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Appendix A.

PROGRAMME

```
C     ANALYSIS OF HELICAL STAIR WITH CENTRAL ANGLE 720 DEGREES
      DIMENSIØN INDEX (6,2), D(6,6), DW(6), X(6)
      READ (2,1) N, PHI
      READ (2,2) W, HT, RØ , RI, H
    1 FØRMAT (I4, F12.7)
    2 FØRMAT (F6.0 , F6.2 , 2F4.1 , F6.2)
    3 FØRMAT (6F13.4 /)
      R1     =   2./3.*(RØ **3 -RI**3) / (RØ **2 - RI**2)
      R2     =   (RØ + RI) / 2.
      B      =   RØ - RI
      RHØ    =   R1/R2
      HPB    =   H/B
      ALPHA  =   ATAN (HT/(4. *PHI*R2))
      GAMMA  =   28./9./(16./3. - 3.36 *HPB* (1. - HPB**4/12.))
      BETA   =   HPB**2
      SA     =   SIN (ALPHA)
      CA     =   CØS (ALPHA)
      SA2    =   SA*SA
      CA2    =   CA*CA
      C2A    =   CA2 - SA2
      S2A    =   2.*SA*CA
      TA     =   SA/CA
      TA2    =   TA*TA
      PI2    =   PHI*PHI
      PI3    =   PI2 * PHI
      D(1,1)=    TA2 * (10.67*PI3 + PHI) + BETA * (6.*PHI*C2A + SA2
                 *TA2 * (10.67 * PI3 - PHI)) + GAMMA * SA2 * (10.67
                 *PI3 + 11.*PHI)
      D(2,2)=    TA2 * (10.67*PI3 - PHI) + BETA *(2.*PHI*C2A + SA2
                 *TA2*(10.67*PI3 + PHI)) + GAMMA * SA2 * (10.67*
                 PI3 + 5.*PHI)
      D(3,3)=    2.*PHI* (1. + 3.* (BETA*SA2 + GAMMA*CA2))
      D(4,4)=    2.*PHI* (1. + (BETA*SA2 + GAMMA*CA2))
      D(5,5)=    D(4,4)
      D(6,6)=    4.*PHI* (BETA*CA2 + GAMMA*SA2)
      D(1,2)=    4.*PI2* (-TA2+SA2*(BETA*TA2 + GAMMA))
      D(1,3)=    PHI* (-TA + 1.5*S2A*(BETA*(2. - TA2) - 3.*GAMMA))
      D(1,4)=    -PHI*(TA - S2A/2.* (BETA*(2. + TA2) - GAMMA))
      D(1,5)=    -4.*PI2*(TA + S2A/2.*(BETA*TA2 + GAMMA))
      D(1,6)=    -4.*PHI*(BETA*CA2*(1. - TA2) + 2. * SA2*GAMMA)
      D(2,3)=    4.*PI2*(TA + S2A/2.* (BETA*TA2 + GAMMA))
      D(2,4)=    D(2,3)
```

```
      D(2,5)=    PHI*(TA + S2A/2.*(BETA*(2. - TA2) - 3.*GAMMA))
      D(2,6)=    0.0
      D(3,4)=    2.*PHI*(1. + BETA*SA2 + GAMMA*CA2)
      D(3,5)=    0.0
      D(3,6)=    -2.*PHI*S2A*(BETA - GAMMA)
      D(4,5)=    0.0
      D(4,6)=    0.0
      D(5,6)=    0.0
      DW(1) =    - 4.*PI2*(RHØ*TA + S2A/2.*(BETA*(TA2 * (4. + RHØ )
                 - 2.) + GAMMA*(6. + RHØ)))
      DW(2) =    - PHI*(3.*TA*RHØ + TA * BETA * (8.*SA2 - 4.*CA2
                 +RHØ * (SA2 - 2.*CA2)) + GAMMA*S2A/2.*(12. + 3.
                 *RHØ))
      DW(3) =    8.*PI2*(BETA*SA2 + GAMMA*CA2)
      DW(4) =    0.0
      DW(5) =    2.*PHI*(RHØ + (2. + RHØ) * (BETA*SA2 + GAMMA*CA2))
      DW(6) =    - 4.*PI2*S2A*(BETA - GAMMA)
      DØ 4 J=    1, 5
      M     =    J + 1
      DØ 4 I=    M, 6
 4    D(I,J)=    D (J, I)
      WRITE (3,5)
 5    FØRMAT (30H THE FLEXIBILITY MATRIX)
      WRITE (3,3) ((D(I, J), J=1, N), I=1, N )
      WRITE (3,6)
 6    FØRMAT (50H THE FLEXIBILITY OF THE UNIT UNIFØRM LØAD)
      WRITE (3,3) DW
      DØ  7  I  =  1, N
 7    INDEX (I,1) = 0
      II  =  0
 8    AMAX   =    -1.
      DØ 13 I  = 1,N
      IF (INDEX (I, 1))    13, 9, 13
 9    DØ 12 J  =  1,N
      IF (INDEX(J, 1))     12, 10, 12
 10   TEMP  =  ABS (D (I, J))
      IF (TEMP - AMAX)     12, 12, 11
 11   IRØW  =  I
      IRØL  =  J
      AMAX  =  TEMP
 12   CØNTINUE
 13   CØNTINUE
      IF (AMAX)   24, 27, 14
 14   INDEX (ICØL, 1)  =  IRØW
      IF (IRØW - ICØL) 15, 17, 15
 15   DØ 16 J  =  1, N
      TEMP  =  D(IRØW , J)
      D (IRØW, J)  =  D(ICØL, J)
 16   D (ICØL, J)  =  TEMP
      II  =  II + 1
```

```
      INDEX (II, 2)  =  ICØL
17    PIVØT  =  D(ICØL, ICØL)
      D(ICØL, ICØL)  =  1.0
      PIVØT  =  1./PIVØT
      DØ 18 J  =  1, N
18    D (ICØL, J)  =  D (ICØL, J) * PIVØT
      DØ 21 I  =  1, N
      IF (I - ICØL) 19, 21, 19
19    TEMP  =  D(I, ICØL)
      D(I, ICØL)  =  0.0
      DØ 20 J  =  1, N
20    D(I, J)  =  D(I, J) - D(ICØL, J) * TEMP
21    CONTINUE
      GØ TØ  8
22    ICØL  =  INDEX (II, 2)
      IRØW  =  INDEX (ICØL, 1)
      DØ 23 I  =  1, N
      TEMP  =  D(I, IRØW)
      D(I, IRØW)  =  D(I, ICØL)
23    D(I, ICØL)  =  TEMP
      II  =  II - 1
24    IF (II) 22, 25, 22
25    WRITE (3, 26)
26    FØRMAT (25H THE INVERSE OF MATRIX)
      WRITE (3, 3) ((D(I, J), J=1, N), I=1, N)
      GØ TØ 29
27    WRITE (3, 28)
28    FØRMAT (12H ZERO PIVØT)
      GØ TØ 40
29    DØ 31 I  =  1, N
      X(I)      =  0.0
      DØ 30   K = 1, N
30    X (I)  =  X (I) + D(I, K) * DW(K)
31    CØNTINUE
      WRITE (3, 32) HPB
32    FØRMAT (38H THE REDUNDANTS AT DEPTH/WIDTH  =  ,F6.3)
      DØ  33  I  =  1, N
33    X(I)  =  X(I)*W
      WRITE (3, 34)
34    FØRMAT (1Hb, 8X, 5HX1/R2, 8X, 5HX2/R2, 8X, 5HX3/R2, 5X,
     8HX4/R2**2, 5X, 8HX5/R2**2, 5X, 8HX6/R2**2)
      WRITE (3, 3) X
      ANGLE  =  0.0
35    C  =  PHI/180.*ANGLE
      TEMP  =  C
      IF (C - 2.*PHI) 37, 37, 36
36    C  =  C - 2.*PHI
      IF (C.GT.2.*PHI)  GØ TØ 36
37    SC  =  SIN (C)
```

```
      CC  =  CØS (C)
      C   =  TEMP
      RM  =  - R2**2*(C*SA/CA*(X(1)*CC + X(2)*SC) + (X(3) + X(4))*
             SC - X(5)*CC) + R1*R2*(1. - CC)*W
      TM  =  R2**2*(SA*((1. - C*SC - CC)*X(1) + (C*CC - SC)*X(2)-
             X(6)) + CA*((CC - 1.)*X(3) + CC*X(4) + SC*X(5) + (C -
             RHØ*SC)*W))
      SM  =  R2**2*(CA*((CC - 1. - C*SC*TA2)*X(1) + (SC + C*CC*TA2)
             *X(2) + X(6)) + SA*((CC - 1.)*X(3) + CC*X(4) + SC*X(5)
             + (C - RHØ*SC)*W))
      RQ  =  - R2*(SC*X(1) - CC*X(2))
      TQ  =  R2*(CA*(CC*X(1) + SC*X(2)) - SA*(X(3) - C*W))
      SQ  =  R2*(SA*(CC*X(1) + SC*X(2)) + CA*(X(3) - C*W))
      WRITE (3, 38) ANGLE
38    FØRMAT (5H    AT, F8.1, 10H   DEGREES)
      WRITE (3, 3) RM, TM, SM, RQ, TQ, SQ
      IF (720. - ANGLE) 40, 40, 39
39    ANGLE  =  ANGLE + 10.
      GØ TØ 35
40    STØP
      END
```

