two cases, Wibulswas[13] obtained laminar flow Nu_T and Nu_{H1} for an equilateral triangular and right-angled isosceles triangular ducts for fully developed velocity profile and simultaneously developing flow for the fluid with Prandtl number, Pr = 0.72. He neglected the transverse velocities u and v, as well as the axial viscous and thermal diffusion, $\mu(\partial^2 w/\partial z^2)$ and $k(\partial^2 t/\partial z^2)$ respectively. Results were obtained by a modification of the numerical method. Some experimental results were also obtained, to compare with the available theory, for an equilateral triangular duct for fully developed velocity profile and simultaneously

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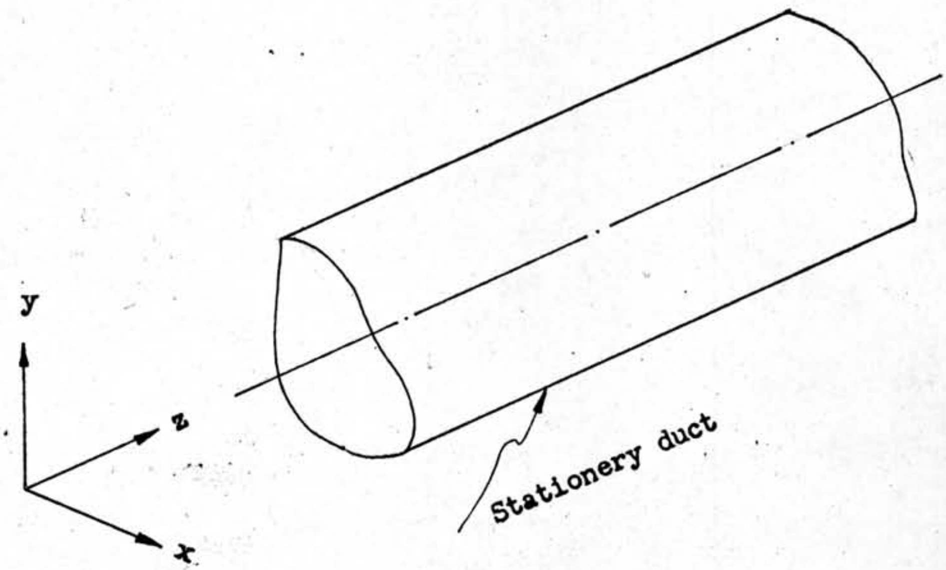
[12] Kutateladze S.S., and Borishanskii U.M., "A Concise Encyclopedia of Heat Transfer," (Pregaman Press Ltd., 1966), p. 107.

[13] Wibulswas P., "Laminar Flow Heat Transfer in Non-Circular Ducts." Ph.D. Thesis, London University (1966), pp. 86-107.



Cartesian Co-ordinates Adapted in Theoretical Analyses.

developing profiles with constant heat input per unit length of the duct. Results show a closed agreement with the theoretical values.

In this paper an experiment was investigated, particularly for the third case, that is, simultaneously developing velocity and temperature profiles, for an equilateral triangular and right-angled isosceles triangular ducts with the thermal boundary condition of constant wall temperature, to provide some data and compare with the available theory.[14]

1.2 Theory

Consider a steady state, simultaneously developing laminar flow in a stationery duct. Also assume that the fluid is incompressible and the fluid properties ρ, C_p, k are constant, independent of fluid temperature, and the body forces, viz., gravity, centrifugal, electromagnetic etc. do not exist. The applicable differential momentum or Navier-Stokes equation in the z-direction can be written as

$$\frac{\mu}{\rho}\left\{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right\} = \frac{1}{\rho}\frac{\partial p}{\partial z} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z},$$

where u, v, and w are the velocities in the x-, y-, and z-direction respectively, μ is the viscosity, ρ is the density and p is the pressure.

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[14] Ibid.

Montgomery and Wibulswas[15] obtained an approximate solution for the development of the velocity profile in rectangular ducts by using the following assumptions:

(1) The term $\partial^2 w/\partial z^2$ is neglected in comparison with the terms $\partial^2 w/\partial x^2$ and $\partial^2 w/\partial y^2$.

