

CHAPTER III

NUMERICAL SOLUTION OF ELECTRIC FIELD

We shall calculate the electric field numerically in a special case from the Berkeley Physics Course formula, equations (2.41) and (2.42) and compare the result with that from the Feynman formula, equations (2.45) and (2.46), letting

$$q = 1, \quad a = 1, \quad x = -1(0.1)1, \quad v = c \sin 60^\circ$$

$$\text{i.e. } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 2$$

The Berkeley Physics Course formula

$$E_x = \frac{-\gamma qx}{\left[(\gamma x)^2 + y^2 \right]^{3/2}} = \frac{-2x}{\left[4x^2 + 1 \right]^{3/2}} \quad \dots (3.1)$$

$$E_y = \frac{-\gamma qy}{\left[(\gamma x)^2 + y^2 \right]^{3/2}} = \frac{-2}{\left[4x^2 + 1 \right]^{3/2}} \quad \dots (3.2)$$

The accuracy of E_x and E_y

The values tabulated in table 3.1 are accurate to the number of decimal places stated because the calculations were done on an electronics desk calculator which has eight significant figures.

x	-2x	$(4x^2 + 1)^{3/2}$	E_x	E_y
-1.0	2.0	11.1803	0.1789	-0.1789
-0.9	1.8	8.7307	0.2062	-0.2291
-0.8	1.6	6.7170	0.2382	-0.2978
-0.7	1.4	5.0926	0.2749	-0.3927
-0.6	1.2	3.8114	0.3148	-0.5247
-0.5	1.0	2.8284	0.3536	-0.7071
-0.4	0.8	2.1002	0.3809	-0.9523
-0.3	0.6	1.5860	0.3783	-1.2610
-0.2	0.4	1.2494	0.3202	-1.6008
-0.1	0.2	1.0606	0.1886	-1.8857
0	0	1.0000	0	-2.0000
0.1	-0.2	1.0606	-0.1886	-1.8857
0.2	-0.4	1.2494	-0.3202	-1.6008
0.3	-0.6	1.5860	-0.3783	-1.2610
0.4	-0.8	2.1002	-0.3809	-0.9523
0.5	-1.0	2.8284	-0.3536	-0.7071
0.6	-1.2	3.8114	-0.3148	-0.5247
0.7	-1.4	5.0926	-0.2749	-0.3927
0.8	-1.6	6.7170	-0.2382	-0.2978
0.9	-1.8	8.7307	-0.2062	-0.2291
1.0	-2.0	11.1803	-0.1789	-0.1789

Table 3.1 Electric field from equation (3.1), (3.2)

Feynman's formula *

From equations (2.45) and (2.46)

$$E_x = -q \left[\frac{x_R}{r_R^3} + \frac{r_R}{c} \frac{d}{dt} \left(\frac{x_R}{r_R^3} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{x_R}{r_R} \right) \right] \quad \dots(2.45)$$

$$E_y = -q \left[\frac{y_R}{r_R^3} + \frac{r_R}{c} \frac{d}{dt} \left(\frac{y_R}{r_R^3} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{y_R}{r_R} \right) \right] \quad \dots (2.46)$$

where x_R , y_R , r_R are given in (2.43) and (2.44) as

$$x_R = \left(1 - \frac{v^2}{c^2}\right)^{-1} \left\{ vt - \frac{v}{c} \left(v^2 t^2 + a^2 \left(1 - \frac{v^2}{c^2}\right) \right)^{1/2} \right\}$$

$$y_R = a$$

$$r_R = \left(1 - \frac{v^2}{c^2}\right)^{-1} \left[v^2 t^2 + \frac{v^2}{c^2} \left\{ v^2 t^2 + a^2 \left(1 - \frac{v^2}{c^2}\right) \right\} - 2 \frac{v^2 t}{c} \left\{ v^2 t^2 + a^2 \left(1 - \frac{v^2}{c^2}\right) \right\}^{1/2} + a^2 \left(1 - \frac{v^2}{c^2}\right)^2 \right]^{1/2}$$

If $a = 1$, $v = c \sin 60^\circ$

$$\text{i.e. } \frac{v^2}{c^2} = \frac{3}{4} = 0.75 \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4} = 0.25$$

and we have $x = vt$.

