CHAPTER V

RESULTS

5.1 Paired t-Test

The paired t-tests are conducted to see if the market price and the theoretical price from each model are significantly different. Table5.1 represents the test results.

In general, there are significant differences between all the model prices and the market prices. All the models tend to overprice both series A and series B warrants and the degree of overpricing changes from model to model. The mean theoretical price of the Galai-Schneller model has the highest value whereas the mean theoretical price of the Dennis-Rendleman model has the lowest value for both of warrants series. For series A warrants, the Darsinos-Satchell model has higher mean model price than the Lim-Terry model. For series B warrants, it is unclear to justify the second and the third ranks. In general, the results entail that the standard model underestimates the warrant price more than the multiple warrants models.

5.2 Pricing Error Statistics

The results for MAE, MAPE, and RMSE are as shown in table 5.2 and 5.3.

5.2.1 Mean Absolute Error (MAE)

For series A warrants (except for SICCO, STEC, and SPALI), the Dennis-Rendleman model has the lowest MAE, followed by the Lim-Terry model, the Darsinos-Satchell model, and the Galai-Schneller model, respectively. For series B warrants, the Galai-Schneller model still has the highest value for all firms. The Dennis-Rendleman model has the lowest value for eight firms whereas the second and the third ranks are robust.

5.2.2 Mean Absolute Percentage Error (MAPE)

For series A warrants, the results are slightly different to those of MAEs. The Galai-Schneller model and the Darsinos-Satchell model have the highest and the second highest MAPEs for 8 firms. For 6 out of 10 firms, the Dennis-Rendleman model and

Table 5.1: Paired t-Statistics

Series A Warrant

	Statistics	Market Price	GS	LT	DS	DR
APC	mean	0.8384	2.0809	1.0582	1.9690	0.9493
	t-stat		18.39*	13.37*	18.80*	7.12*
BTC	mean	1.6142	4.5679	1.8719	4.0192	1.8111
	t-stat		22.02*	10.66*	20.99*	9.48*
ESTAR	mean	0.2641	0.4858	0.3391	0.4781	0.3144
	t-stat		39.47*	28.40*	38.85*	20.98*
JAS	mean	0.2569	0.6701	0.3248	0.6414	0.2793
	t-stat		30.46*	18.73*	29.98*	8.12*
QH	mean	0.4356	0.7478	0.5112	0.7347	0.4784
	t-stat		42.43*	18.25*	41.37*	12.00*
QH2	mean	0.3296	0.5154	0.3629	0.4987	0.3349
	t-stat		51.29*	23.21*	48.95*	4.19*
SICCO	mean	3.3296	3.5335	3.0043	3.5184	2.9019
	t-stat		7.32*	-14.36*	6.83*	-18.95*
SPORT	mean	2.0756	2.8110	2.2248	2.7562	2.1986
	t-stat		34.84*	19.91*	34.10*	16.44*
STEC	mean	10.3904	12.0370	11.1145	12.0342	11.1370
	t-stat		24.85*	11.94*	24.82*	12.27*
SPALI	mean	1.6302	1.3410	1.1640	1.3366	1.1500
	t-stat		-31.99*	-79.59*	-32.81*	-83.18*

Series B Warrant

	Statistics	Market Price	GS	LT	DS	DR
APC	mean	0.7951	2.1132	1.1132	1.3534	1.0574
	t-stat		21.08*	21.02*	22.18*	16.30*
BTC	mean	1.1703	3.8290	1.4300	1.6830	1.2771
	t-stat		21.04*	8.61*	10.98*	4.14*
ESTAR	mean	0.1723	0.3617	0.2233	0.2582	0.2177
	t-stat		29.20*	14.64*	19.91*	12.82*
JAS	mean	0.3433	0.7592	0.407	0.5268	0.4003
	t-stat		32.46*	23.29*	23.87*	28.19*
QH	mean	0.6620	1.1451	0.8780	0.8718	0.7853
	t-stat		41.64*	28.74*	25.97*	19.18*
QH2	mean	0.3645	0.5516	0.3864	0.3791	0.3456
	t-stat		60.53*	8.93*	5.97*	-7.77*
SICCO	mean	4.0990	4.9920	4.4028	4.3836	4.2367
	t-stat		26.94*	11.35*	10.70*	5.12*
SPORT	mean	0.9261	1.2589	0.7892	0.8059	0.5760
	t-stat		9.88*	-6.44	-5.36	-19.25*
STEC	mean	3.7524	3.9401	3.4047	3.4050	3.3147
	t-stat		2.43*	-5.71	-5.70	-7.19*
SPALI	mean	1.2002	1.5452	1.3652	1.3497	1.2375
	t-stat		22.77*	14.50*	13.37*	3.85*