INPUT DATA

W = uniform load per unit length of horizontal projection of center line of step (Kg/m)

HT = vertical distance between the supported ends. (m.)

RØ, RI = external and internal raduis of the helical stair (m.)

H = depth of stair section (m.)

Appendix B.

PROGRAMME

```
C     ANALYSIS ØF VERTICAL DEFLECTIØN ØF THE HELICAL STAIR
      READ (2,1)  X1, X2, X3, X4, X5, X6
1     FØRMAT (6F12.5)
      READ (2,2) W, HT, RØ , RI, H, E, G
2     FØRMAT (5F8.2,  2F12.2)
      WRITE (3,3)
3     FØRMAT (11H         ANGLE, 10X, 10HDEFLECTIØN)
      R1     =  2./3. * (RØ**3 - RI**3) / (RØ**2 - RI**2)
      R2     =  (RØ + RI) / 2.
      B      =  RØ - RI
      RHØ    =  R1 / R2
      HPB    =  H / B
      ALPHA  =  ATAN (HT / (4.*3.141593*R2))
      EIR    =  E * 1./12.* B* H **3
      EIS    =  E * 1./12.* H* B**3
      GJ     =  G * 1./16.* B* H**3*(16./3. - 3.36* HPB* (1. - HPB**
                4/12.))
      CA     =  CØS (ALPHA)
      SA     =  SIN (ALPHA)
      TA     =  SA / CA
      CA2    =  CA * CA
      SA2    =  SA * SA
      TA2    =  TA * TA
      PHI    =  3.141593
      PI2    =  PHI * PHI
      ANGLE  =  0.0
4     F      =  PHI * ANGLE / 180.
      TEMP   =  F
      IF (F - 2.*PHI) 6, 6, 5
5     F      =  F - 2.*PHI
      IF (F.GT.2.*PHI) GØ TØ 5
6     SF     =  SIN (F)
      CF     =  CØS (F)
      F      =  TEMP
      FO     =  R2**3* R1*(SF * (4.*PHI - F) / 2. - CF + 1.) / EIR
                / CA + (R2**4*( TA*SA / EIS + CA / GJ )) * (8.*PI2
                - 1*F / 2. + (4.*PHI + RHØ*(2.*PHI - F/2.)) * SF +
                (1. - CF) *(1. + RHØ))
      F1     =  -R2**3*TA/CA * ((F - 4.*PHI) * CF/4. - SF *(1. + 16.
                *PI2 - F*F) / 4.) / EIR + R2**3*SA*((SF*(.75 + 4.*
                PI2 - F*F/4.)- CF*(.75*F + PHI))* TA2 - CF*( 2.*PHI
```

```
            - F/2.) - 4. *PHI*(1. - TA2) - 1.5*SF + F) /EIS
            + R2**3*SA*(8.*PHI- F +(9./4. + 4.*PI2 - F*F/4.)
            *SF + (PHI - 5./4.)*CF) / GJ
      F2  = -R2**3*TA/CA*(CF*(4.*PI2 -1./8. - F*F/4.) + (F/4.
            + PHI) * SF + CF/8.) / EIR + R2**3*SA* (TA2*(1.-CF*
            (1.+ 4.*PI2 - F*F/4.) + (PHI - .75*F)*SF) - 1. + CF
               - SF * (2.*PHI - F/2.)) / EIS + R2**3*SA*(2.-
            (2. + 4. *PI2 - F*F/4.)*CF + (3.*PHI - 5.*F/4.)*SF)
            / GJ
      F3  = R2**3/CA* ((F/2. - 2.*PHI)*CF -SF/2.) / EIR + (R2
            **3*(TA*SA/EIS + CA/GJ))* ((F/2. - 2.*PHI) * (2.
            +CF) - 1.5*SF)
      F4  = R2**2/CA*(CF*(F/2.-2.*PHI) - SF/2.) / EIR + (R2**
            2*(TA*SA/EIS + CA/GJ)) * ((F/2. - 2.*PHI)*CF-SF/2.)
      F5  = R2**2/CA*((F/2. - 2.*PHI)*SF)/EIR + (R2**2*(TA*SA
            /EIS + CA/GJ)) * ((F/2. - 2.*PHI)*SF - 1. + CF)
      F6  = R2**2*SA*(1./EIS - 1./GJ)*(4.*PHI - F + SF)
      DFEC = (FO*W + F1*X1 + F2*X2 + F3*X3 + F4*X4 + F5*X5 +
            F6*X6) * 0.01
      WRITE (3,7)  ANGLE, DFEC
    7 FØRMAT (F10.1, F20.6)
      IF (720. - ANGLE) 9, 9, 8
    8 ANGLE=  ANGLE + 10.
      GØ TØ 4
    9 STØP
      END
```

INPUT DATA

X1,X2,X3 = redundant forces at the lower support (Kg)

X4,X5,X6 = redundant moments at the lower support (Kg - m)

W = uniform load per unit length of horizontal project -ion of center line of step (Kg/m)

HT = vertical distance between the supported ends (m.)

RØ,RI = external and internal radius of the helical stair (m.)

H = depth of stair section (m.)

E = modulus of elasticity of concrete (K.S.C.)

G = shearing modulus of elasticity of concrete (K.S.C.)

Appendix C.