Therefore,

$$x_R = 4 \left\{ x - \sin 60^\circ \left(x^2 + \frac{1}{4} \right)^{1/2} \right\} = 4 \left\{ x - 0.866025 \left(x^2 + 0.25 \right)^{1/2} \right\} \quad \dots(3.3)$$

$$r_R = 4 \left\{ x^2 + 0.75(x^2 + 0.25) - 1.732050x \left(x^2 + 0.25 \right)^{1/2} + 0.0625 \right\}^{1/2} \quad \dots(3.4)$$

From Numerical Mathematical Analysis by J.B. Scarborough, page 134, we have the numerical differentiation formula

$$\left(\frac{dy}{dx}\right)_{x_0} = \frac{1}{\Delta x} \left[\frac{\Delta y_{-1} + \Delta y_0}{2} - \frac{1}{3!} \frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2} + \frac{4}{5!} \frac{\Delta^5 y_{-3} + \Delta^5 y_{-2}}{2} + \dots \right] \quad (3.5)$$

$$\left(\frac{d^2y}{dx^2}\right)_{x_0} = \frac{1}{(\Delta x)^2} \left[\Delta^2 y_{-1} - \frac{1}{12} \Delta^4 y_{-2} + \frac{8}{6!} \Delta^6 y_{-3} + \dots \right] \quad (3.6)$$

E_x and E_y are calculated in the following tables step by step.

x	ct	x_R	r_R
-1.1	-1.270171	-8.585688	8.643729
-1.0	-1.154701	-7.872982	7.936237
-0.9	-1.039231	-7.166509	7.235943
-0.8	-0.923761	-6.468025	6.544873
-0.7	-0.808291	-5.779931	5.865801
-0.6	-0.692821	-5.105549	5.202560
-0.5	-0.577351	-4.449489	4.560477
-0.4	-0.461880	-3.818106	3.946890
-0.3	-0.346410	-3.219900	3.371611
-0.2	-0.230940	-2.665475	2.846886
-0.1	-0.115470	-2.166351	2.386018
0	0	-1.732050	2.000000
0.1	0.115470	-1.366351	1.693198
0.2	0.230940	-1.065475	1.461246
0.3	0.346410	-0.819900	1.293151
0.4	0.461880	-0.618106	1.175610
0.5	0.577351	-0.449489	1.096378
0.6	0.692821	-0.305549	1.045642
0.7	0.808291	-0.179931	1.016063
0.8	0.923761	-0.068025	1.002316
0.9	1.039231	+0.033491	1.000567
1.0	1.154701	+0.127018	1.008042
1.1	1.270171	+0.214312	1.022715

Table 3.2 x_R and r_R calculated from (3.3) and (3.4)

From this table $\Delta(ct) = 0.115470$

x_R	r_R	x_R/r_R	x_R/r_R^3	$1/r_R$	$1/r_R^3$
-8.585688	8.643729	-0.993285	-0.013294	0.115691	0.001548
-7.872982	7.936237	-0.992030	-0.015751	0.126004	0.002001
-7.166509	7.235943	-0.990404	-0.018916	0.138199	0.002639
-6.468025	6.544873	-0.988258	-0.023071	0.152791	0.003567
-5.779931	5.865801	-0.985361	-0.028638	0.170480	0.004955
-5.105549	5.202560	0.981353	-0.036257	0.192213	0.007101
-4.449489	4.560477	-0.975663	-0.046912	0.219275	0.010543
-3.818106	3.946890	-0.967371	-0.062099	0.253364	0.016264
-3.219900	3.371611	-0.955003	-0.084010	0.296594	0.026091
-2.665475	2.846886	-0.936277	-0.115522	0.351261	0.043340
-2.166351	2.386018	-0.907936	-0.159481	0.419108	0.073617
-1.732050	2.000000	-0.866025	-0.216506	0.500000	0.125000
-1.366351	1.693198	-0.806965	-0.281474	0.590598	0.206005
-1.065475	1.461246	-0.729155	-0.341486	0.684347	0.320501
-0.819900	1.293151	-0.634033	-0.379152	0.773305	0.462437
-0.618106	1.175610	-0.525775	-0.380429	0.850622	0.615475
-0.449489	1.096378	-0.409976	-0.341066	0.912094	0.758786
-0.305549	1.045642	-0.292212	-0.267259	0.956350	0.874684
-0.179931	1.016063	-0.177086	-0.171532	0.984191	0.953319
-0.068025	1.002316	-0.067868	-0.067555	0.997689	0.993084
+0.033491	1.000567	+0.033472	+0.033434	0.999433	0.998301
+0.127018	1.008042	+0.126005	+0.124002	0.992022	0.976257
+0.214312	1.022715	+0.209552	+0.200347	0.977790	0.934838

