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APPENDIX

APPENDIX A

A.1 Stress-strain Relations for Plane Stress of Orthotropic Material[3]

The stress-strain relations for orthotropic laminae can be written in matrices as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} \qquad (A.1)$$

A plane stress state is defined by

$$\sigma_3 = 0, \quad \tau_{23} = 0, \quad \tau_{31} = 0$$

Therefore, the stress-strain relations above is reduced for plane stress as

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \qquad (A.2)$$

where the Q_{ij}, the so-called reduced stiffness, may be written in terms of the engineering constants as

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} \qquad (A.3)$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12}$$

The Poisson's ratio, $\nu_{ij} = -\varepsilon_j/\varepsilon_i$, is for transverse strain in the j-direction when stressed in the i-direction.

A.2 Resultant Moments for Orthotropic Material[3]

The implications of the Kirchhoff or the Kirchhoff-Love hypothesis on the laminate displacements u, v, and w in the x-, y-, and z-directions being that the displacement, u, at any point z through the laminate thickness is

$$u = u_o - zw_{,x} \qquad (A.4)$$

where u_o is the displacement of a point on the reference plane in the x-direction.

By similar reasoning, the displacement, v, in the y-direction is

$$v = v_o - zw_{,y} \qquad (A.5)$$

For small strains(linear elasticity), the strains are defined in terms of displacements as

$$\varepsilon_1 = \varepsilon_{xx} = u_{,x} = -zw_{,xx}$$

$$\varepsilon_2 = \varepsilon_{yy} = v_{,y} = -zw_{,yy} \tag{A.6}$$

$$\gamma_{12} = \gamma_{xy} = u_{,y} + v_{,x} = -2zw_{,xy}$$

From Eq.(A.2), stresses, in terms of displacements, are

$$\sigma_1 = \sigma_{xx} = Q_{11}\varepsilon_1 + Q_{12}\varepsilon_2 = -\frac{zE_1}{1 - \nu_{12}\nu_{21}}(w_{,xx} + \nu_{21}w_{,yy})$$

$$\sigma_2 = \sigma_{yy} = Q_{12}\varepsilon_1 + Q_{22}\varepsilon_2 = -\frac{zE_2}{1 - \nu_{12}\nu_{21}}(w_{,yy} + \nu_{12}w_{,xx}) \tag{A.7}$$

$$\tau_{12} = \tau_{xy} = G_{12}\gamma_{12} = -2G_{12}zw_{,xy}$$

The moment resultants acting on a laminate are obtained by integration of the stresses in each layer or lamina through the laminate thickness,

$$\begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} \cdot zdz \tag{A.8}$$

That is,

$$M_{xx} = \int_{-h/2}^{h/2} \sigma_{xx}\, zdz$$

$$= -\int_{-h/2}^{h/2} \frac{z^2 E_1}{1 - \nu_{12}\nu_{21}}(w_{,xx} + \nu_{21}w_{,yy})dz$$

$$= -\frac{E_1 h^3}{12(1 - \nu_{12}\nu_{21})}(w_{,xx} + \nu_{21}w_{,yy})$$

$$= -D_x(w_{,xx} + \nu_{21}w_{,yy})$$

Similarly,

$$M_{yy} = -D_y(w_{,yy} + \nu_{12} w_{,xx})$$

$$M_{xy} = -D_{xy} w_{,xy}$$

where $D_x = \dfrac{E_1 h^3}{12(1 - \nu_{12}\nu_{21})}$

$$D_y = \frac{E_2 h^3}{12(1 - \nu_{12}\nu_{21})}$$

$$D_{xy} = \frac{G_{12} h^3}{6}$$

Therefore,

$$\sigma_{xx} = -\frac{12z}{h^3}(D_x w_{,xx} + D_1 w_{,yy})$$

$$\sigma_{yy} = -\frac{12z}{h^3}(D_y w_{,yy} + D_1 w_{,xx}) \qquad (A.9)$$

$$\tau_{xy} = -\frac{12z}{h^3} D_{xy} w_{,xy}$$

APPENDIX B

GALERKIN'S METHOD [15]

This method was developed by I.G. Bubnov and applied by B.G. Galerkin for the series solution of some problems of engineering mechanics. Consider the variation of the following equation,

$$\iint_A L \cdot \delta w \cdot dx \cdot dy = 0 \tag{B.1}$$

in which the quantity L is the expression in term of w and their derivatives. The most effective way to integrate the fundamental differential equations of the theory of flexible plates is to represent the deflection $w(x,y)$ in the form of a series

$$w = c_1\eta_1 + c_2\eta_2 + \ldots\ldots\ldots + c_n\eta_n = \sum_{i=1}^{n} c_i\eta_i \tag{B.2}$$

where η_i are the selected quantities, independent of each other and are functions of the coordinates x and y,

c_i are some parametric quantities to be determined.