PROGRAMME

```
C    , ANALYSIS ØF HELICAL STAIR BY MØRGAN METHØD
       READ (2, 1) W, A, RØ, RI,  H, PHI
   1   FØRMAT (4F10.2, F10.3, F10.2)
   2   FØRMAT (//5H  MV=,F13.4, 26X, 4H   H=,F13.4)
   3   FØRMAT (//10H     ANGLE, 15X, 5H    MRF, 15 X, 5H   TF, 15X,
       5H  NNF)
   4   FØRMAT (F10.0, 3F20.4)
   5   FØRMAT (//10H     ANGLE, 15X, 5H, SHF,15X, 5H   PNE, 15X,
       5H   SNF)
       THETA  =  3.14159*A/180.
       PHI    =  3.14159*PHI/180.
       B      =  RØ - RI
       R1     =  2./3.*(RØ**3 - RI**3)/(RØ**2 - RI**2)
       R2     =  (RØ + RI)/2.
       CK1    =  1./3. - 3.36/16.*H/B*(1. - H**4/B**4/12.)
       CK     =  THETA*CØS(2.*THETA)/4. - SIN(2.*THETA)/8.
       CM     =  THETA/2. - SIN(2.*THETA)/4.
       CN     =  THETA*CØS(THETA) - SIN(THETA)
       CGI1   =  36.*CK1/7.
       CGI2   =  36.*CK1*H**2/(7.*B**2)
       CS     =  CØS (PHI)**2 + CGI2*SIN(PHI)**2
       B1     =  CGI1*(CM + 0.5* SIN(2.*THETA)) + CS*CM
       C1     =  - CGI1*CK*R2*SIN(PHI)/CØS(PHI) + CS*CK*R2*SIN(PHI)
                 /CØS(PHI)
       C1     =  C1+R2*SIN(PHI)*CØS(PHI)*CM*(1. - CGI2)
       D1     =  W*R1*(CGI1*R1*(CM + 0.5*SIN(2.*THETA) - SIN(THETA)))
       D1     =  D1 + W*R1*(R1*CM*CS + CN*R2*CS)
       B2     =  - CGI1*CK + CS*CK + (CS - CGI2)*CM
       C3     =  CGI1*R2/2.*SIN(PHI)/CØS(PHI)
       C4     =  C3*(THETA**3/3. - THETA**2*SIN(2.*THETA)/2. - 2.*CK)
       C5     =  CS*R2/2.*SIN(PHI)/CØS(PHI)
       C6     =  C5*(THETA**3/3. + THETA**2*SIN(2.*THETA)/2. + 2.*CK)
       C7     =  (CS - CGI2)*2.*CK*R2*SIN(PHI)/CØS(PHI)
       C8     =  CM*R2*CØS(PHI)**2*(SIN(PHI)/CØS(PHI) + CGI2*CØS(PHI)
                 /SIN(PHI))
       C2     =  C4 + C6 + C7 + C8
       D3     =  CGI1*R1*(CN - CK) + CS*CK*R1 + CS*R2*(THETA**2*SIN
                 (THETA) + 2.*CN)
       D2     =  W*R1*(D3 + (CS - CGI2)*(CM*R1 + CN*R2))
       T      =  (C1*D2 - C2*D1)/(B1*C2 - B2*C1)
       H      =  (B2*D1 - B1*D2)/(B1*C2 - B2*C1)
       WRITE (3, 2) T, H
       WRITE (3, 3)
```

```
      ANGLE  =  0.0
      THETA  =  0.0
20    TRF    =  T*CØS(THETA) + H*R2*THETA*SIN(PHI)/CØS(PHI)*SIN(THETA)
      TRF    =  TRF - W*R1**2*(1.-CØS(THETA))
      TNF    =  T*SIN(THETA)*SIN(PHI)
      TNF    =  TNF - H*R2*THETA*SIN(PHI)/CØS(PHI)*CØS(THETA)*SIN(PHI)
      TNF    =  TNF - H*R2*SIN(THETA)*CØS(PHI)
      TNF    =  TNF + (W*R1**2*SIN(THETA) - W*R1*R2*THETA)*SIN(PHI)
      TF     =  (T*SIN(THETA) - H*R2*THETA*CØS(THETA)*SIN(PHI)/CØS
                (PHI))*CØS(PHI)
      TF     =  TF + (W*R1**2*SIN(THETA) - W*R1*R2*THETA)*CØS(PHI)
      TF     =  TF + H*R2*SIN(THETA)*SIN(PHI)
      WRITE (3, 4)  ANGLE, TRF, TF,TNF
      IF (ANGLE - A) 30, 50, 50
30    THETA  =  THETA + 0.174533
      ANGLE  =  ANGLE + 10.0
      IF  (ANGLE - A) 20, 20, 40
40    THETA  =  3.14159*A/180.
      ANGLE  =  A
      GØ TØ 20
50    WRITE (3, 5)
      ANGLE  =  0.0
      THETA  =  0.0
60    PNF    =  -H*SIN (THETA)*CØS(PHI) - W*R1*THETA*SIN(PHI)
      SNF    =  W*R1*THETA*CØS(PHI) - H*SIN(THETA)*SIN(PHI)
      SHF    =  H*CØS(THETA)
      WRITE (3, 4) ANGLE, SHF, PNF, SNF
      IF (ANGLE - A) 70, 90, 90
70    THETA  =  THETA + 0.174533
      ANGLE  =  ANGLE + 10.0
      IF (ANGLE - A) 60, 60, 80
80    THETA  =  3.14159*A/180.
      ANGLE  =  A
      GØ TØ 60
90    STØP
      END
```

INPUT DATA

W = uniform load per unit length of horizontal projection of center line of load (Kg/m)

A = half arc subtended by helix (Degrees)

RØ, RI = external and internal radius of the helical stair(m.)

H = depth of stair section (m.)

PHI = slope made by tangent to helix center-line with respect to horizontal plane (Degrees)

Appendix D

PROGRAMME

```
C       ANALYSIS ØF HELICAL STAIR BY BERGMAN METHOD
        READ (2, 1) W, R, B, H , THETA
  1     FØRMAT (5F10.2)
        WRITE (3, 2)
  2     FØRMAT (7H   ANGLE, 15X, 6HMØMENT, 14X, 7HTØRSIØN, 16X, 5H
        SHEAR)
        HPB     =  H/B
        RK      =  3.1333/(16./3. - 3.36*HPB * (1. - HPB**4/12.))
        THETA   =  THETA*3.1415927/180.
        CC      =  CØS (THETA)
        SC      =  SIN (THETA)
        U       =  2.*((RK + 1.0)*SC - RK*THETA*CC)/((RK + 1.0)*THETA
                   - (RK - 1.0)*SC*CC)
        ANGLE   =  0.0
  3     ALPHA   =  ANGLE*3.1415927/180.
        CA      =  CØS (ALPHA)
        SA      =  SIN (ALPHA)
        BM      =  W*R**2*(U*CA - 1.0)
        TA      =  W*R**2*(U*SA - ALPHA)
        VA      =  W*R*ALPHA
        WRITE (3, 4) ANGLE, BM, TA, VA
  4     FØRMAT (F7.1, 3(12X, F9.2))
        IF (ANGLE .EQ.360.) GØ TØ 5
        ANGLE  =  ANGLE + 10.0
        GØ  TØ 3
  5     STØP
        END
```

INPUT DATA

W = uniform load per unit length of horizontal projection of center line of step (Kg/m)

R = radius of center line of step (m.)

B = width of stair section (m.)

H = depth of stair section (m.)

THETA = half angle subtending the helical stair (Degrees)

Appendix E

PROGRAMME

```
C       ANALYSIS ØF HELICAL STAIR BY HØLME METHOD
        READ (2, 1) W, BD, H, HT, R, ALPHA
   1    FØRMAT (F10.1, 5F10.2)
        WRITE (3, 2)
   2    FØRMAT (7H  ANGLE, 4X, 14HNØRMAL  MØMENT, 2X, 18HTANGENTIAL
        MØMENT, 2X, 16HBINØRMAL  MØMENT, 5X, 13HNØRMAL SHEAR, 2X,
        18HTANGENTIAL  THRUST, 3X, 15HBINØRMAL  SHEAR)
        ALPHA  =  ALPHA*3.1415927/180.
        ERS    =  (H/BD)**2
        ERJ    =  0.7*(1. + ERS)
        THETA  =  ATAN (HT/(2.*ALPHA*R))
        CC     =  CØS (THETA)
        SC     =  SIN (THETA)
        GAMMA  =  ERJ*CC**2 + ERS*SC**2
        RHØ    =  (ERS - ERJ)*CC**2
        SA     =  SIN (ALPHA)
        CA     =  CØS (ALPHA)
        S2A    =  2.*SA*CA
        C2A    =  CA*CA - SA*SA
        A      =  - GAMMA*(ALPHA/2. - S2A/4.) - (ALPHA/2. + S2A
                  /4.)
        B      =  (GAMMA - 1.)/8.*(S2A - 2.*ALPHA*C2A) + ERS*
                  (ALPHA/2. - S2A/4.) + (ALPHA/2. + S2A/4.)
        E      =  GAMMA*(SA - ALPHA*CA) + SA
        C      =  (1. - GAMMA)/8.*(S2A - 2.*ALPHA*C2A) - RHØ*
                  (ALPHA/2. - S2A/4.)
        D      =  GAMMA/2.*(ALPHA**3/3. + ALPHA**2*S2A/2. + ALPHA
                  *C2A/2. - S2A/4.) + (RHØ + ERS)/8.*(S2A - 2.*
                  ALPHA*C2A) + ERS*CC**2/SC**2*(ALPHA/2. - S2A/4.)
                  + (ALPHA**3/3. - ALPHA**2*S2A/2.)/2.
        F      =  GAMMA*(ALPHA**2*SA + 2.*ALPHA*CA - 2.*SA) + (
                  RHØ - 1.)*(SA - ALPHA*CA)
        C1     =  (B*F - D*E)/(A*D - B*C)
        C2     =  (C*E - A*F)/(A*D - B*C)
        PHE    =  0.0
   3    F      =  PHE*3.1415927/180.
        CF     =  CØS(F)
        SF     =  SIN(F)
        RM     =  W*R*R/CC*(1. - C1*CF - C2*F*SF + C2*CF)
        TM     =  W*R*R*(F - C1*SF + C2*F*CF)
        BM     =  W*R*R*SC/CC*(F - C1*SF + C2*F*CF + C2*SF/SC**2)
        VN     =  W*R*C2*CF/SC
        FT     =  W*R*(F*SC/CC + C2*SF*CC/SC)
        VB     =  W*R*(F - C2*SF)
        WRITE (3, 4)  PHE, RM, TM, BM, VN, FT, VB
```

```
4       FØRMAT (F6.1, 6F18.2)
        IF (PHE .EQ.360.) GØ  TØ  5
        PHE    =  PHE + 10.
        GØ TØ  3
5       STØP
        END
```

