

## CHAPTER 3



### PREDICTION OF THE DISCHARGE CHARACTERISTICS

#### 3.1 Introduction.

When the heated water is discharged as a bouyant jet from the outlet into the flowing ambient fluids, the dilution ratio  $S_o$  which is defined as the ratio between the density difference at any point and density difference at nozzle outlet depends on many parameters. These parameters are usually expressed in the form of dimensionless groups. The following sections describe the effects of these dimensionless groups on the dilution process.

#### 3.2 Dimensional analysis.

The parameters described in Chapter 2 for geometric, kinematic and dynamic similarities, can be incorporated and formulated as follows:

$$S_o = \frac{\Delta \rho}{(\Delta \rho)_o} = f_1 \left( F, \frac{(\Delta \rho)_o}{\rho_a}, \frac{Y_o}{d}, \frac{U}{V}, \frac{X}{d}, \frac{L}{d} \right) \quad (3.1)$$

where

$\Delta \rho$  = density difference between the hot plume and ambient water at any point

$(\Delta \rho)_o$  = density difference between the water at the nozzle outlet and ambient water

$U$  = Jet velocity

$V$  = ambient current velocity

$\rho_a$  = ambient density

- $x$  = coordinates measured in the direction of the current  
 $d$  = nozzle diameter  
 $Y_o$  = depth of ambient flow over nozzle  
 $L$  = length characterizing the geometry or shape of the outlet structure  
 $F$  = Densimetric Froude Number =  $\frac{U}{\sqrt{\frac{\Delta \rho}{\rho_a} g D}}$

For the range of temperature investigated in this study, it can be assumed that

$$\frac{\Delta \rho}{(\Delta \rho)_o} = \frac{\Delta T}{(\Delta T)_o} \quad (3.2)$$

where  $\Delta T$  = temperature difference between the hot plume and ambient water at any point

$(\Delta T)_o$  = temperature difference between the water at the nozzle outlet and ambient water

For small enough values of  $\frac{\Delta \rho}{\rho_a}$ , Boussinesq's assumption (8) allows the omission of this parameter. The equation (3.1) when combined with equation (3.2) can then be reduced to:

$$S_o = \frac{\Delta T}{(\Delta T)_o} = f_2 \left( F, \frac{Y_o}{d}, \frac{U}{V}, \frac{x}{d}, \frac{L}{d} \right) \quad (3.3)$$

Thus the temperature dilution achieved will be a function of the jet Densimetric Froude Number, the submergence ratio, the ratio between the velocity of jet and velocity of ambient fluid, the distance along the ambient current, and the geometry of the outlet structure.

### 3.3 Consideration of a jet.

The principal mechanism by which dilution of a submerged buoyant jet is achieved is the entrainment of the surrounding fluid into the jet.

The total dilution,  $S'_o$ , is the integrated effect of entrainment along the jet trajectory from the nozzle to the ocean surface. Hence, two factors are important in determining the dilution ratio of submerged bouyant jets, first, the rate of entrainment, and second, the length of the jet from the nozzle to the water surface. The rate of entrainment decreases with increasing Densimetric Froude Number,  $F$ , and  $U/V$  ( for  $U/V \geq 1$  ) (12).

The length of the jet trajectory increases with increasing values of  $F$  because smaller bouyancy forces or higher velocities produce a jet which continues for a great distance in the original direction before rising to the free surface. The ambient current generally tends to bend the jet in the horizontal direction and consequently increasing its trajectory length. Also, greater submergence, defined by the parameter  $Y_o/d$ , results in increasing the jet length.

It can be expected, therefore, that the dilution,  $S'_o$  for a submerged, round, vertical, bouyant jet will increase with  $Y_o/d$  and generally decrease with increasing  $U/V$  (i.e. increase with  $V/U$ ). The effects of increasing  $F$ , on the other hand, will initially lower  $S'_o$ , as the entrainment rate will be decreased. However, as  $F$  is further increased, the increase in the length of the jet trajectory will counter-balance the decrease in entrainment rate and eventually improve the total jet dilution.

