CHAPTER 4

THE EFFECT OF EXTERNAL NOISE

4.1 Introduction

When an external noise $n_i(t)$ is added to the input signal $x(t)$, then the input of the linear system is

$$x_i(t) = x(t) + n_i(t) \tag{4.1}$$

The output response $y_i(t)$ of this system can be written as

$$y_i(t) = \int_0^{\infty} h(u)x_i(t-u)du \tag{4.2}$$

The crosscorrelation between the input $x(t)$ and the output response $y_i(t)$ becomes

$$\phi_{xy_i}(\tau) = \frac{1}{T}\int_0^{T} x(t)y_i(t+\tau)dt \tag{4.3}$$

From eqns. (4.2) and (4.3), we obtain

$$\phi_{xy_i}(\tau) = \frac{1}{T}\int_0^{T}\int_0^{\infty} h(u)x(t)x_i(t+\tau-u)dudt \tag{4.4}$$

From eqns. (4.1) and (4.4), we have

$$\phi_{xy_i}(\tau) = \int_0^{\infty} h(u)\phi_{xx}(\tau-u)du + \int_0^{\infty} h(u)\phi_{xn_i}(\tau-u)du \tag{4.5}$$

It can be seen that the first integral term on the right-hand side of eqn. (4.5) is the same as eqn. (1.3), therefore the second integral term must be evaluated, and for this case the crosscorrelation function $\phi_{xn_i}(\tau)$ is required. However there are

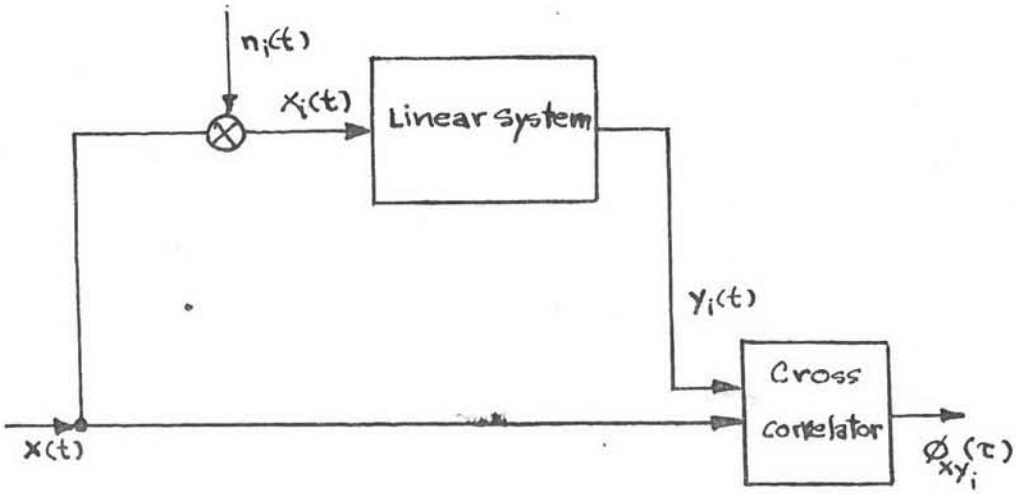

Figure 4.1

The cross correlation between input signal $x(t)$ and output response $y_i(t)$

two interested forms of the external noise to be considered, they are the time polynomial function and the white noise signals.

4.2 Determination of $\phi_{xn_i}(\tau)$ Effected by the Time Polynomial Function Noise Signal

The time polynomial function signal $n_i(t)$ may be written in the form

$$n_i(t) = b_0+b_1t+b_2t^2+\ldots\ldots+b_mt^m \quad (4.6)$$

$$= \sum_{i=0}^{m} b_i t^i \quad (4.7)$$

where b_0, b_1, b_2, ... , b_m are coefficients of time varible t of the order 0,1,2,...,m respectively, and m is the positive integer.

Since the input signal x(t) is periodic with period T, the crosscorrelation function $\phi_{xn_i}(\tau)$ will be determined over the same period. Hence, we have

$$\phi_{xn_i}(\tau) = \frac{1}{T}\int_0^T x(t)n_i(t+\tau)dt \quad (4.8)$$

From eqns. (4.7) and (4.8), we obtain

$$\phi_{xn_i}(\tau) = \frac{1}{T}\int_0^T x(t) \sum_{i=0}^{m} b_i(t+\tau)^i dt \quad (4.9)$$

Rearrange the eqn. (4.9), (see Appendix E), we have

$$\phi_{xn_i}(\tau) = \frac{1}{T}\int_0^T x(t) \sum_{i=0}^{m} t^i \sum_{j=i}^{m} {}^jC_i b_j \tau^{j-i} dt \quad (4.10)$$

where $${}^jC_i = \frac{j!}{(j-i)!i!}$$

$$\phi_{xn_i}(\tau) = \sum_{i=0}^{m} \frac{1}{T}\int_0^T x(t)t^i dt \sum_{j=i}^{m} {}^jC_i b_j \tau^{j-i}$$

$$= \sum_{i=0}^{m} M_i \sum_{j=i}^{m} {}^jC_i b_j \tau^{j-i} \tag{4.11}$$

where M_i is the time moment of the input signal $x(t) = \frac{1}{T}\int_0^T x(t)t^i dt$