INPUT DATA

W = uniform load per unit true length of the helix center line (Kg/m)

BD = width of stair section (m.)

H = depth of stair section (m.)

HT = vertical distance between the supported end of the helix (m.)

R = the radius of the center line of step (m.)

ALPHA = half arc of the helix (Degrees)

Appendix F

PROGRAMME

```
C         ANALYSIS ØF HELICAL STAIR BY SCORDELIS METHØD
          READ (2, 1) W, B, H, R, PHE, HT, FX, FR
    1     FØRMAT (F10.2, 3F6.2, F7.1, F6.2, 2F10.5)
          WRITE (3, 2)
    2     FØRMAT (8H  ANGLE, 5X, 14HRADIAL  MØMENT, 4X, 15HLATERAL
          MØMENT, 12X, 7HTØRSION, 6X, 13HRADIAL  FØRCE, 5X, 14HLATE
          -RAL  FØRCE, 3X, 17HTANGENTIAL  FØRCE)
          FE      =  PHE*3.1415927/180.
          ALPHA   =  ATAN (HT/R/FE/2.)
          SA      =  SIN(ALPHA)
          CA      =  CØS(ALPHA)
          TA      =  SA/CA
          ANGLE   =  0.0
    3     THETA   =  ANGLE*3.1415927/180.
          CC      =  CØS(THETA)
          SC      =  SIN (THETA)
          RMW     =  -R*R*(1. - CC)
          SMW     =  -R*R*(THETA - SC)*SA
          TMW     =  -R*R*(THETA - SC)*CA
          RMX     =  -R*THETA*SC*TA
          SMX     =  R*(SC*CA + THETA*CC*SA*TA)
          TMX     =  -R*SA*(SC - THETA*CC)
          RMR     =  CC
          SMR     =  SC*SA
          TMR     =  SC*CA
          RMT     =  -R*(1. - CC)
          SMT     =  R*SC*SA
          TMT     =  R*SC*CA
          ECT     =  B*B/12./R
          RF      =  W*FX*CC
          SF      =  W*(R*THETA*CA + FX*SC*SA)
          TF      =  W*(-R*THETA*SA + FX*SC*CA)
          RM      =  W*(RMW + ECT*RMT + FX*RMX + FR*RMR)
          SM      =  W*(SMW + ECT*SMT + FX*SMX + FR*SMR)
          TM      =  W*(TMW + ECT*TMT + FX*TMX + FR*TMR)
          WRITE (3, 4) ANGLE, RM, SM, TM, RF, SF, TF
    4     FØRMAT (F7.2 , 6F19.3)
          IF (ANGLE.EQ.PHE) GØ TØ 5
          ANGLE   =  ANGLE + 10.0
          GØ TØ 3
    5     STØP
          END
```

INPUT DATA

W = uniform load per unit length of horizontal projection of center line of step (Kg/m.)

B = width of stair section (m.)

H = depth of stair section (m.)

R = radius of center line of step (m.)

PHE = half arc of helix (Degrees)

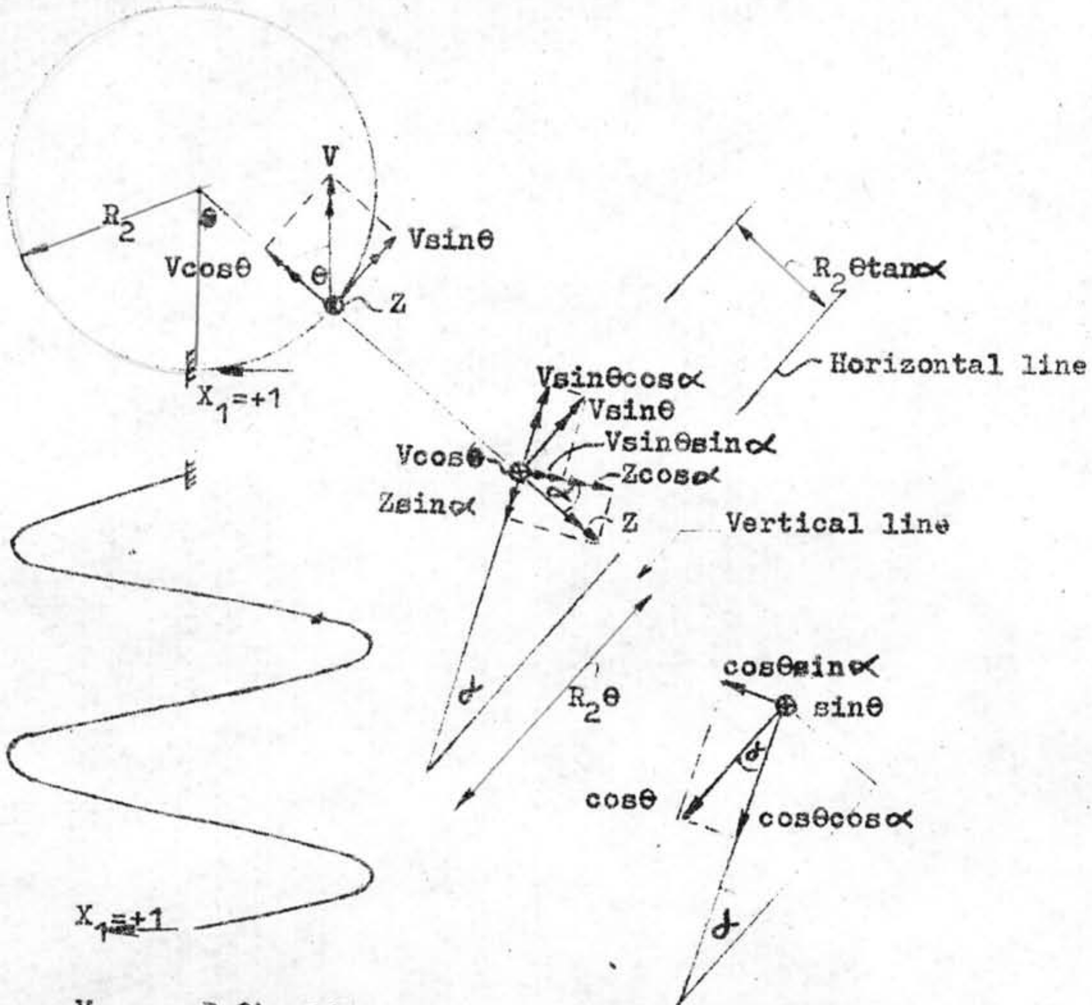
HT = vertical distance between the supported end of the helix (m.)