### 3.4 Prediction equation of the temperature distribution.

Armstrong (2) predicted the temperatures in the vicinity of a heated discharge in coastal waters by assuming that the temperature

distribution of the plume is linear, steady state and non-dispersive.

The equation has the form:

$$\frac{d\alpha}{dt} = -\frac{Q}{A} \frac{d\alpha}{dx} + E \frac{d^2\alpha}{dx^2} + S_c \quad (3.4)$$

where:

$\alpha$  = concentration of water quality characteristics

$t$  = time

$Q$  = total flow of entrained ambient water

$A$  = cross-sectional area of the entrained ambient water

$E$  = dispersion coefficient

$S_c$  = source or sink within the hot plume

$x$  = coordinates measured in the direction of the current

The solution of equation (3.4) is (for details, see Appendix. C)

$$T = (T_R - T_a) e^{-kx/U} + T_a \quad (3.5)$$

$$\Delta T = (T_R - T_a) e^{-kx/U}$$

where:

$T_a$  = ambient temperature

$T_R$  = base temperature along the centre-line of the nozzle

$U$  = velocity of ambient current

$x$  = distance in the direction of the current

$\Delta T$  = temperature rise above ambient temperature

$$k = \lambda / \rho C_p D$$

$\lambda$  = heat dissipation rate

$\rho$  = water density

$C_p$  = specific heat of water

3.5 Application of the predicted equation to the present problem.

To obtain the temperature distribution for the problem under consideration, the equation (3.5) is used together with the following assumptions:

1. The average plume width is 10 metres.
2. The average ambient current velocity is 1 m/sec .
3. The depth of the discharge outlet from the water surface is 4 metres.
4. The heat dissipation rate ( $\lambda$ ) is 58.6 cal/cm<sup>2</sup>.hr.°C (11)
5. The ambient temperature is 30°C

Then,

$$\begin{aligned}\text{Net tidal flow} &= (\text{cross-sectional area}) \times (\text{velocity}) \\ &= (10 \times 4) \times 1 \\ &= 40 \text{ m}^3/\text{sec}\end{aligned}$$

The average temperature of the plume may be calculated from the following expression:

$$\begin{aligned}\text{Temperature} &= \frac{(\text{mass of ambient water} \times \text{temperature}) + (\text{mass of hot water} \times \text{temperature})}{\text{sum of the hot and cold water}}\end{aligned}$$

$$\begin{aligned}T_{\text{avg}} &= \frac{(40 \text{ m}^3/\text{sec})(30^\circ\text{C}) + (35 \text{ m}^3/\text{sec})(40^\circ\text{C})}{(40 + 35) \text{ m}^3/\text{sec}} \\ &= 34.6^\circ\text{C}\end{aligned}$$

and

$$\begin{aligned}k &= \frac{\lambda}{\rho C_p D} \\ &= \frac{(58.6 \text{ cal/cm}^2 \cdot \text{hr} \cdot ^\circ\text{C})(1/60 \text{ sec/hr})}{(1 \text{ gm/cm}^3)(1 \text{ cal/gm} \cdot ^\circ\text{C})(4 \text{ m.})(100 \text{ cm/m})} \\ &= 0.0244 \text{ sec}^{-1}\end{aligned}$$



Substituting these values into equation (3.5) by assuming that  $T_R$  be the same as  $T_{avg}$ .

$$\begin{aligned} T &= ( 34.6 - 30.0 ) e^{-0.0244x} \\ &= 4.6 e^{-0.0244x} \end{aligned} \quad ( 3.6 )$$

From the equation (3.6), the temperatures at the plume surface which is in contact with air can be calculated for various values of  $x$ . Table 3.1 illustrates the predicted temperature rise above ambient temperature at the various distances from the point of discharge in the direction of the ambient current.

TABLE 3.1

Predicted temperature rise above ambient temperature  
at the various distances from the point of discharge

Distance ( m )	Temperature rise(°C)
0	4.60
20	2.82
50	1.36
100	0.40
150	0.12
200	0.04