CHAPTER 2

PSEUDORANDOM BINARY SIGNAL

2.1 Introduction

There are several types of pseudorandom binary signal, such as maximum-length sequence, quadratic residue, and twin prime signal. It is known that their auto correlation function are similar to those of white noise. In this chapter the maximum-length sequence binary signal, or m-sequence binary signal will be discussed. The m-sequence binary signal is a periodic function signal with a period of discrete 2^n-1 intervals, where n is the positive integer. The reason of using this signal is that it is easier generated than the other types of pseudorandom signal.

2.2 Properties of the M-sequence Binary Signal

The m-sequence binary signal can be generated[7,14] simply by a set of shift registers and the modulo-two adders with proper feedback. This generator is controled by a clock unit with period of length Δt . Its properties[7,8,9] are summarized here for easy referrence.

(a) For an n-stage shift register, the length of period of m-sequence produced from these shift register is equal to $T = 2^n-1$ digits or $(2^n-1)\Delta t$.

(b)The m-sequence must contain all 2^n-1 non zero sub-sequence of n digits once and once only, and has 2^{n-1} ones and $2^{n-1}-1$ zeros. The number of all ones cannot exceed the number of all zeros more than one.

(c)The m-sequence has the"shift and add" property. If an m-sequence is added by modulo-two addition to the same m-sequence but delayed by r digits from the former. The resulting sequence is again the same as original m-sequence but delayed now be q digits, where both r and q are positive integers in the range $1 \leq r, q \leq 2^n-2$.

(d)Succesive occurences of one of the states in the binary sequence are called runs. If these runs are tabulated, it is found that there are 2^{n-1} runs in the m-sequence, of which one half are of length 1 digit, one quarter of length 2 digits, one eigth of length 3 digits, and so on, provided that the number of runs of a given length so indicated is greater than one. There are equal number of runs of either state, except that there is a run of n ones but no true run of n zeros and that there is also a run of n-1 zeros but no true run of n-1 ones.

(e)If m-sequence X is any binary sequence of ones and zeros state and X' is defined by

$$X' = (I \oplus D)X = D^{k_s}X$$

Where I is the identily operator.

D is a delay unit

$\oplus$ is the symbol of module-two adder

k_s is the value of q when r=1

The sequence X will have a one in each position corresponding to the start of a run of ones or zeros in X, and zero other where.

(f)For an m-sequence X, the value of k_s-2 is equal to the number of digits between the start of a run of n-1 zeros and the start of the run of **n** ones. These are not included the start digits of both runs.

(g)It will be assumed that the two states of the m-sequence previously defined as states of one and zero, will be chosen to correspond to the amplitude levels of +a and -a respectively. Then the number of zero crossing in now sequence is 2^{n-1}.

(h)The m-sequence X of state +a and -a there is the auto correlation function shown in figure 2.1[7,8] where

$$\phi_{xx}(\tau) = a^2\left(1-\frac{|\tau|2^n}{(2^n-1)\Delta t}\right) \qquad 0 \le |\tau| \le \Delta t$$

$$= \frac{-a^2}{2^n-1} \qquad \tau > \Delta t$$

(i)The power density spectrum of an m-sequence is a line spectrum, and has an envelope to the shape of $\left(\frac{\sin\theta}{\theta}\right)^2$. The first value of zero is at the frequency of the clock pulse $\frac{2\pi}{\Delta t}$. Its shape is shown in figure 2.2[7] and see the proof in Appendix C.