FX, FR = redundant force and moment at mid point of center line of the step. (Kg, Kg/m.)

APPENDIX G

Analysis of internal forces caused by the redundants and external load

$X_1 = +1$ (all other forces are zero)



$V = R_2\theta\tan\alpha \qquad Z = R_2(1-\cos\theta)$

$m_{t1} = -V\sin\theta\cos\alpha + Z\sin\alpha$

$= R_2\sin\alpha(1 - \theta\sin\theta - \cos\theta)$

$m_{r1} = -V\cos\theta$

$= -R_2\theta\tan\alpha\cos\theta$

$m_{s1} = -Z\cos\alpha - V\sin\theta\sin\alpha$

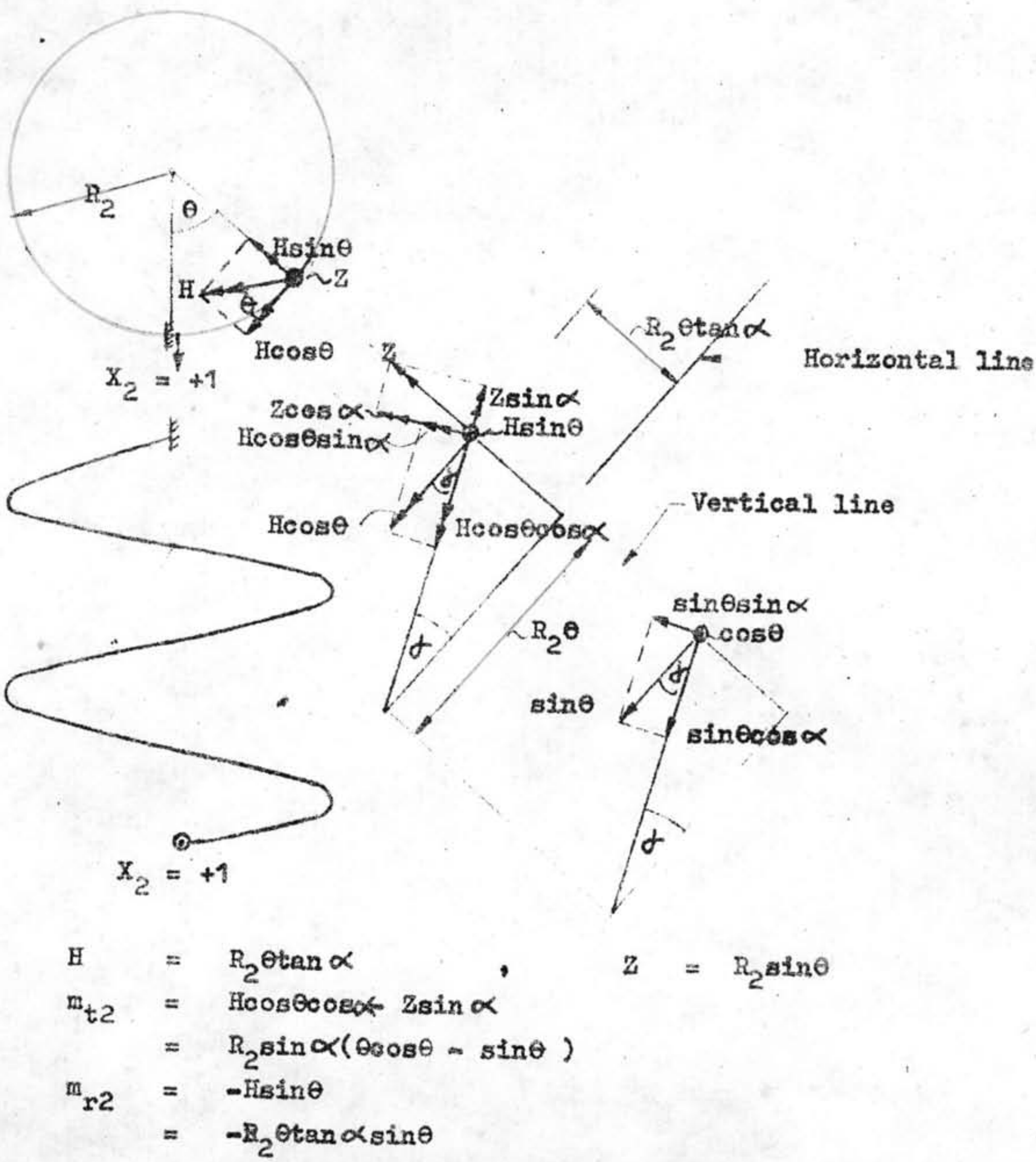
$= R_2\cos\alpha(\cos\theta - 1 - \theta\sin\theta\tan^2\alpha)$

$Q_{t1} = \cos\theta\cos\alpha$

$Q_{r1} = -\sin\theta$

$Q_{s1} = \cos\theta\sin\alpha$

$X_2 = +1$ (all other forces are zero)



$$H = R_2\theta\tan\alpha \quad , \quad Z = R_2\sin\theta$$

$$m_{t2} = H\cos\theta\cos\alpha - Z\sin\alpha$$

$$= R_2\sin\alpha(\theta\cos\theta - \sin\theta)$$

$$m_{r2} = -H\sin\theta$$

$$= -R_2\theta\tan\alpha\sin\theta$$

$$m_{s2} = H\cos\theta\sin\alpha + Z\cos\alpha$$

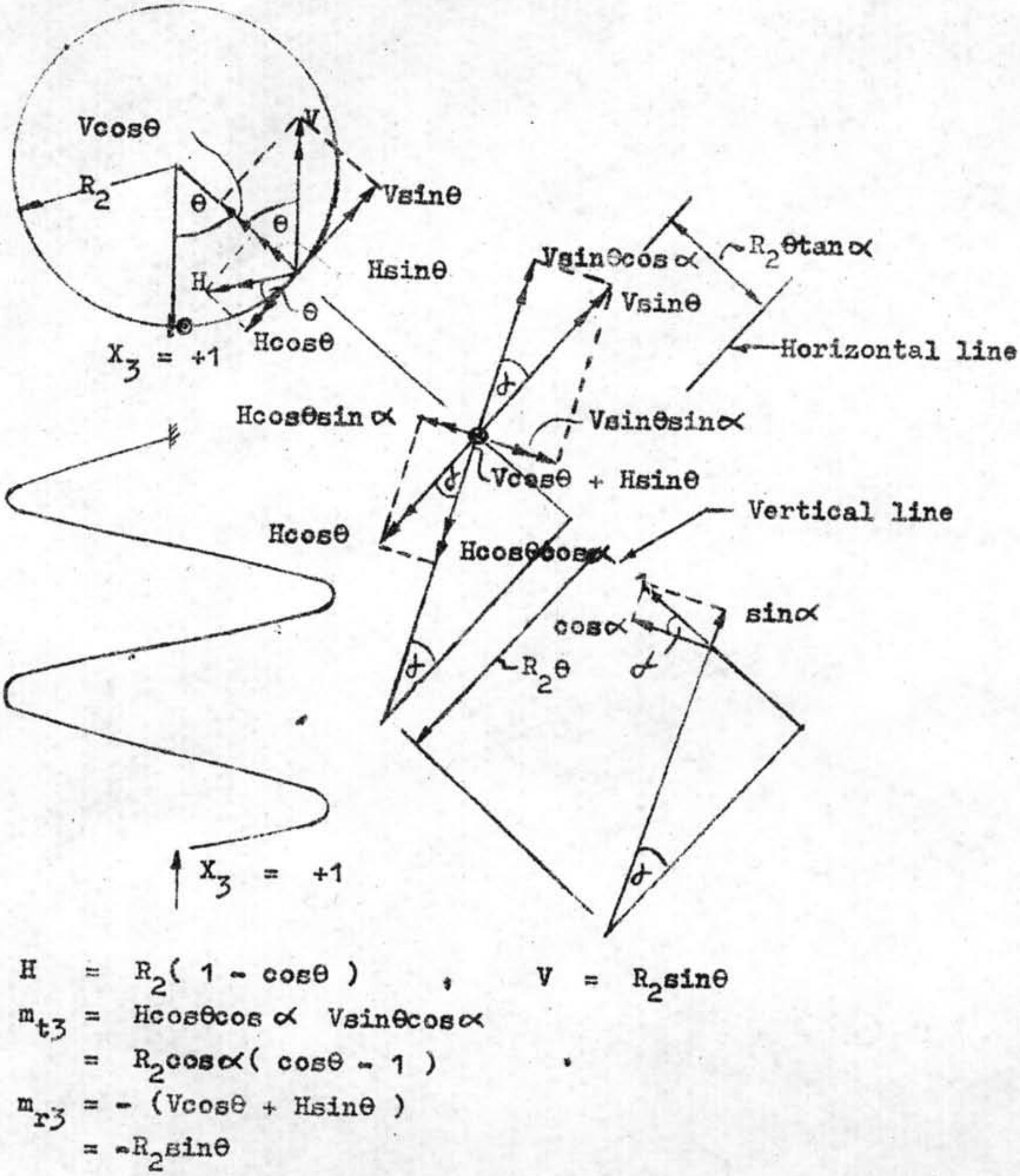
$$= R_2\cos\alpha(\theta\cos\theta\tan^2\alpha + \sin\theta)$$