Making use of expression (B.2), δw is expressed in terms of the variation of the parameters c_i

$$\delta w = \sum_{i=1}^{n} \eta_i \delta c_i \tag{B.3}$$

Substitute series (B.3) into expression (B.1), then

$$\sum_{i=1}^{n} \int_A \int L \cdot \eta_i \, \delta c_i \cdot dx \cdot dy = 0 \tag{B.4}$$

But the variations δc_i are arbitrary, consequently, equation(B.4) is satisfied if each equation of the following type is separately satisfied

$$\int_A \int L \cdot \eta_i \, dx \cdot dy = 0 \quad ; \quad i = 1, 2, \ldots\ldots, n \tag{B.5}$$

After integration with respect to x and y we obtain a set of algebraic equations containing the unknown parameters c_i. The number of these equations is equal to n. Solving this system of equations, the parameters c_i are found. In buckling problem, L is the buckling equation and the determination of c_i is indeterminate which results in a set of equations whose eigen value is the critical buckling load.

APPENDIX C

Appendix C. shows the flow chart and the computer program used in the computations where

$$\begin{aligned} \text{UN} &= \nu_{\bullet} \\ \text{AN} &= n \\ \text{AK} &= k \\ \text{AJ} &= \beta \\ \text{AL} &= \alpha \\ \text{ALAMDA(N,L)} &= \lambda(n,\alpha) \end{aligned}$$