* Significant at the 5% level for a one-tail test.

the Lim-Terry model have the lowest and the second lowest MAPEs. The results are similar to those of MAE for series B warrants.

5.2.3 Root Mean Square Error (RMSE)

The results resemble to those of MAEs. The results of series A warrants show that the Galai-Schneller model has the highest RMSE, followed by the Darsinos-Satchell model, the Lim-Terry model and the Dennis-Rendleman model. For series B warrant, unlike series A results, the Dennis-Rendelman model has the lowest RMSE. The Galai-Schneller model underperforms all other models.

Upon closer inspection of pricing error statistics of series A, the degree of mispricing does not differ much when comparing the Galai-Schneller model with the Darsinos-Satchell model and the Lim-Terry model with the Dennis-Rendleman model. However, when comparing the first pair with the second pair, there is much difference in mispricing warrants. For series B warrants, the statistics from the Lim-Terry model, the Dennis-Rendleman model and the Darsinos-Satchell model do not differ much. Nevertheless, the degree of mispricing from the Galai-Schneller model is much higher than the degree of mispricing from the other three models.

5.3 Analysis of Pricing Errors

MAPEs are further examined to understand more about the behavior of each model. The paired t-tests are also conducted here to compare MAPE between each model and see if it is significantly larger than each other. The results are reported in table 5.4.

For both series, the Lim-Terry model and the Darsinos-Satchell model provide best estimates when warrants are in-the-money. The Galai-Schneller model, in turn, produces the most accurate approximation for at-the-money warrants. The Darsinos-Satchell model performs best when series A warrants are at-the-money and series B warrants are out-of-money.

From a ranking perspective, the Dennis-Rendleman model has the lowest MAPE, followed by the Lim-Terry model, the Darsinos-Satchell model, and the Galai-Schneller

