CHAPTER III

NUMERICAL EXAMPLES AND RESULTS

Several problems were solved to assess the accuracy of the proposed method of analysis and to demonstrate its versatility. In Example 1 a thin-walled rectangular box-section under free warping was analysed and checked against the solution obtained by the theory of thin-walled elastic structures (10). Results of analyses of several 30-story framed-tubes by the present method and by a three dimensional frame analysis program (ETABS, Ref. 11) are included in Examples 2 - 4.

3.1 Example 1

A uniform thin-walled cantiliver of the closed rectangular box-section with no axial restraint at the base (Fig. 5) was subjected to a point torque T at the free end. The shear modulus and thickness of the box-section were G and t, respectively.

In this case, we found that the solution of the present analysis is the same as that given by the theory of thin-walled elastic structures (10) when the correct displacement functions were assumed. The detail of the analysis is given in the Appendix.

3.2 Example 2

A 30 - story framed-tube, whose typical floor plan is shown in Fig. 6, was analysed for a uniformly distributed torque of 162 kip-ft per unit height. The story height was 12 feet and the spacing of columns in all frame panels was 12 feet. Table 1 lists the sectional properties of the members. The modulus of elasticity and shear rigidity were 5×10^5 kips / ft^2 and 2.07×10^5 kips / ft^2, respectively. The stiffness factor[1] and shear lag parameter[2] were 0.12 and 0.067, respectively.

The results of analysis by the proposed method and by using the computer program ETABS (11) are shown in Figs. 7 - 10, and will be discussed in conjunction with Example 3.

3.3 Example 3

A 30 - story framed-tube whose properties were similar to that of Example 2 except for the spacing of columns in the short panel was studied for the same loading. Fig. 11 depicts the typical floor plan. The stiffness factor for the shorter and longer panels were 0.21 and 0.12, respectively, and the corresponding shear lag parameters were 0.074 and 0.067. The results obtained

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[1] Stiffness factor, SF., as defined by Khan and Amin (5), is SF. = $12I_b h / A_c d^3$.

[2] Shear lag parameter, SL., has been defined by Moselhi, el at. (9), as G_{zs} / E_z.

are shown in Figs. 12 - 15. The results by ETABS are again included for comparison.

Figs. 7 and 12 show the variation of the angle of twist along the height for Examples 2 and 3, respectively. It is seen that the solutions by the proposed method and by the computer program ETABS agree reasonably well. The proposed method overestimates the angle of twist at the top by about 10 % for Example 2 and about 7 % for Example 3.

Figs. 8 and 13 show the variation of axial forces along the height in the corner columns. The proposed method underestimates the maximum axial force at the base by about 25 % for Example2 and about 26 % for Example 3.

The distribution of the column axial forces is shown in Figs. 9 and 14. It is seen that results for the axial forces in the columns by the proposed method are worse for columns in the interior of the structure as well as in the upper stories, owing to the inherent assumptions of a linear variation of the axial displacement along the cross section of the equivalent tube. Figs. 10 and 15 show the shear force distribution in the columns and the spandrel beams. It is seen that about 80 % or more of the predicted shear forces in the interior columns and the spandrel beams can be expected to have an error of less than 25 %. Bending moments in the corner columns of framed panel 1 are shown in Tables 2 and 3. Reasonable agreement is also obtained in general except for the first story where the assumption that the inflection points occur at the mid heights of the columns is not valid.

3.4 Example 4

A 30 - story framed - tube with a step-wise variation of the member sections at every 10-story height of the structure is considered in this example. The center line dimensions, material properties and the loading were the same as in Example 2. Table 1 lists the sectional properties of the members. The stiffness factor and the shear lag parameter are tabulated in Table 4.

In this example we employed the assumed displacement fields in Eqs. (31), (32) and (33). The results of analyses are shown in Figs. 17 - 20.

Fig. 17 discloses that the twisting angle predicted by the present method and ETABS are in good agreement with a maximum discrepancy of less than 10 % . The variation of axial force in the corner column by both methods is shown in Fig. 18. Reasonable agreement in the solutions is observed in general, although there are considerable discrepancies in the top stories. The proposed method underestimates the maximum axial force at the base by about 23 % .

Figs. 19 and 20 show the distribution of the column axial forces and shear forces in the members, respectively, and Table 5 containts the bending moments in the corner columns. Similar observations can be made as in the previous examples.