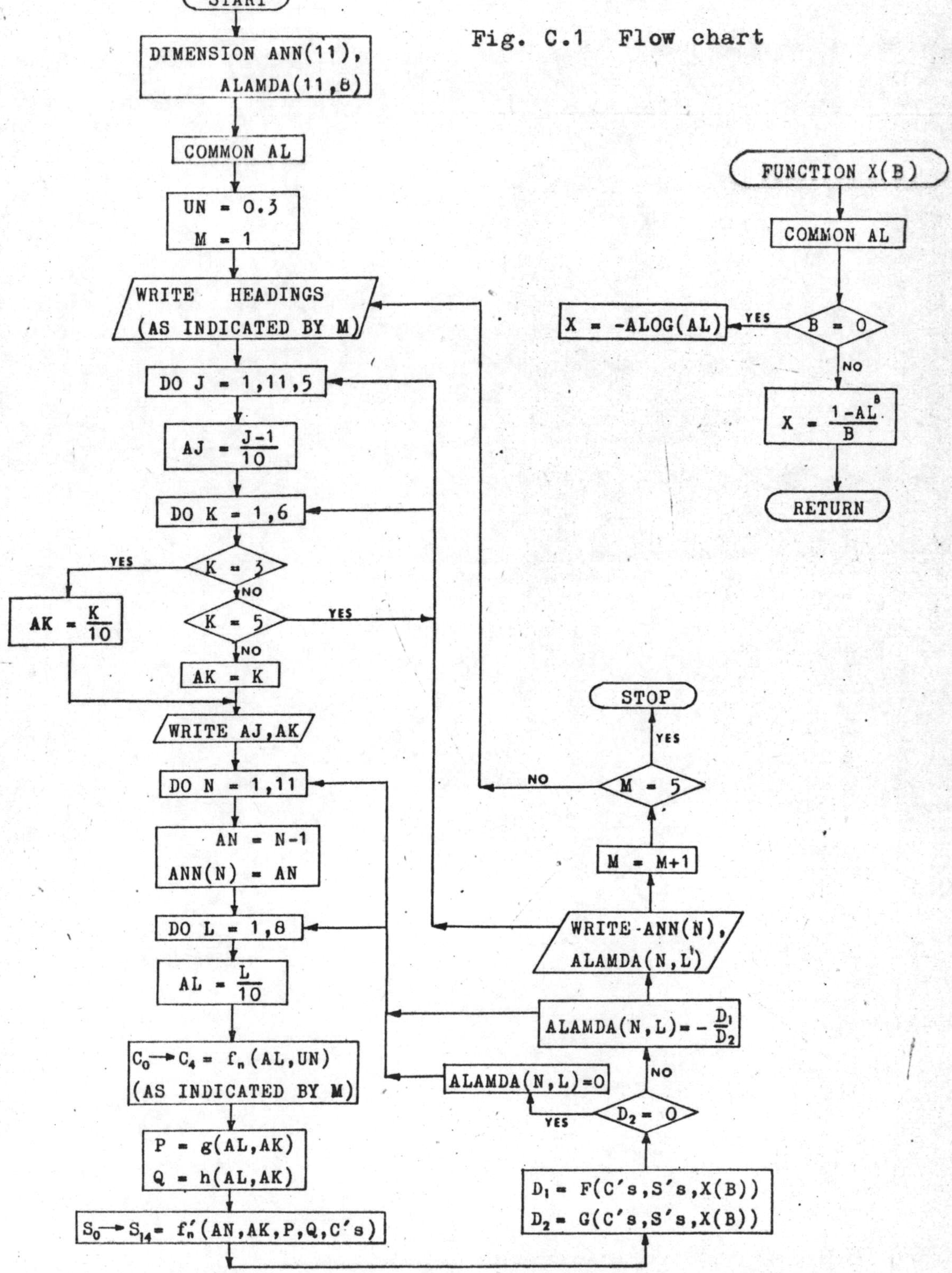

Fig. C.1 Flow chart

```
C B015583 ATHIKOM BANGVIWAT
C TO FIND THE CRITICAL LOADS FOR UNSYMMETRIC BUCKLING PLATES.
      DIMENSION ANN(11),ALAMDA(11,8)
      COMMON AL
      UN = 0.3
      M = 1
  100 WRITE(3,250)
      GO TO 140
  110 WRITE(3,260)
      GO TO 140
  120 WRITE(3,270)
      GO TO 140
  130 WRITE(3,280)
  140 DO 240 J=1,11,5
      AJ = J
      AJ = (AJ-1.)/10.
      DO 240 K=1,6
      IF(K.EQ.3) GO TO 150
      IF(K.EQ.5) GO TO 240
      AK = K
      GO TO 160
  150 AK=K
      AK = AK/10.
  160 WRITE(3,290) AJ, AK
      DO 230 N=1,11
      AN = N
      AN = AN-1.
      ANN(N) = AN
      DO 230 L=1,8
      AL = L
      AL = AL/10.
      GO TO (170,180,190,200),M
  170 C0 = 1.
      C1 = -C0*2.*(AL**8-2.*AL**6+2.*AL**2-1.)/(AL**8-3.*AL**6
     *       +3.*AL**4-AL**2)
      C2 = -C0*(3.*AL**8-4.*AL**6+1.)/(AL**8-2.*AL**6+AL**4)-C
     *       1*(2.*AL**6-3.*AL**4+1.)/(AL**6-2.*AL**4+AL**2)
      C3 = -C0*(AL**8-1.)/(AL**8-AL**6)-C1*(AL**6-1.)/(AL**6-A
     *       L**4)-C2*(AL**4-1.)/(AL**4-AL**2)
      C4 = -C0-C1-C2-C3
      GO TO 210
  180 C0 = 1.
      C1 = -C0*2.*((13.+UN)*AL**10-(33.+3.*UN)*AL**8+(18.+2.*U
     *       N)*AL**6+(14.+2.*UN)*AL**4-(15.+3.*UN)*AL**2+(3.+UN
     *       ))/((13.+UN)*AL**10-(44.+4.*UN)*AL**8+(54.+6.*UN)*A
     *       L**6-(28.+4.*UN)*AL**4+(5.+UN)*AL**2)
      C2 = -C0*(3.*AL**8-4.*AL**6+1.)/(AL**8-2.*AL**6+AL**4)-C
     *       1*(2.*AL**6-3.*AL**4+1.)/(AL**6-2.*AL**4+AL**2)
      C3 = -C0*(AL**8-1.)/(AL**8-AL**6)-C1*(AL**6-1.)/(AL**6-A
     *       L**4)-C2*(AL**4-1.)/(AL**4-AL**2)
      C4 = -C0-C1-C2-C3
      GO TO 210
  190 C0 = 1.
      C1 = -C0*2.*((3.+UN)*AL**10-(15.+3.*UN)*AL**8+(14.+2.*UN
```

```
     *     )*AL**6+(18.+2.*UN)*AL**4-(33.+3.*UN)*AL**2+(13.+UN
     *     ))/((5.+UN)*AL**10-(28.+4.*UN)*AL**8+(54.+6.*UN)*AL
     *     **6-(44.+4.*UN)*AL**4+(13.+UN)*AL**2)
      C2 = -C0*(AL**8-4.*AL**2+3.)/(AL**8-2.*AL**6+AL**4)-C1*(
     *     AL**6-3.*AL**2+2.)/(AL**6-2.*AL**4+AL**2)
      C3 = -C0*(AL**8-1.)/(AL**8-AL**6)-C1*(AL**6-1.)/(AL**6-A
     *     L**4)-C2*(AL**4-1.)/(AL**4-AL**2)
      C4 = -C0-C1-C2-C3
      GO TO 210
  200 C0 =1.
      C1 = -C0*2.*((39.+16.*UN+UN**2)*AL**10-(165.+48.*UN+3.*U