Table 5.2: Pricing Error Statistics of Series A Warrant

		MAE				MAPE				RMSE			
		GS	LT	DS	DR	GS	LT	DS	DR	GS	LT	DS	DR
APC	mean	1.2425	0.2877	1.1306	0.2623	1.1624	0.2889	1.0714	0.3068	1.9171	0.4180	1.7228	0.3543
	SD	1.4615	0.3035	1.3013	0.2385	0.6474	0.1816	0.5969	0.1145	7.2266	0.3118	5.7138	0.2361
BTC	mean	2.9541	0.3891	2.4066	0.3503	1.4163	0.2061	1.1316	0.1964	4.3158	0.6229	3.6065	0.5253
	SD	3.1491	0.4869	2.6886	0.3918	1.1016	0.1847	0.9699	0.1504	29.9978	0.8072	22.0310	0.5406
ESTAR	mean	0.2216	0.0782	0.2140	0.0591	0.7673	0.2624	0.7388	0.1985	0.2639	0.1007	0.2560	0.0791
	SD	0.1434	0.0635	0.1406	0.0526	0.3293	0.1673	0.3265	0.1362	0.0794	0.0137	0.0752	0.0102
JAS	mean	0.4133	0.0801	0.3845	0.0537	1.4673	0.2630	1.3530	0.1879	0.5427	0.1160	0.5084	0.0749
	SD	0.3520	0.0840	0.3328	0.0523	0.4826	0.1588	0.4678	0.1149	0.5726	0.0262	0.4963	0.0119
QH	mean	0.3122	0.0813	0.2991	0.0577	1.0227	0.3170	0.9781	0.2279	0.3400	0.1072	0.3271	0.0781
	SD	0.1349	0.0699	0.1325	0.0528	1.2464	0.5567	1.1861	0.4591	0.0928	0.0170	0.0879	0.0101
QH2	mean	0.1858	0.0403	0.1690	0.0268	0.6070	0.1520	0.5452	0.1131	0.2100	0.0511	0.1931	0.0341
	SD	0.0981	0.0315	0.0935	0.0212	0.3972	0.2329	0.3540	0.2094	0.0440	0.0039	0.0383	0.0017
SICCO	mean	0.5298	0.5239	0.5237	0.5730	0.2147	0.2071	0.2125	0.2196	0.6942	0.6300	0.6848	0.6869
	SD	0.4490	0.3500	0.4417	0.3790	0.6141	0.4369	0.6087	0.4027	0.8790	0.4646	0.8576	0.5118
SPORT	mean	0.7376	0.1698	0.6833	0.1472	0.3475	0.0851	0.3219	0.0739	0.8215	0.1978	0.7636	0.1789
	SD	0.3623	0.1017	0.3414	0.1017	0.0969	0.0522	0.0936	0.0522	0.7117	0.0527	0.6291	0.0528
STEC	mean	1.6466	0.7241	1.6438	0.7466	0.1593	0.0707	0.1590	0.0728	1.6796	0.7851	1.6769	0.8062
	SD	0.3379	0.3093	0.3377	0.3103	0.0362	0.0336	0.0362	0.0336	1.2131	0.5626	1.2108	0.5740
SPALI	mean	0.2935	0.4662	0.2976	0.4802	0.2058	0.3112	0.2084	0.3201	0.3107	0.4719	0.3144	0.4856
	SD	0.1022	0.0739	0.1017	0.0728	0.1100	0.1055	0.1101	0.1067	0.0553	0.0623	0.0556	0.0638
Total	mean					0.8649	0.2259	0.7835	0.1941				
	SD					0.8008	0.2768	0.7284	0.2392				

Table 5.3: Pricing Error Statistics of Series B Warrant

		MAE				MAPE				RMSE			
		GS	LT	DS	DR	GS	LT	DS	DR	GS	LT	DS	DR
APC	mean	1.3181	0.3249	0.5584	0.2892	1.4075	0.3530	0.6149	0.2846	1.9346	0.4676	0.7979	0.4490
	SD	1.4174	0.3366	0.5705	0.3438	0.5515	0.2340	0.3230	0.2031	7.3552	0.3745	1.1432	0.4006
BTC	mean	2.6733	0.5318	0.7710	0.5019	1.7137	0.4210	0.5555	0.4449	4.2020	0.8196	1.3073	0.6732
	SD	3.2444	0.6241	1.0565	0.4490	1.6922	0.3544	0.5826	0.2776	31.8557	1.4274	3.7712	0.8391
ESTAR	mean	0.1896	0.0763	0.0981	0.0761	0.9324	0.3890	0.4731	0.3981	0.2528	0.1034	0.1407	0.1021
	SD	0.1674	0.0698	0.1009	0.0682	0.6562	0.2691	0.4088	0.2647	0.0874	0.0165	0.0296	0.0165
JAS	mean	0.4159	0.0667	0.1836	0.0582	1.0843	0.1593	0.4290	0.1479	0.5323	0.0964	0.2710	0.0775
	SD	0.3324	0.0697	0.1995	0.0511	0.3437	0.1118	0.2950	0.0943	0.5377	0.0180	0.1492	0.0091
QH	mean	0.4831	0.2206	0.2161	0.1362	0.7364	0.342	0.3369	0.2131	0.5277	0.2561	0.2566	0.1703
	SD	0.2124	0.1302	0.1386	0.1024	0.3083	0.2117	0.2305	0.1650	0.2058	0.0631	0.0663	0.0342
QH2	mean	0.1874	0.0532	0.051	0.0396	0.5469	0.142	0.1402	0.0995	0.2050	0.0700	0.0681	0.0682
	SD	0.0832	0.0455	0.0444	0.0556	0.2000	0.0823	0.0910	0.0879	0.0338	0.0121	0.0110	0.0166
SICCO	mean	0.9487	0.5145	0.4958	0.4812	0.2206	0.117	0.1136	0.1121	1.1871	0.7008	0.6892	0.6489
	SD	0.7142	0.4763	0.4791	0.4358	0.1474	0.1035	0.1035	0.0931	1.9064	0.8149	0.8328	0.7398
SPORT	mean	0.5466	0.4464	0.46	0.4787	0.5820	0.4925	0.5061	0.5363	0.8720	0.5263	0.5495	0.5584
	SD	0.6801	0.2790	0.3003	0.2878	0.5723	0.2230	0.2381	0.2422	1.6881	0.3418	0.3930	0.3999
STEC	mean	1.4185	1.0686	1.0689	1.0792	0.4207	0.3503	0.3503	0.3570	1.5339	1.2479	1.2481	1.2744
	SD	0.5845	0.6453	0.6453	0.6787	0.2252	0.2728	0.2728	0.2862	1.6177	1.4265	1.4267	1.5428
SPALI	mean	0.3451	0.1721	0.1594	0.1071	0.2800	0.1365	0.1257	0.0870	0.3952	0.2195	0.2064	0.1290
	SD	0.1932	0.1366	0.1315	0.0720	0.1277	0.1004	0.0971	0.0568	0.1450	0.0546	0.0494	0.0168
Total	mean					0.8592	0.2951	0.3825	0.2797				
	SD					0.8558	0.2562	0.3644	0.2528				

