การโก่งงอของแผ่นวงแหวนทำด้วยวัสดุแบบออโธทรอปปิค

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BUCKLING OF ORTHOTROPIC ANNULAR PLATES

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บทคัดย่อ

บทความนี้เป็นการวิเคราะห์ปัญหาเกี่ยวกับการโก่งงอของแผ่นวงแหวนกลมบางทำด้วยวัสดุแบบออโธทรอปปิค(orthotropic)ซึ่งรับแรงกดในแนวรัศมีที่ขอบนอกและขอบในอย่างสม่ำเสมอ การรองรับที่ขอบนอกและขอบในของแผ่นวงแหวนเป็นแบบยึดแน่นและแบบรองรับธรรมดาสลับกันรวมได้ทั้งสิ้น ๔ กรณี สมการแสดงการโก่งงอได้จากการใช้วิธีวาริเอชั่น(variational method) การแก้สมการแสดงการโก่งงอกระทำได้โดยการใช้วิธีของกาเลอร์คิน(Galerkin's method)พร้อมด้วยโพลีโนเมียล(Polynomial)ยกกำลัง ๘ ซึ่งเป็นไปตามเงื่อนไขที่ขอบทุกกรณี ผลของการวิเคราะห์แสดงเป็นกราฟอยู่ในรูปของตัวแปรไร้มิติซึ่งมีค่าต่างๆกันของอัตราส่วนแรงกดที่ขอบ อัตราส่วนของรัศมีภายในและภายนอก และอัตราส่วนของความแข็งเกร็ง(rigidity ratio) ผลการวิจัยแสดงว่าการโก่งงอเกือบทั้งหมดเป็นแบบไม่มีสมมาตรกับจุดศูนย์กลางและจำนวนคลื่นในแนวเส้นรอบวงเพิ่มขึ้นเมื่ออัตราส่วนของรัศมีเพิ่มขึ้น

Thesis Title Buckling of Orthotropic Annular Plates

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ABSTRACT

Buckling of thin polar orthotropic annular plates subjected to uniform in-plane radial edge pressures has been analyzed for all possible combinations of clamped and simply supported edge conditions. The variational method is used for deriving the buckling equation. The Galerkin's method with simple polynomial of eight order satisfying all boundary conditions as admissible functions is employed in the analysis. Results are represented in the dimensionless parameters for various ratios of edge pressures, inner to outer radii, and rigidity ratios. The important conclusion is that most of buckling modes are non-symmetric and that number of the circumferential half-waves increases with the increase in the radius ratio.

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NOTATION

A	=	area;
D_x , D_y , D_r , D_θ	=	flexural rigidities in x, y, r, and θ directions, respectively;
D_{xy} , $D_{r\theta}$	=	shear rigidities;
D_1	=	$\nu_\theta D_r$;
E	=	modulus of elasticity in tension and compression;
G	=	modulus of elasticity in shear;
I	=	total potential energy;
M_{xx} , M_{yy}	=	bending moments per unit length about x and y-axes, respectively;
M_{xy}	=	twisting moment per unit length;
P	=	surface pressure load;
T	=	potential energy of deformation of the middle surface;
U	=	potential energy of deformation;
V	=	potential energy of deformation due to bending;
W	=	potential energy of loading;
a	=	inner radius of the plate;
b	=	outer radius of the plate;

h	=	plate thickness;
k	=	rigidity ratio $(= \sqrt{D_\theta/D_r})$;
n	=	number of half-waves on the circumference;
u, v, w	=	displacements in x, y, and z directions, respectively;
x, y, z	=	cartesian ares;
α	=	radius ratio $(= a/b)$;
β	=	pressure ratio $(= P_i/P_o)$;
σ	=	normal stress;
τ	=	shear stress;
ε	=	normal strain;
γ	=	shear strain;
ν	=	Poisson's ratio;
λ	=	dimensionless critical load parameter.

Subscripts

cr	=	critical value;
i	=	at inner edge;
o	=	at outer edge;
r	=	in radial direction.