     *     N**2)*AL**8+(126.+32.*UN+2.*UN**2)*AL**6+(126.+32.*
     *     UN+2.*UN**2)*AL**4-(165.+48.*UN+3.*UN**2)*AL**2+(39
     *     .+16.*UN+UN**2))/((65.+18.*UN+UN**2)*AL**10-(308.+7
     *     2.*UN+4.*UN**2)*AL**8+(486.+108.*UN+6.*UN**2)*AL**6
     *     -(308.+72.*UN+4.*UN**2)*AL**4+(65.+18.*UN+UN**2)*AL
     *     **2)
      C2 = -C0*((15.+3.*UN)*AL**8-(28.+4.*UN)*AL**6+(13.+UN))/
     *     ((9.+UN)*AL**8-(22.+2.*UN)*AL**6+(13.+UN)*AL**4)-C1
     *     *((14.+2.*UN)*AL**6-(27.+3.*UN)*AL**4+(13.+UN))/((9
     *     .+UN)*AL**6-(22.+2.*UN)*AL**4+(13.+UN)*AL**2)
      C3 = -C0*(AL**8-1.)/(AL**8-AL**6)-C1*(AL**6-1.)/(AL**6-A
     *     L**4)-C2*(AL**4-1.)/(AL**4-AL**2)
      C4 = -C0-C1-C2-C3
  210 P  =-(1.-AJ*AL**(AK+1.))/(1.-AL**(2.*AK))
      Q  = AL**(AK+1.)*(AL**(AK-1.)-AJ)/(1.-AL**(2.*AK))
      S0 =-C0*(2.*AN**2+2.*AN**2*AK**2-AN**4*AK**2)
      S1 =-C1*(2.*AN**2+2.*AN**2*AK**2-AN**4*AK**2)
      S2 = C2*(72.-8.*AK**2-18.*AN**2-2.*AN**2*AK**2+AN**4*AK*
     *     *2)
      S3 = C3*(600.-24.*AK**2-50.*AN**2-2.*AN**2*AK**2+AN**4*A
     *     K**2)
      S4 = C4*(2352.-48.*AK**2-98.*AN**2-2.*AN**2*AK**2+AN**4*
     *     AK**2)
      S5 = C0*AN**2*AK*P
      S6 =-C0*AN**2*AK*Q
      S7 = C1*(AN**2*AK-2.*AK-2.)*P
      S8 =-C1*(AN**2*AK-2.*AK+2.)*Q
      S9 = C2*(AN**2*AK-4.*AK-12.)*P
      S10=-C2*(AN**2*AK-4.*AK+12.)*Q
      S11= C3*(AN**2*AK-6.*AK-30.)*P
      S12=-C3*(AN**2*AK-6.*AK+30.)*Q
      S13= C4*(AN**2*AK-8.*AK-56.)*P
      S14=-C4*(AN**2*AK-8.*AK+56.)*Q
      D1 = S0*C0*X(-3.)+(S1*C0+S0*C1)*X(-1.)+(S2*C0+S1*C1+S0*C
     *     2)*X(1.)+(S3*C0+S2*C1+S1*C2+S0*C3)*X(3.)+(S4*C0+S3*
     *     C1+S2*C2+S1*C3+S0*C4)*X(5.)+(S4*C1+S3*C2+S2*C3+S1*C
     *     4)*X(7.)+(S4*C2+S3*C3+S2*C4)*X(9.)+(S4*C3+S3*C4)*X(
     *     11.)+S4*C4*X(13.)
      D2 = S5*C0*X(AK-2.)+S6*C0*X(-AK-2.)+(S7*C0+S5*C1)*X(AK)+
     *     (S8*C0+S6*C1)*X(-AK)+(S9*C0+S7*C1+S5*C2)*X(AK+2.)+(
     *     S10*C0+S8*C1+S6*C2)*X(-AK+2.)+(S11*C0+S9*C1+S7*C2+S
     *     5*C3)*X(AK+4.)+(S12*C0+S10*C1+S8*C2+S6*C3)*X(-AK+4.
     *     )+(S13*C0+S11*C1+S9*C2+S7*C3+S5*C4)*X(AK+6.)+(S14*C
```

```
     *      0+S12*C1+S10*C2+S8*C3+S6*C4)*X(-AK+6.)+(S13*C1+S11*
     *      C2+S9*C3+S7*C4)*X(AK+8.)+(S14*C1+S12*C2+S10*C3+S8*C