Table 3.3 $\frac{x_R}{r_R}$, $\frac{x_R}{r_R^3}$, $\frac{1}{r_R}$ and $\frac{1}{r_R^3}$ where x_r and r_R are from (3.3) and (3.4)

$f = \frac{x_R}{r^3}$	Δf	$\Delta^2 f$	$\Delta^3 f$	$\frac{1^{st} \text{ term of (3.7)}}{c}$	$\frac{2^{nd} \text{ term of (3.7)}}{c}$	$\frac{1}{c} \frac{d}{dt} \left(\frac{x_R}{r^3} \right)$
-0.013294	- 2457					
-0.015751	- 3165	- 708	- 282	-0.024344		
-0.018916	- 4155	- 990	- 422	-0.031697	-0.000508	-0.031189
-0.023071	- 5567	- 1412	- 640	-0.042098	-0.000766	-0.041332
-0.028638	- 7619	- 2052	- 984	-0.057097	-0.001172	-0.055925
-0.036257	-10655	- 3036	- 1496	-0.079129	-0.001790	-0.077339
-0.046912	-15187	- 4532	- 2192	-0.111899	-0.002662	-0.109237
-0.062099	-21911	- 6724	- 2877	-0.160639	-0.003658	-0.156981
-0.084010	-31512	- 9601	- 2846	-0.231328	-0.004130	-0.227198
-0.115522	-43959	-12447	- 619	-0.326799	-0.002501	-0.324298
-0.159481	-57025	-13066	+ 5123	-0.437274	+0.003250	-0.440524
-0.216506	-64968	- 7943	+12899	-0.528245	+0.013006	-0.541251
-0.281474	-60012	+ 4956	+17390	-0.541180	+0.021859	-0.563039
-0.341486	-37666	+22346	+14043	-0.422958	+0.022685	-0.445643
-0.379152	- 1277	+36389	+ 4251	-0.168628	+0.013203	-0.181831
-0.380429	+39363	+40640	- 6196	+0.164913	-0.001404	+0.166317
-0.341066	+73807	+34444	-12524	+0.490040	-0.013510	+0.503550
-0.267259	+95727	+21920	-13670	+0.734104	-0.018904	+0.753008
-0.171532	+103977	8250	-11238	+0.864744	-0.017976	+0.882720
-0.067555	+100989	- 2988	- 7433	+0.887529	-0.013475	+0.901004
+0.033434	+ 60568	-10421	- 3802	+0.829467	-0.008108	+0.837575
+0.124002	+ 76345	-14223		+0.722755		
+0.200347						

Table 3.4 $\frac{1}{c} \frac{d}{dt} \left(\frac{x_R}{r^3} \right)$ calculated by using the formula (3.5)

Thus : $\left(\frac{df}{dt} \right)_{t_0} = \frac{c}{0.230940} (\Delta f_{-1} + \Delta f_0) - \frac{c}{1.385640} (\Delta^3 f_{-2} + \Delta^3 f_{-1}) + \dots$ (3.7)

$x_R / r_R = \xi$	$\Delta \xi$	$\Delta^2 \xi$	$\Delta^3 \xi$	$\Delta^4 \xi$	$\frac{1^{\text{st}} \text{ term of (3.8)}}{c^2}$	$\frac{2^{\text{nd}} \text{ term of (3.8)}}{c^2}$	$\frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{x_R}{r_R} \right)$
-0.993285	1255						
-0.992030	1626	371					
-0.990404	2146	520	149				
-0.988258	2897	751	231	82	-0.039001	0.000512	0.038489
-0.985361	4008	1111	360	129	0.056326	0.000806	0.055520
-0.981353	5690	1682	571	211	0.083327	0.001319	0.082008
-0.975663	8292	2602	920	349	0.126153	0.002181	0.123972
-0.967371	12368	4076	1474	554	0.195155	0.003462	0.191693
-0.955003	18726	6358	2282	808	0.305708	0.005050	0.300658
-0.936277	28341	9615	3257	975	0.476862	0.006094	0.470768
-0.907936	41911	13570	3955	698	0.721143	0.004362	0.716781
-0.866025	59060	17149	3579	-376	1.017775	-0.002350	1.020125
-0.806965	77810	18750	1601	-1978	1.286207	-0.012362	1.298569
-0.729155	95122	17312	-1438	-3039	1.406285	-0.018994	1.425279
-0.634033	108258	13136	-4176	-2738	1.298432	-0.017112	1.315544
-0.525775	115799	7541	-5595	-1419	0.985225	-0.008869	0.994094
-0.409976	117764	1965	-5576	+19	0.565589	0.000119	0.565470
-0.292212	115126	-2638	-4603	+973	0.147379	0.006081	0.141298
-0.177086	109218	-5908	-3270	+1333	-0.197855	0.008331	-0.206186
-0.067868	101340	-7878	-1970	+1300	-0.443111	0.008125	-0.451236
+0.033472	92533	-8807	-929	+1041	-0.590865	0.006506	-0.597371
+0.126005	83547	-8986	-179	+750	-0.660542	0.004688	-0.665230
+0.209552					-0.673967		