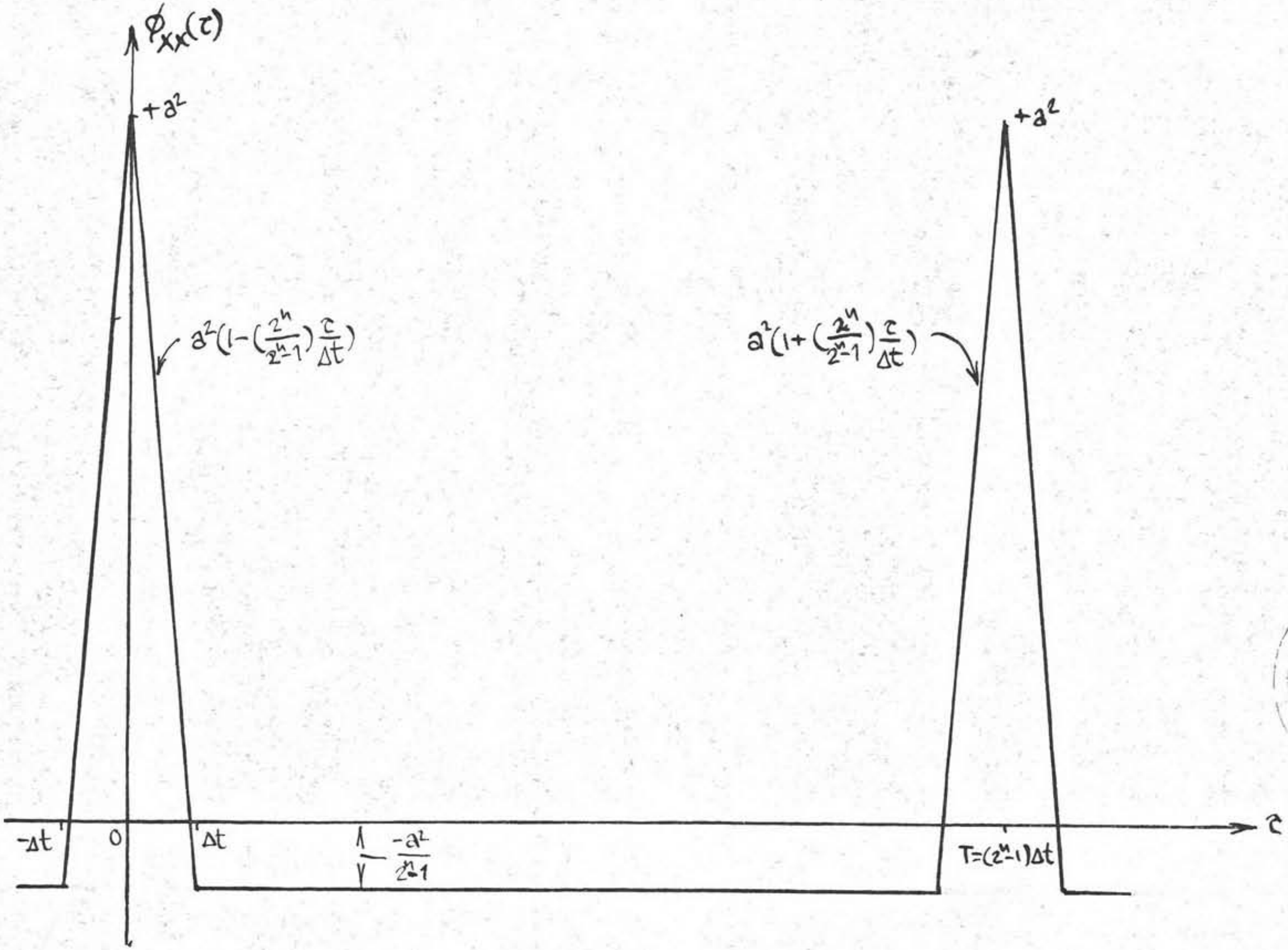

Figure 2.1 The Auto-correlation function of m-sequence x with states +a and -a

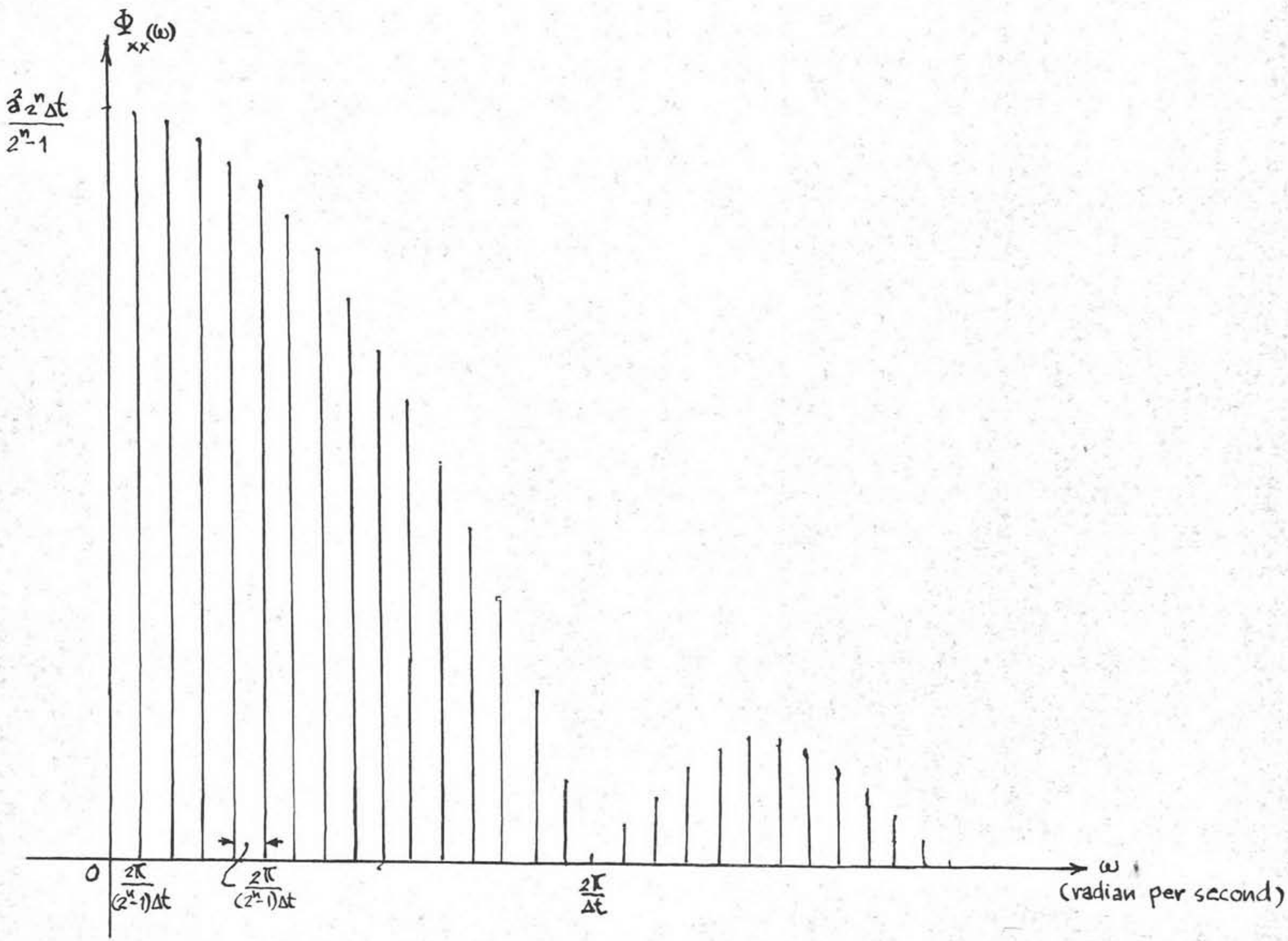

Figure 2.2 The Power Density Spectrum of m-sequence x with states +a and -a

(j)The cross correlation of an m-sequence X whose two states are +a and -a, and a similar sequence $\overline{X}$ whose two states are +a and 0 corresponded to sequence X respectively has the form shown infigure 2.3[7,8]

$$\bar{x}(t) = \frac{1}{2}(x(t) + a)$$

$$\phi_{x\bar{x}}(\tau) = \frac{1}{2}\left(\phi_{xx}(\tau) + \frac{a^2}{2^n-1}\right)$$

2.3 Summary

Some advantages and disadvantages of pseudorandom binary signal[17] are.

(a) simple to apply

(b) inaccurate

(c) insensitive for disturbances

(d) complicated data processing

This signal is suitable to use for determination of correlation function.