Table 5.4: MAPE Analysis

Series A Warrant				
	GS	LT	DS	DR
In-the-money	0.8764	0.2185	0.7919	0.1733
At-the-money	0.8002	0.3152	0.7449	0.3537
Out-of-the-money	NA	NA	NA	NA

Series B Warrant				
	GS	LT	DS	DR
In-the-money	0.9976	0.2432	0.3643	0.2067
At-the-money	0.7336	0.3134	0.3822	0.2813
Out-of-the-money	0.7838	0.3107	0.3528	0.2831

model for in-the-money series A warrants and all cases of series B warrants, respectively. Nevertheless, for the case that series A warrants are at-the-money, the Lim-Terry model outperforms the Dennis-Rendleman model.

5.4 The Wilcoxon Signed-Rank Test

The statistic comparison of the model performance based upon MAE, MAPE, and RMSE was accomplished by the performing the Wilcoxon Signed-Rank test. Results are reported in table 5.5.

5.4.1 Series A Warrant

Closer inspection of table 5.5 suggests the following. The results from all the measures are uniformly consistent. The pricing errors of the Galai-Schneller model are significantly larger than those of others, indicating that the Galai-Schneller model performs worst. The pricing errors of the Darsinos-Satchell model are significantly larger than those of the Lim-Terry model and the Dennis-Rendleman model. However, when comparing the pricing errors of the Dennis-Rendleman model with the pricing errors of the Lim-Terry model, the p-value reveals the insignificance. Hence, it is unable to judge which model is better. All in all, for series A warrant pricing, the

Galai-Schneller model performs worst. The Darsinos-Satchell model ranks the third. The conclusion on the better performance model between the Lim-Terry model and the Dennis-Rendleman model cannot be drawn.

5.4.2 Series B Warrant

The pricing errors of the Galai-Schneller model are significantly larger than the pricing errors of the Lim-Terry model, the Darsinos-Satchell model and the Dennis-Rendleman model, indicating that the Galai-Schneller model performs worst. The pricing errors of the Darsinos-Satchell model is not significantly larger than the pricing error of the Lim-Terry model but is significantly larger than the pricing error of the Dennis-Rendleman model. This entails that the Darsinos-Satchell model has similar performance to the Lim-Terry model. However, it underperforms the Dennis-Rendleman model. The pricing errors of the Dennis-Rendleman model is significantly larger than the pricing errors of the Lim-Terry model. In general, in pricing series B warrants, the Galai-Schneller model performs worst whereas the Darsinos-Satchell model performs worse than the Dennis-Rendleman model. The Lim-Terry model and the Dennis-Rendleman model show similar performance.