Table 3.5 $\frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{x_R}{r_R} \right)$ calculated from the formula (3.6)

Thus : $\left(\frac{d^2 \xi}{dt^2} \right) = \frac{c^2 \Delta^2 \xi}{\Delta \xi^2} - \frac{c^2 \Delta^4 \xi}{\Delta \xi^4} \dots$ (3.8)

$\frac{1}{r_R} = g$	Δg	$\Delta^2 g$	$\Delta^3 g$	$\frac{1^{st} \text{ term of (3.9)}}{c}$	$\frac{2^{nd} \text{ term of (3.9)}}{c}$	$\frac{1}{c} \frac{d}{dt} \left(\frac{1}{r_R} \right)$
0.001548	453					
0.002001	638	185	105	0.004724		
0.002639	928	290	170	0.006781	0.000199	0.006582
0.003567	1388	460	298	0.010029	0.000338	0.009691
0.004955	2146	758	538	0.015303	0.000604	0.014699
0.007101	3442	1296	983	0.024197	0.001098	0.023099
0.010543	5721	2279	1827	0.039677	0.002028	0.037649
0.016264	9827	4106	3316	0.067325	0.003712	0.063613
0.026091	17249	7422	5606	0.117243	0.006439	0.110804
0.043340	30277	13028	8078	0.205794	0.009876	0.195918
0.073617	51383	21106	8516	0.353599	0.011976	0.341623
0.125000	81005	29622	3869	0.573258	0.008939	0.564319
0.206005	114496	33491	-6051	0.846545	-0.001575	0.848120
0.320501	141936	27440	-16338	1.110384	-0.016158	1.126542
0.462437	153038	11102	-20829	1.277276	-0.026823	1.304099
0.615475	143311	-9727	-17686	1.283230	-0.027796	1.311026
0.758786	115898	-27413	-9850	1.122409	-0.019873	1.142282
0.874684	78635	-37263	-1607	0.842353	-0.008269	0.850622
0.953319	39765	-38870	+4322	0.512688	+0.001960	0.510728
0.993084	5217	-34548	+7287	0.194778	0.008378	0.186400
0.998301	-22044	-27261	+7886	-0.072863	0.010951	-0.083814
0.976257	-41419	-19375		-0.274803		
0.934838						

Table 3.6 $\frac{1}{c} \frac{d}{dt} \left(\frac{1}{r_R} \right)$ calculated from the formula (3.5),

$$\text{Thus : } \frac{1}{c} \frac{d}{dt} \left(\frac{1}{r_R} \right)_{to} = \frac{c}{0.230940} (\Delta g_1 + \Delta g_0) - \frac{c}{1.385640} (\Delta^2 g_{-2} + \Delta^2 g_{-1}) + \dots (3.9)$$

