CHAPTER I

INTRODUCTION

In [7], H.E. Vaughan determined all solutions of the functional equation

$$(S) \qquad g(x - y) = g(x)g(y) + f(x)f(y) ,$$

where f and g are functions from $\mathbb{R}$ into $\mathbb{R}$. If we consider f and g to be functions on a group (G,o) into a field F, the equation (S) is a special case of the functional equation

$$(A) \qquad g(xoy^{-1}) = g(x)g(y) + f(x)f(y) .$$

The purpose of this study is to determine all solutions of (A). In Chapter III, we characterize all solutions of (A) on an abelian group. Chapter IV deals with characterization of continuous solutions of (A) on an abelian topological group. In Chapter VI, we illustrate how our results can be applied to the case where $G = \mathbb{R}^n$ and $G = \mathbb{R}^* = \mathbb{R} - \{0\}$. In doing so we need the knowledge of homomorphisms on $\mathbb{R}^n$ into the unit circle $\triangle$, and homomorphisms on $\mathbb{R}^*$ into $\mathbb{C}^*$. Chapter V deals with the study of such homomorphisms.