     *      4)*X(-AK+8.)+(S13*C2+S11*C3+S9*C4)*X(AK+10.)+(S14*C
     *      2+S12*C3+S10*C4)*X(-AK+10.)+(S13*C3+S11*C4)*X(AK+12
     *      .)+(S14*C3+S12*C4)*X(-AK+12.)+S13*C4*X(AK+14.)+S14*
     *      C4*X(-AK+14.)
      IF(D2.EQ.0.) GO TO 220
      ALAMDA(N,L)=-D1/D2
      GO TO 230
  220 ALAMDA(N,L) = 0.
  230 CONTINUE
      WRITE(3,300)
      WRITE(3,310) (ANN(N),(ALAMDA(N,L),L=1,8),N=1,11)
  240 CONTINUE
      M = M+1
      IF(M.EQ.5) STOP
      GO TO (100,110,120,130),M
  250 FORMAT(//40X,'CRITICAL LOADS FOR UNSYMMETRIC BUCKLING'/,
     *40X,'OF POLAR ORTHOTROPIC ANNULAR PLATES.',//20X,'CASE',
     *' : BOYH EDGES ARE CLAMPED.',//)
  260 FORMAT(//40X,'CRITICAL LOADS FOR UNSYMMETRIC BUCKLING'/,
     *40X,'OF POLAR ORTHOTROPIC ANNULAR PLATES.',//20X,'CASE',
     *' : INNER EDGE IS SIMPLY SUPPORTED',/27X,'OUTER EDGE I',
     *'S CLAMPED.',//)
  270 FORMAT(//40X,'CRITICAL LOADS FOR UNSYMMETRIC BUCKLING'/,
     *40X,'OF POLAR ORTHOTROPIC ANNULAR PLATES.',//20X,'CASE',
     *' : INNER EDGE IS CLAMPED',/27X,'OUTER EDGE IS SIMPLY ',
     *'SUPPORTED.',//)
  280 FORMAT(//40X,'CRITICAL LOADS FOR UNSYMMETRIC BUCKLING'/,
     *40X,'OF POLAR ORTHOTROPIC ANNULAR PLATES.',//20X,'CASE',
     *' : BOTH EDGES ARE SIMPLY SUPPORTED.',//)
  290 FORMAT(/////28X,'PRESSURE RATIO : PI/PO = ',F5.1,/28X,
     *'RIGIDITY RATIO :      K = ',F5.1,/)
  300 FORMAT(42X,'LAMDA : CRITICAL LOAD PARAMETER',/16X,'*',
     *42X,'A/B',/12X,'N',3X,'*',7X,'0.1',8X,'0.2',8X,'0.3',8X,
     *'0.4',8X,'0.5',8X,'0.6',8X,'0.7',8X,'0.8',/10X,49('* '))
  310 FORMAT(10X,F4.0,2X,'*',1X,8F11.1)
      STOP
      END


C     FUNCTION SUBPROGRAM
      FUNCTION  X(B)
      COMMON AL
      IF(B) 320,330,320
  320 X = (1.-AL**B)/B
      RETURN
  330 X = -ALOG(AL)
      RETURN
      END
```

AUTOBIOGRAPHY

Mr. Athikom Bangviwat, born on Nov. 15, 1954, in Bangkok, Thailand, finished his pre-university studies at Trium Udom Secondary School. Intending for the engineering, he pursued in the Department of Mechanical Engineering, Chulalongkorn University where he received, in the first year, an honour certificate for good study record in 1974. Having formally completed the B.Eng. in 1977, he was further enrolled in the master degree program.