$\frac{1}{r_R} = \xi$	$\Delta \xi$	$\Delta^2 \xi$	$\Delta^3 \xi$	$\Delta^4 \xi$	$\frac{1^{\text{st}} \text{ term of (3.10)}}{c^2}$	$\frac{2^{\text{nd}} \text{ term of (3.10)}}{c^2}$	$\frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{1}{r_R} \right)$
0.115691							
-.126004	10313	1882					
0.138199	12195	2397	515	185	0.179780	0.001157	0.178623
0.152791	14592	3097	700	247	0.232281	0.001544	0.230737
0.170480	17689	4044	947	338	0.303308	0.002113	0.301195
0.192213	21733	5329	1285	413	0.399685	0.002582	0.397103
0.219275	27062	7027	1698	416	0.527039	0.002600	0.524439
0.253364	34089	9141	2114	182	0.685593	0.001138	0.684455
0.296594	43230	11437	2296	-553	0.857797	-0.003457	0.861254
0.351261	54667	13180	1743	-1878	0.988525	-0.011738	1.000263
0.419108	67847	13045	-135	-3204	0.978400	-0.020025	0.998425
0.500000	80892	9706	-3339	-3216	0.727969	-0.020100	0.748069
0.590598	90598	3151	-6555	-1387	0.236331	-0.008669	0.245000
0.684347	93749	-4791	-7942	+1092	-0.359334	+0.006825	-0.366159
0.773305	88958	-11641	-6850	+2646	-0.873097	+0.016538	-0.889635
0.850622	77317	-15845	-4204	+2833	-1.188405	+0.017707	-1.206112
0.912094	61472	-17216	-1371	+2172	-1.291233	+0.013575	-1.304808
0.956350	44256	-16415	+801	+1271	-1.231156	+0.007944	-1.239100
0.984191	27841	-14343	+2072	+517	-1.075752	+0.003232	-1.078984
0.997689	13498	-11754	+2589	+10	-0.881572	+0.000063	-0.881635
0.999433	1744	-9155	+2599	-265	-0.686643	-0.001657	-0.684986
0.992022	-7411	-6821	+2334		-0.511588		
0.977790	-14232						

Table 3.7 $\frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{1}{r_R} \right)$ which calculated from the formula (3.6),

$$\text{Thus: } \left(\frac{d^2 \xi}{dt^2} \right) = \frac{c^2 \Delta^2 \xi_{i-1}}{c^2} - \frac{c^2 \Delta^2 \xi_{i-2}}{c^2} + \dots \quad (3.10)$$

x	$\frac{x_R}{r_R^3}$	$\frac{1}{c} \frac{d}{dt} \left(\frac{x_R}{r_R^3} \right)$	$\frac{r_R}{c} \frac{d}{dt} \left(\frac{x_R}{r_R^3} \right)$	$\frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{x_R}{r_R^3} \right)$	E_x
-1.1	-0.013294				
-1.0	-0.015751	-0.024344	-0.193200		
-0.9	-0.018916	-0.031189	-0.225682	0.038489	+0.206109
-0.8	-0.023071	-0.041332	-0.270513	0.055520	0.238064
-0.7	-0.028638	-0.055925	-0.328045	0.082008	0.274675
-0.6	-0.036257	-0.077339	-0.402361	0.123972	0.314646
-0.5	-0.046912	-0.109237	-0.498173	0.191693	0.353392
-0.4	-0.062099	-0.156981	-0.619587	0.300658	0.381028
-0.3	-0.084010	-0.227198	-0.766024	0.470768	0.379266
-0.2	-0.115522	-0.324298	-0.923240	0.716781	0.321981
-0.1	-0.159481	-0.440524	-1.051099	1.020125	0.190455
0	-0.216506	-0.541251	-1.082502	1.298569	0.000439
0.1	-0.281474	-0.563039	-0.953337	1.425279	-0.190468
0.2	-0.341486	-0.445643	-0.651194	1.315544	-0.322864
0.3	-0.379152	-0.181831	-0.235135	0.994094	-0.379807
0.4	-0.380429	+0.166317	+0.195524	0.565470	-0.380565
0.5	-0.341066	+0.503550	+0.552081	0.141298	-0.352313
0.6	-0.267259	+0.753008	+0.787377	-0.206186	-0.313932
0.7	-0.171532	+0.882720	+0.896900	-0.451236	-0.274132
0.8	-0.067555	+0.901004	+0.903091	-0.597371	-0.238165
0.9	+0.033434	+0.837575	+0.838050	-0.665230	-0.206254
1.0	+0.124002	+0.722755	+0.728568		
1.1	+0.200347				

Table 3.8 E_x calculated from the formula (2.45)