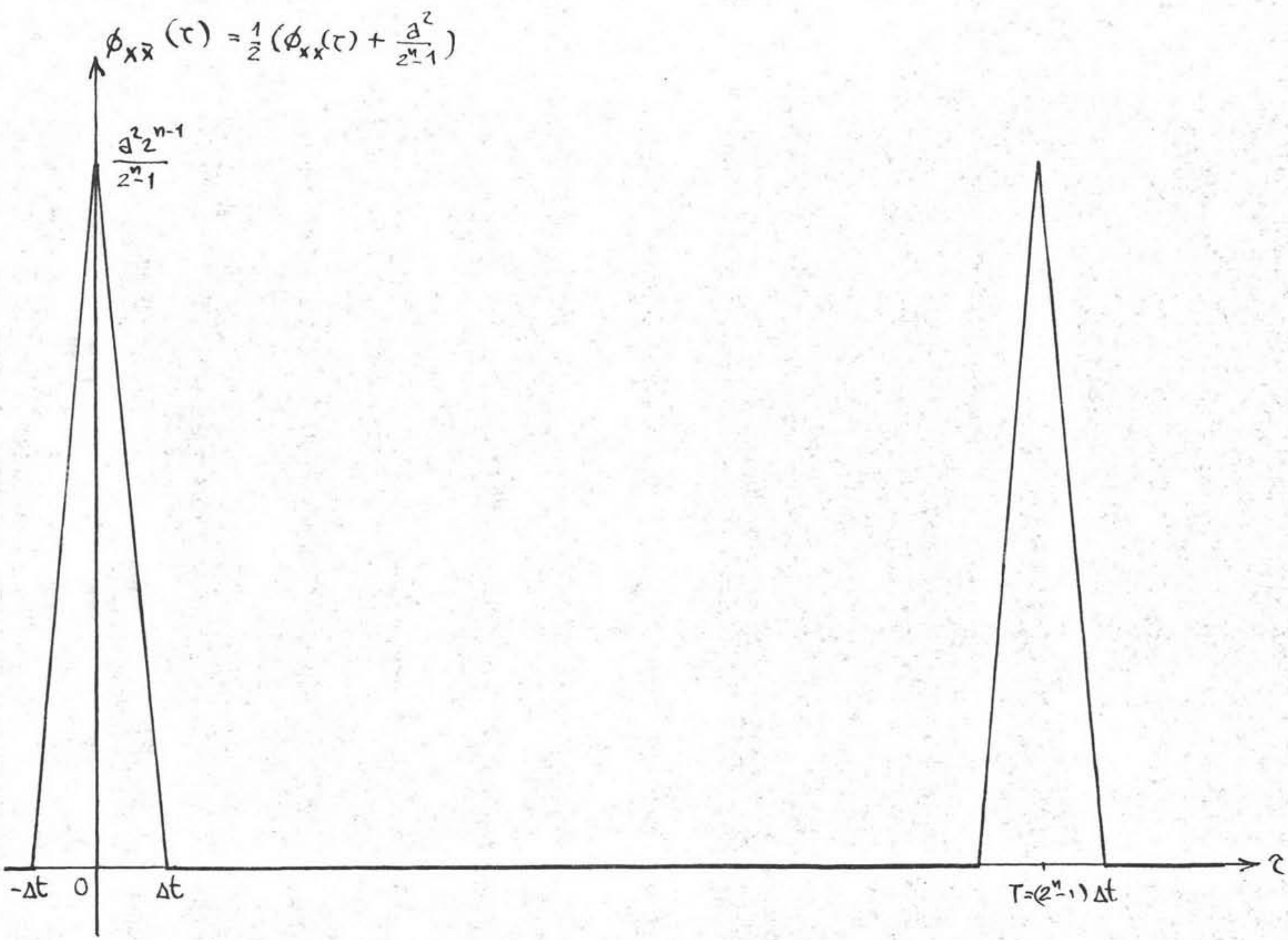

Figure 2.3 The cross-correlation function of $x(t)$ and $\bar{x}(t)$, $\bar{x}(t) = \frac{1}{2}(x(t)+a)$