Since the eqn. (4.11) is not convenient to determine the crosscorrelation function $\phi_{xn_i}(\tau)$. Hence, it may be rearranged as (see also Appendix E)

$$\phi_{xn_i}(\tau) = A_0 + A_1\tau + A_2\tau^2 + \dots + A_m\tau^m \tag{4.12}$$

where $A_i = \sum_{j=i}^{m} {}^jC_i b_j M_{j-i}$

$$M_{j-i} = \frac{1}{T}\int_0^T x(t)t^{j-i} dt$$

In practice, it is simpler to delay the input signal $x(t)$ rather than the external noise $n_i(t)$. The crosscorrelation between the input signal and the external noise can be expressed as

$$\phi_{xn_i}(\tau) = \frac{1}{T}\int_0^T x(t-\tau)n_i(t)dt \tag{4.13}$$

By replacing $t = t+\tau$, it can be shown that eqn. (4.13) is reduced to eqn. (4.8).

From eqns. (4.7) and (4.13), we obtain

$$\phi_{xn_i}(\tau) = \frac{1}{T}\int_0^T x(t-\tau)\sum_{i=0}^{m} b_i t^i dt$$

$$\phi_{xn_i}(\tau) = \sum_{i=0}^{m} b_i \left\{ \frac{1}{T}\int_0^T x(t-\tau)t^i dt \right\}$$

$$= b_0 M_0(\tau) + b_1 M_1(\tau) + \dots + b_m M_m(\tau) \tag{4.14}$$

where $M_i(\tau) = \frac{1}{T}\int_0^T x(t-\tau)t^i dt$, is the new time moment of $x(t)$.

It is seen that the result of $\phi_{xn_i}(\tau)$ in eqn. (4.14) which is the modified form of $\phi_{xn_i}(\tau)$ in eqn.(4.8), will give the same result as obtained by eqn.(4.12).

4.3 Determination of $\phi_{xn_i}(\tau)$ Effected by the White Noise Signal

Normally, it is difficult to predict the waveform of the white noise, but its properties are well-known. Thus the crosscorrelation function $\phi_{xn_i}(\tau)$ may be determined as follow:
From eqn.(4.8), it can be rewritten as

$$\phi_{xn_i}(\tau) = \frac{1}{T}\int_0^T x(t)\, n_i(t+\tau)\, dt \tag{4.15}$$

The square value of $\phi_{xn_i}(\tau)$ is

$$\phi^2_{xn_i}(\tau) = \left\{\frac{1}{T}\int_0^T x(t)\, n_i(t+\tau)\, dt\right\}^2 \tag{4.16}$$

and see Appendix F, we have

$$\phi^2_{xn_i}(\tau) = \phi_{xx}(\tau)\, \phi_{n_i n_i}(\tau) \tag{4.17}$$

Since the autocorrelation function of the white noise $n_i(t)$ is known to be $K\delta(0)$, the eqn.(4.17) becomes

$$\phi^2_{xn_i}(\tau) = K\delta(0)\, \phi_{xx}(\tau) \tag{4.18}$$

Substituting for the value of $\phi_{xx}(\tau)$ from property h, we have

$$\left.\begin{aligned} \phi^2_{xn_i}(\tau) &= Ka^2 &&, \text{ for } \tau = 0 \\ &= 0 &&, \text{ for } \tau \neq 0 \end{aligned}\right\} \tag{4.19}$$

Thus the value of $\phi_{xn_i}(\tau)$ is

$$\left.\begin{aligned} \phi_{xn_i}(\tau) &= a\sqrt{K} \qquad , \text{ for } \tau = 0 \\ &= 0 \qquad , \text{ for } \tau \neq 0 \end{aligned}\right\} \quad (4.20)$$

or it can be rewritten in the form

$$\phi_{xn_i}(\tau) \leq a\sqrt{K} \qquad , \text{ for all range of } \tau \qquad (4.21)$$

The approximation value of $\phi_{xn_i}(\tau)$ will be assumed constant for all range of τ as its peak value at $\tau = 0$, thus

$$\phi_{xn_i}(\tau) \doteq a\sqrt{K} \qquad , \text{ for all range of } \tau \qquad (4.22)$$

In this case, the error due to the above assumption, for $\tau \neq 0$, is neglected when the signal-to-noise ratio is large. The modified form of $\phi_{xn_i}(\tau)$ for the white noise will have the same result as shown in eqn.(4.22)