x	$1/r_R^3$	$\frac{r_R}{c} \frac{d}{dt} \left(\frac{1}{r_R^3} \right)$	$\frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{1}{r_R} \right)$	E_y
-1.1	0.001548			
-1.0	0.002001	0.037491		
-0.9	0.002639	0.047627	0.178623	-0.228889
-0.8	0.003567	0.063427	0.230737	-0.297731
-0.7	0.004955	0.086222	0.301195	-0.392372
-0.6	0.007101	0.120174	0.397103	-0.524378
-0.5	0.010543	0.171698	0.524439	-0.706680
-0.4	0.016264	0.251074	0.684455	-0.951793
-0.3	0.026091	0.373588	0.861254	-1.260933
-0.2	0.043340	0.557757	1.000263	-1.601360
-0.1	0.073617	0.815119	0.998425	-1.887161
0	0.125000	1.128638	0.748069	-2.001707
0.1	0.206005	1.436035	0.245000	-1.887040
0.2	0.320501	1.646155	-0.366159	-1.600497
0.3	0.462437	1.686397	-0.889635	-1.259199
0.4	0.615475	1.541256	-1.206112	-0.950619
0.5	0.758786	1.252373	-1.304808	-0.706351
0.6	0.874684	0.889446	-1.239100	-0.525030
0.7	0.953319	0.518932	-1.078984	-0.393267
0.8	0.993084	0.186832	-0.881635	-0.298281
0.9	0.998301	-0.083862	-0.684986	-0.229453
1.0	0.976257	-0.277013		
1.1	0.934838			

Table 3.9 E_y calculated from the formula (2.46)

x	ct	x_R	From table 3.1 (Berkeley's)		From table 3.8;3.9 (Feynman's)		$E = \sqrt{E_x^2 + E_y^2}$
			E_x	E_y	E_x	E_y	
-1.0	-1.15	-7.87	0.1789	-0.1789			0.2530
-0.9	-1.04	-7.17	0.2062	-0.2291	0.21	-0.23	0.3082
-0.8	-0.92	-6.47	0.2382	-0.2978	0.24	-0.30	0.3813
-0.7	-0.81	-5.78	0.2749	-0.3927	0.27	-0.39	0.4794
-0.6	-0.69	-5.11	0.3148	-0.5247	0.31	-0.52	0.6119
-0.5	-0.58	-4.45	0.3536	-0.7071	0.35	-0.71	0.7906
-0.4	-0.46	-3.82	0.3809	-0.9523	0.38	-0.95	1.0257
-0.3	-0.35	-3.22	0.3783	-1.2610	0.38	-1.26	1.3165
-0.2	-0.23	-2.67	0.3202	-1.6008	0.32	-1.60	1.6325
-0.1	-0.12	-2.17	0.1886	-1.8857	0.19	-1.89	1.9778
0	0	-1.73	0	-2.0000	0	-2.00	2.0000
0.1	0.12	-1.37	-0.1886	-1.8857	-0.19	-1.89	1.9778
0.2	0.23	-1.07	-0.3202	-1.6008	-0.32	-1.60	1.6325
0.3	0.35	-0.82	-0.3783	-1.2610	-0.38	-1.26	1.3165
0.4	0.46	-0.62	-0.3809	-0.9523	-0.38	-0.95	1.0257
0.5	0.58	-0.45	-0.3536	-0.7071	-0.35	-0.71	0.7906
0.6	0.69	-0.31	-0.3148	-0.5247	-0.31	-0.53	0.6119
0.7	0.81	-0.18	-0.2749	-0.3927	-0.27	-0.39	0.4794
0.8	0.92	-0.68	-0.2382	-0.2978	-0.24	-0.30	0.3813
0.9	1.04	+0.03	-0.2062	-0.2291	-0.21	-0.23	0.3082
1.0	1.15	+0.13	-0.1789	-0.1789			0.2530

Table 3.10 Summary of the results.

The accuracy of E_x and E_y

The electric field calculated from the equation (2.45) and (2.46)

$$E_x = -q \left[\frac{x_R}{r_R^3} + \frac{r_R}{c} \frac{d}{dt} \left(\frac{x_R}{r_R^3} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{x_R}{r_R} \right) \right] \dots(2.45)$$

$$E_y = -q \left[\frac{y_R}{r_R^3} + \frac{r_R}{c} \frac{d}{dt} \left(\frac{y_R}{r_R^3} \right) + \frac{1}{c^2} \frac{d^2}{dt^2} \left(\frac{y_R}{r_R} \right) \right] \dots(2.46)$$

is accurate to one decimal place for the following reason.

Formulas (3.5) and (3.6) were used to calculate the first and second derivatives in (2.45) and (2.46). For all cases calculations showed that the third term in formulas (3.5) and (3.6) were less than 0.005 and the higher order terms were even smaller. And in the second terms of E_x and E_y , when the first derivative which is accurate to two decimal places is multiplied by r_R (which is less than 10 and greater than 1), the result has an accuracy of only one decimal place.

Since the first terms and the third terms of E_x and E_y have accuracy greater than one decimal place, it follows that E_x and E_y are accurate to one decimal place.