$$Q_{t2} = \sin\theta\cos\alpha$$

$$Q_{r2} = \cos\theta$$

$$Q_{s2} = \sin\theta\sin\alpha$$

$X_3 = +1$ (all other forces are zero)



$H = R_2(1 - \cos\theta)$, $V = R_2 \sin\theta$

$m_{t3} = H\cos\theta\cos\alpha \quad V\sin\theta\cos\alpha$

$= R_2\cos\alpha(\cos\theta - 1)$.

$m_{r3} = -(V\cos\theta + H\sin\theta)$

$= -R_2\sin\theta$

$m_{s3} = H\cos\theta\sin\alpha - V\sin\theta\sin\alpha$

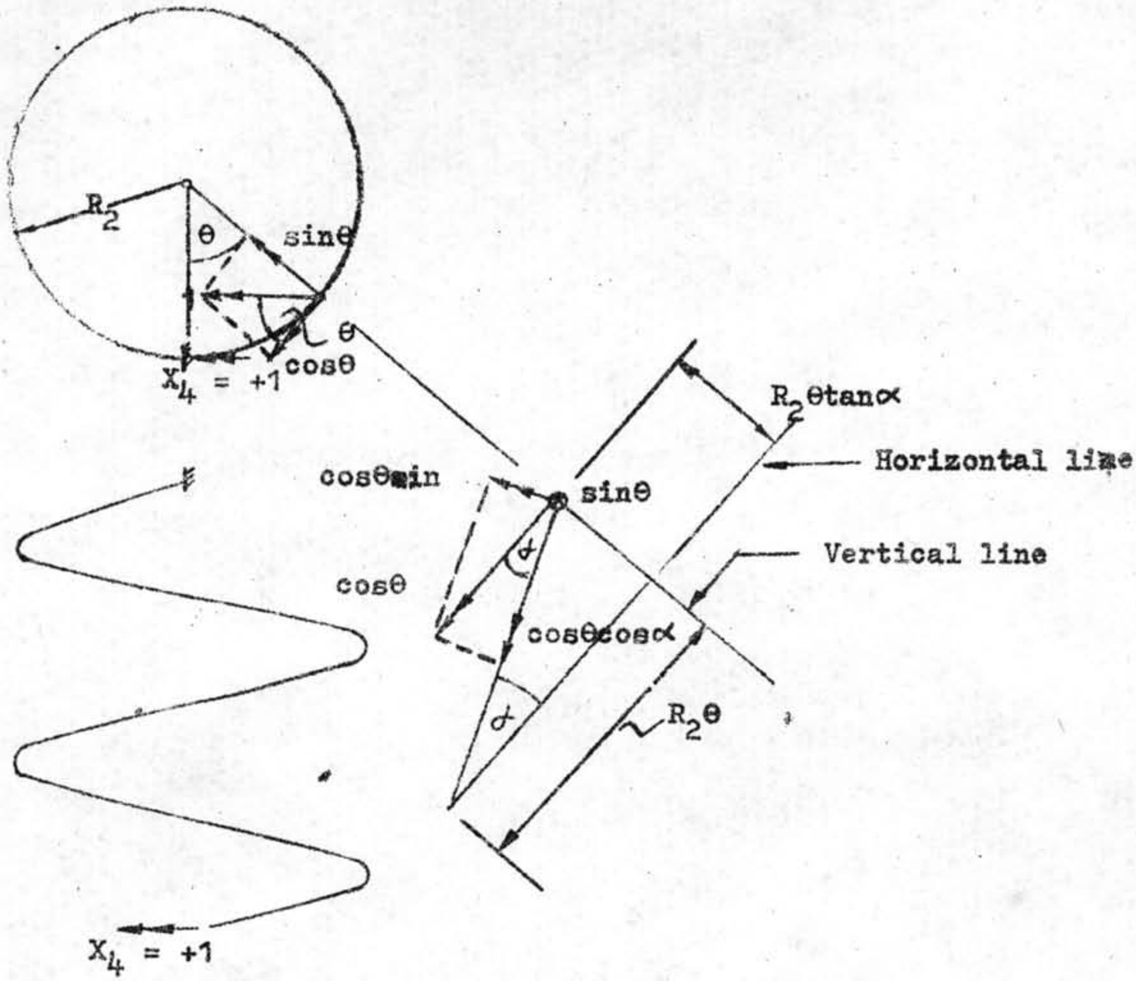
$= R_2\sin\alpha(\cos\theta - 1)$

$Q_{t3} = -\sin\alpha$

$Q_{r3} = 0$

$Q_{s3} = \cos\alpha$

$X_4 = +1$ (all other forces are zero)