(2) The velocities u and v are assumed to be negligible in comparison with the mainstream velocity w.

(3) The pressure gradient, dp/dz, is a function of z alone.

The equation then simplifies to

$$\frac{\mu}{\rho}\left\{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right\} = \frac{1}{\rho}\frac{dp}{dz} + w\frac{\partial w}{\partial z} \qquad \ldots\ldots (1)$$

The above assumptions will also be applied to solve a flow in triangular ducts.

By neglecting the temperature effect due to viscous friction, the applicable energy equation for an incompressible fluid with invariable physical properties can be written as

$$\alpha\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}\right) = u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} + w\frac{\partial t}{\partial z},$$

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[15]S.R. Montgomery, and P. Wibulswas, "Laminar Flow Heat Transfer for Simultaneously Developing Velocity and Temperature Profiles in Ducts of Rectangular Cross Section," Appl. Sci. Res. 18. (1967), 247-59.

where t = local fluid temperature,

α = thermal fluid diffusivity, $k/Cp\,\rho$.

The conduction term in the axial direction, $\partial^2 t/\partial z^2$, is negligible in comparison with the terms $\partial^2 t/\partial x^2$ and $\partial^2 t/\partial y^2$, and the velocities u and v are zero, the energy equation is therefore reduced to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = \frac{w}{\alpha}\frac{\partial t}{\partial z} \qquad \ldots\ldots (2)$$

Solution of equations (1) and (2) can be obtained by numerical method. The left hand sides of both equations contain second derivatives in two dimensions and these can be replaced by finite different approximations. The numerical solutions for the equilateral and right-angled isosceles triangular ducts are carried out on a computer. Details of theoretical analysis and the computing procedure can be found in the Ph.D. thesis by Wibulswas.[16] The numerical solutions obtained can be put into the following equations:-

For equilateral triangular duct, $Nu_m = 1.59\ Gz^{0.33}$ .... (3a)

For right-angled isosceles triangular duct, $Nu_m = 1.47\ Gz^{0.31}$..(3b)

1.3 Experimental correlation

Consider a length L of a duct bounded by a closed perimeter P and through which fluid could be circulated at various measurable flow rates. The heat absorbed by the fluid