5.5 Regression Analysis of Pricing Errors

The purpose of the regression analysis is to examine whether the prediction errors contain systematic errors from the model input parameters. Each regression is run between the percentage pricing errors as the dependent variable and the degree of moneyness, time-to-maturity, firm volatility, and risk-free rate of interest rate. The results are revealed in table 5.6. The Wald tests are conducted under the null hypothesis that all coefficients are zero. The results show statistical significance, meaning that the independent variables are significantly different from zero. The effect of each independent variable are examined seperately.

5.5.1 Degree of Moneyness $((v - K)/K)$

For both of series A and series B warrants, all coefficients (except for series B coefficient of the Lim-Terry model) are significantly negative, suggesting that the models tend to overprice more when the degree of in-the-money increases.

Table 5.5: Wilcoxon Test Statistics

Series A Warrant			
	MAE	MAPE	RMSE
GS vs. LT	47*	49*	49*
GS vs. DS	51*	49*	51*
GS vs. DR	45*	47*	51*
DS vs. LT	45*	49*	47*
DS vs. DR	45*	47*	47*
DR vs. LT	-23	-27	-29

Series B Warrant			
	MAE	MAPE	RMSE
GS vs. LT	55*	55*	55*
GS vs. DS	55*	55*	55*
GS vs. DR	55*	55*	55*
DS vs. LT	25	27	31
DS vs. DR	45*	45*	43*
DR vs. LT	-37	-23	-33

* Significant at the 5% level for a one-tail test.

5.5.2 Time to Maturity (τ)

As the time to maturity decreases, the absolute percentage pricing errors should decrease due to less uncertainties in stock price. In other words, there is a smaller chance of a high alteration in stock price. Investors will understand the warrant price behavior more as the time passes as well. Consequently, coefficients should be significantly negative. As shown in table 5.6, they are significantly negative for series B warrants and for some models of series A. Hence, it could be inferred that the market learning effect is obvious for the series B warrants analyzed in this study.

5.5.3 Firm Volatility (σ_v)

The coefficients are all significantly negative except for series A coefficient of the Dennis-Rendleman model. This implies that these models tend to overprice the warrants more when the firm volatility is high.

Table 5.6: Results of Regression Tests

Series A Warrant

	GS	LT	DS	DR
C	0.9918*	0.1713*	0.9239*	0.0444
$((v-K)/K)$	-0.0971*	-0.0182*	-0.0701*	-0.0433*
τ	0.0019	-0.0373*	-0.0084	-0.0407*
σ_v	-2.5863*	-0.4135*	-2.4558*	-0.0529
r	-5.8113*	1.8712	-4.4363	3.1537*
R-Squared	0.4443	0.1496	0.4167	0.1372
Wald Test	630.77*	128.07*	613.71*	35.24*
H_0:$\beta_0 = \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$				

Series B Warrant

	GS	LT	DS	DR
C	1.2851*	0.5014*	1.0052*	0.6794*
$((v-K)/K)$	-0.2667*	-0.0188	-0.0497*	-0.0818*
τ	-0.1393*	-0.0532*	-0.0854*	-0.0599*
σ_v	-2.2640*	-1.1454*	-1.7695*	-1.2014*
r	0.0687	7.8021*	5.5458*	4.8565*
R-Squared	0.6627	0.5079	0.6997	0.4822
Wald Test	303.22*	109.22*	218.93*	77.50*
H_0:$\beta_0 = \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$				

* Significant at the 5% level.

5.5.4 Risk-Free Rate of Interest (r)

The series A coefficient of the Galai-Schneller model is significantly negative, suggesting that tis models overprice series A warrants when the interest rate is higher. Series A coefficients of the Dennis-Rendleman model and all series B coefficients (except the Galai-Schneller model) are significantly positive, implying that the models overprice the warrants when the interest rate is lower.

It can be seen that all the R-Squared values of series A warrants are less than the R-Squared of series B warrants, implying in the series A warrant models can capture the model inputs better than series B warrant models. Rely on these results, the subtle slippage effect seems to have less impact on pricing multiple warrants than the cross-dilution effect.