$m_{t4} = \cos\theta\cos\alpha$

$m_{r4} = -\sin\theta$

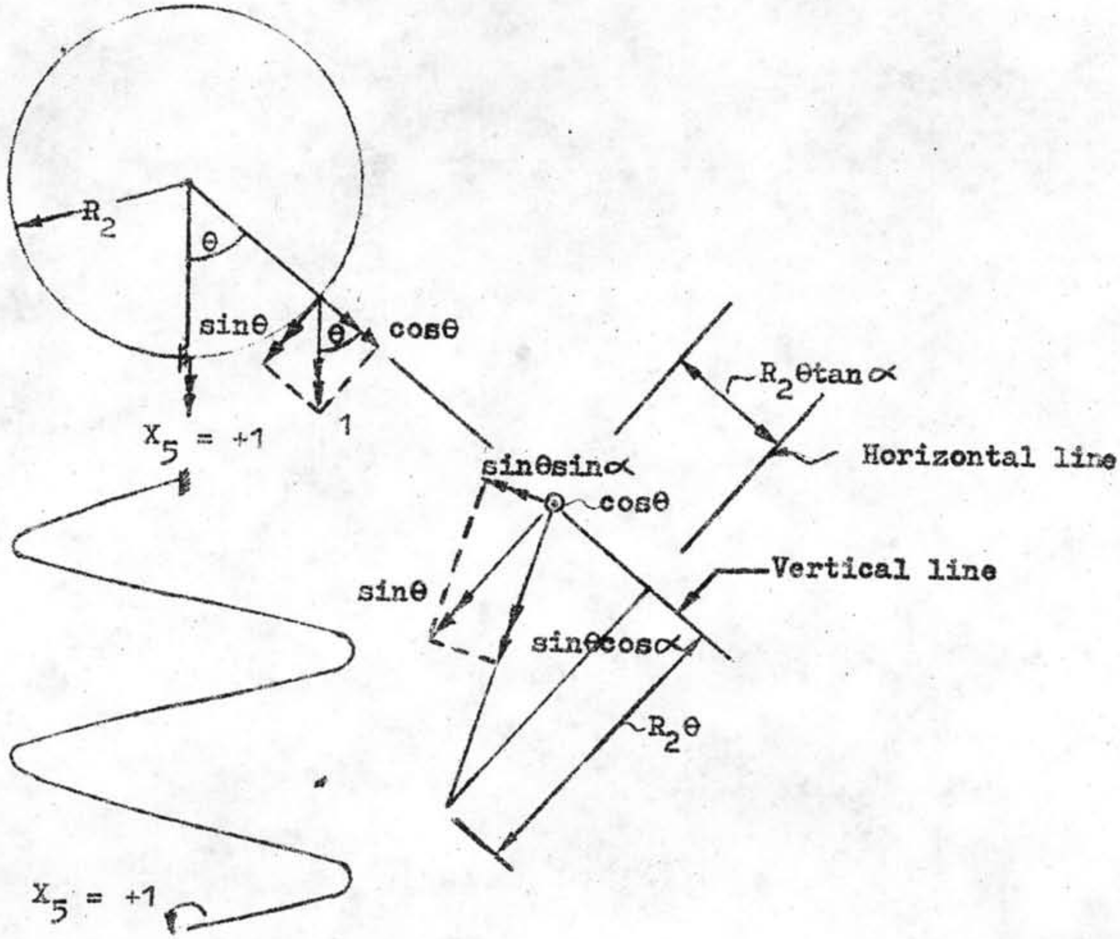
$m_{s4} = \cos\theta\sin\alpha$

$Q_{t4} = 0$

$Q_{r4} = 0$

$Q_{s4} = 0$

$X_5 = +1$ (all other forces are zero)



$m_{t5} = \sin\theta\cos\alpha$

$m_{r5} = \cos\theta$

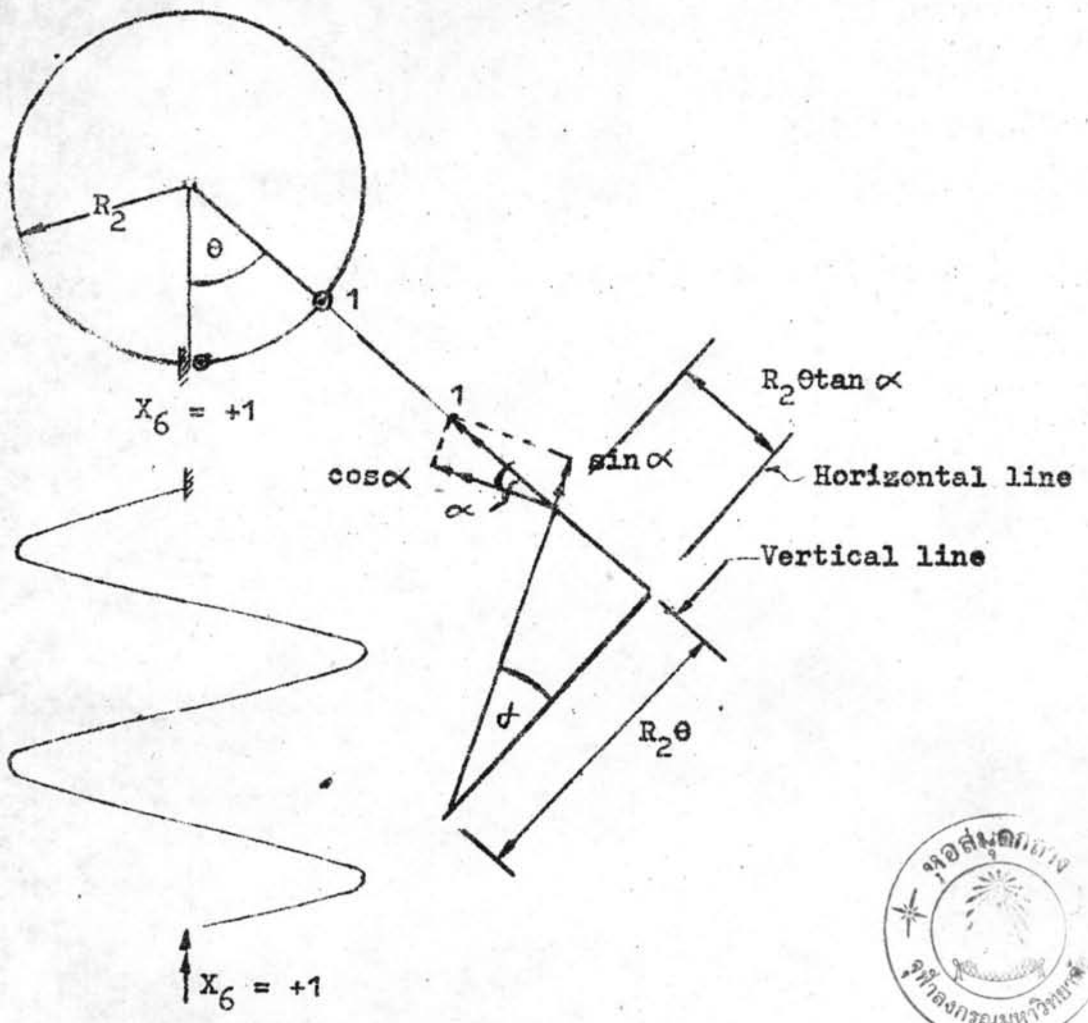
$m_{s5} = \sin\theta\sin\alpha$

$Q_{t5} = 0$

$Q_{r5} = 0$

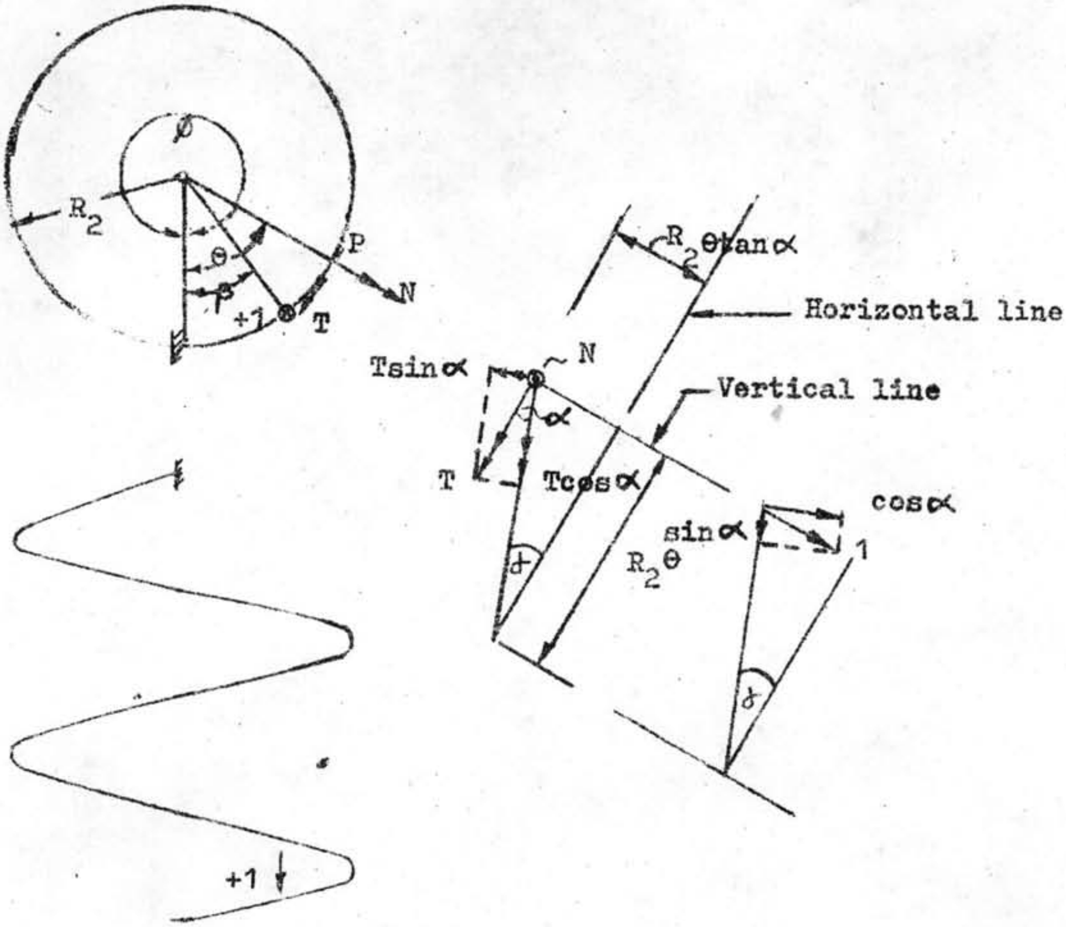
$Q_{s5} = 0$

$X_6 = +1$ (all other forces are zero)



$$m_{t6} = -\sin\alpha$$
$$m_{r6} = 0$$
$$m_{s6} = \cos\alpha$$
$$Q_{t6} = 0$$
$$Q_{r6} = 0$$
$$Q_{s6} = 0$$