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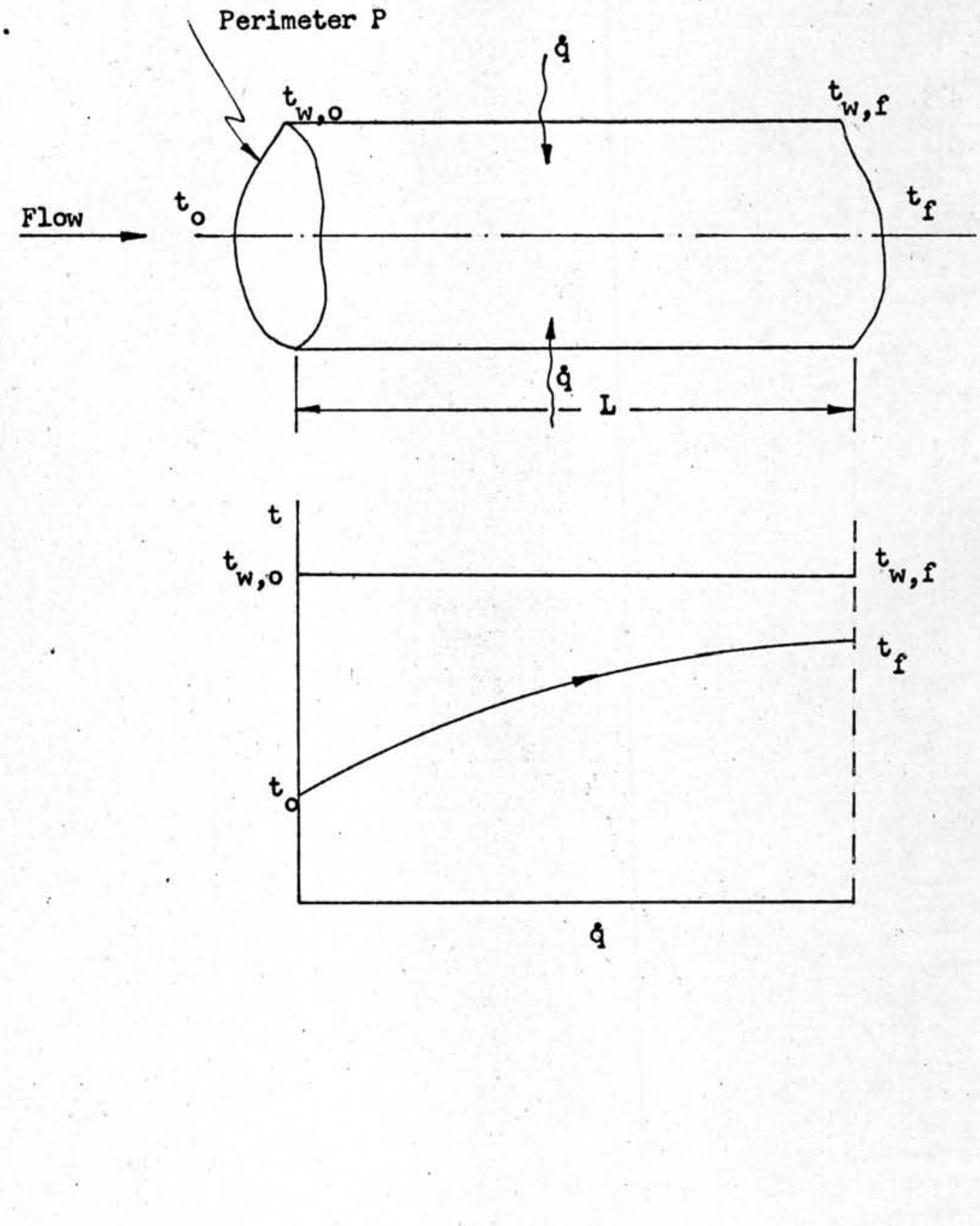
[16] Wibulswas, op. cit.



Schematic Diagram of Fluid and Wall

Temperatures - Constant Wall Temperature Case.

in flow through the duct would be identical with the heat passing into the duct in directions at right angles to its longitudinal axis, thus

$$\dot{q} = \rho w_b A_c C_p (t_f - t_o) = h_l P L \Delta t_l \quad \ldots\ldots (4)$$

where $\dot{q}$ = heat transfer rate,

w_b = average velocity in the direction of duct axis,

A_c = cross sectional area of the duct,

h_l = logarithmic mean heat transfer coefficient,

t_f = final fluid temperature,

t_o = initial fluid temperature,

Δt_l = logarithmic mean temperature difference,

$$= \frac{(t_{w,o} - t_o) - (t_{w,f} - t_f)}{\ln \frac{(t_{w,o} - t_o)}{(t_{w,f} - t_f)}}$$

and where $t_{w,o}$ = initial wall temperature,

$t_{w,f}$ = final wall temperature.

Then eq.(4) may be arranged to obtain the following expressions:-

$$h_l = \frac{\rho w_b A_c C_p (t_f - t_o)}{P L \Delta t_l} \quad \ldots\ldots (5)$$

By definition, the logarithmic mean Nusselt number, Nu_l, is defined as

$$Nu_l = \frac{h_l d_h}{k} \quad \ldots\ldots (6)$$

where d_h = hydraulic diameter of the duct,

$$= 4A_c/P$$

Substitute h_1 in eq.(5) to eq.(6), thus obtaining

$$Nu_1 = \frac{\rho w_b A_c C_p (t_f - t_o)}{P L \Delta t_1} \frac{d_h}{k} \qquad .... \qquad (7)$$

Eq.(7) may be deduced and rearranged to give the following expressions:-

$$Nu_1 = \frac{1}{4} \cdot \frac{w_b d_h^2}{\alpha L} \cdot \frac{(t_f - t_o)}{\Delta t_1}, \qquad .... \qquad (8)$$

where α = thermal diffusivity of fluid,

$$= k/\rho C_p$$

Eq.(8) was used in this work to determine experimental results. The results obtained were then compared with predicted solutions.