Internal forces caused by unit vertical load applied at an angle β from the lower support



$$N = R_2 \sin(\theta - \beta) \quad , \quad T = R_2\ 1 - \cos(\theta - \beta)$$

Internal forces at any section

when $0 \leq \theta \leq \beta$

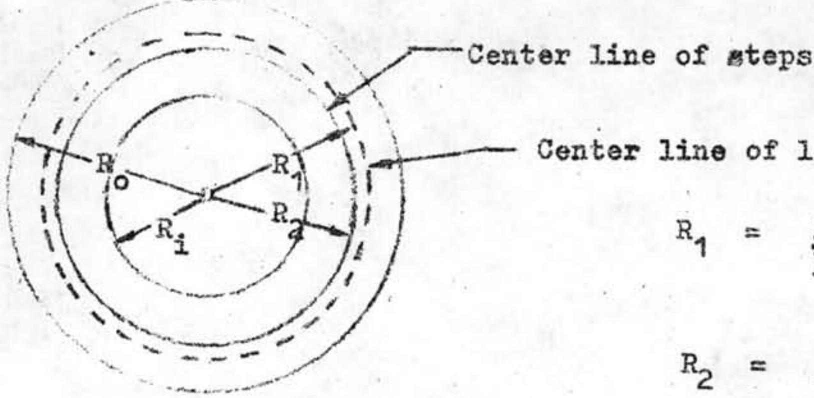
$$m_{tp} = m_{rp} = m_{sp} = 0$$
$$Q_{tp} = Q_{rp} = Q_{sp} = 0$$

when $\beta \leq \theta \leq \emptyset$

$$m_{tp} = T\cos\alpha$$
$$= R_2 \cos\alpha \left[1 - \cos(\theta - \beta)\right]$$
$$m_{rp} = N$$
$$= R_2 \sin(\theta - \beta)$$
$$m_{sp} = T\sin\alpha$$
$$= R_2 \sin\alpha \left[1 - \cos(\theta - \beta)\right]$$
$$Q_{tp} = \sin\alpha$$
$$Q_{rp} = 0$$
$$Q_{sp} = -\cos\alpha$$

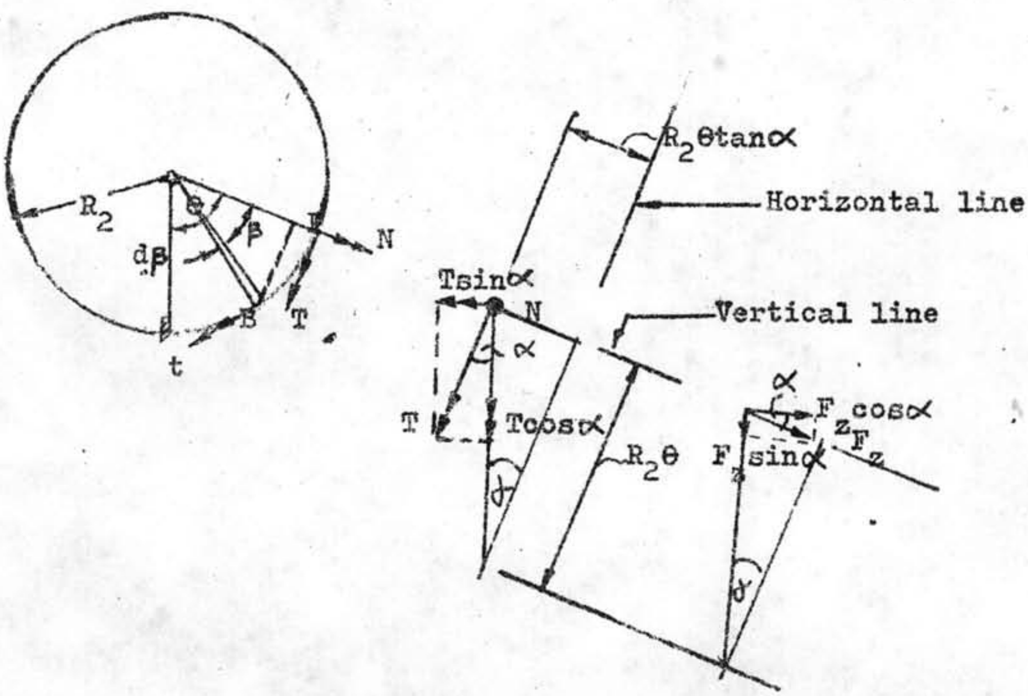
<u>Internal forces due to the uniform load per unit length of horizontal projection of center line of steps</u>

$W = +1$



$$R_1 = \frac{2}{3}\frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)}$$

$$R_2 = \frac{1}{2}(R_o + R_i)$$



Since the center of gravity of the uniform load is not coincided with the center line of steps. Therefore, any small element of the uniform load acting at B will have an additional torque of

$$t = -(R_1 - R_2)R_2 d\beta$$

So the element of uniform load at B causes a radial moment at P of

$$N = R_2^2 \sin\beta d\beta - t\sin\beta$$

and a horizontal-tangential moment at P of

$$T = R_2^2(1-\cos\beta)d\beta + t\cos\beta$$

and a vertical force at P of

$$F_z = R_2 d\beta$$

The moments and forces at P caused by any small element of the uniform load acting at B are given by

$$dm_{tw} = T\cos\alpha$$

$$= R_2^2\cos\alpha(1-\cos\beta)d\beta + t\cos\alpha\cos\beta$$

$$= R_2^2\cos\alpha(1- \frac{R_1}{R_2}\cos\beta)d\beta$$

$$dm_{rw} = N$$

$$= R_1R_2\sin\beta d\beta$$

$$dm_{sw} = T\sin\alpha$$

$$= R_2^2\sin\alpha(1-\frac{R_1}{R_2}\cos\beta)d\beta$$

$$dQ_{tw} = F_z\sin\alpha$$

$$= R_2 d\beta\sin\alpha$$

$$dQ_{rw} = 0$$

$$dQ_{sw} = -F_z d\beta\cos\alpha$$

Thus the moments and forces at P caused by a uniform load over the arc θ is given by

$$m_{tw} = R_2^2\cos\alpha\int_0^\theta(1- \frac{R_1}{R_2}\cos\beta)d\beta$$

$$= R_2^2\cos\alpha\,(\theta - \frac{R_1}{R_2}\sin\theta)$$

$$m_{rw} = R_1R_2\int_0^\theta\sin\beta d\beta$$

$$= R_1R_2(1 - \cos\theta)$$

$$m_{sw} = R_2^2\sin\alpha\int_0^\theta(1 - \frac{R_1}{R_2}\cos\beta)d\beta$$

$$= R_2^2\sin\alpha\,(\theta - \frac{R_1}{R_2}\sin\theta)$$

$$Q_{tw} = R_2\sin\alpha\int_0^\theta d\beta \qquad = R_2\theta\sin\alpha$$

$$Q_{rw} = 0$$

$$Q_{sw} = -R_2\cos\alpha\int_0^\theta d\beta \qquad = -R_2\theta\cos\alpha$$

VITA

Name of student	Mr. Sumit Dechasajja
Qualification	Bechelor of Engineering
Institute	Chulalongkorn University
Graduated year	1972