Chapter VI

Conclusions and Suggestions

6.1. Conclusions

After the simulation runs of well-established multi-objective benchmark problems in Chapter IV and of multi-objective continuum topology optimization a problem in Chapter V, this thesis can be concluded as follows.

6.1.1. MOEA Performance

The conclusions for two successful strategies – co-operative co-evolution and winning score – in the proposed multi-objective evolutionary algorithms (MOEAs) are as follows.

6.1.1.1. Co-operative Co-evolution

Co-operative co-evolutionary multi-objective algorithm (CCMOA), which employs co-operative co-evolution [43] for multi-objective optimization problems (MOOPs), is much more superior to the two well-established MOEAs – fast elitist non-dominated sorting genetic algorithm (NSGA-II), and improved strength Pareto evolutionary algorithm (SPEA-II) – for problems without linkage among decision variables – ZDT1-6, DTZL1-4, and DTLZ7.

In the same way, co-operative co-evolutionary improved compressed-objective genetic algorithm (CCCOGA-II), which is the resulting algorithm from the integration of co-operative co-evolution into improved compressed-objective genetic algorithm (COGA-II), is also much better than its predecessor improved compressed-objective genetic algorithm (COGA-II) for the problems with three-or-more objectives – DTLZ1-4, and DTLZ7. However, for problems with linkage among decision variables – DTLZ5-6, linked DTLZ2, linked DTLZ6, and continuum topology optimization problems, the effectiveness of co-operative co-evolution deteriorates. The CCMOA is also better than NSGA-II and SPEA-II for DTLZ6 and L_1-DTLZ2, regardless of numbers of objectives. For real-world continuum topology optimization problems, it is superior to NSGA-II and SPEA-II for the linear-elastic and thermo-elastic problems. Thus, it can conclude that the co-operative co-evolution can much improve the performances of MOEA for problems without linkage among decision variables. In addition, it can also improve the performances of MOEA for some optimization problems with linkage among decision variables such as DTLZ6, L_1-DTLZ2, the linear-elastic and thermo-elastic problems. However, studies of the effectiveness of co-operative co-evolution for multi-objective optimization with linkage among decision variables should be further investigated.

6.1.1.2. Winning Score

The employment of winning scores in improved compressed-objective genetic algorithm (COGA-II) and co-operative co-evolutionary improved compressed-objective genetic algorithm (CCCOGA-II) is very successful for optimization problems with three-or-more objectives. COGA-II is much superior to NSGA-II and SPEA-II not only for well established benchmark problems in Chapter IV, but also for real-world continuum topology optimization problems in Chapter V. Similarly, due to the effectiveness of the employment of winning, CCCOGA-II is much better than CCMOA for all employed optimization problems – the well-established benchmark problems and continuum topology optimization problems.

Thus, proposed MOEAs – CCMOA, COGA-II, and CCCOGA-II – can improve performances of MOEAs. Comparing the proposed MOEAs against each other, CCMOA is suitable for two-objective optimization problem with low linkage among decision variables while COGA-II is suitable for optimization problems with three-or-more objectives. However, for problems with weak linkage among decision variables, such as DTLZ1-4, DTLZ7, and L_1-DTLZ2, CCCOGA-II is more suitable than COGA-II.

6.1.2. Continuum Topology Optimization

The conclusions for the continuum topology optimization are as follows.

6.1.2.1. Progressive Refinement Run

The progressive refinement run is proposed to solve a continuum topology optimization problem with 2-3 design objectives. Since it can reduce the numbers of possible solutions in an early running stage, it can obtain good solutions for all employed continuum topology optimization problems. The progressive refinement run may be useful for three-dimensional topology optimization problems, such as [85], [106], and [118]-[120], since there are a very large number of possible solutions for such problems of which domain is divided into required number of elements. It is then difficult to search for good solutions by using only the required domain division. Compared to two-dimensional problems, the progressive refinement run for 3D problems can much more reduced the number of possible solutions in an early running stage in order to obtain good solutions in the final running stage, of which domain is divided into the required number of elements.

6.1.2.2. Objective Increasing Run

The objective increasing run, for a continuum topology optimization problem with many design objectives, are used to solve the continuum topology optimization problems. It use only 2-3 objectives in the first optimized run, then the number of optimized objectives is then increased one-by-one to the required number of design objectives.

The additions of 3 geometrically structural objectives into the employed continuum topology optimization problems are used in the studies of MOEA performance for a continuum topology optimization problem with many design objectives. Since the objective increasing run is proposed to reduce the degradation of MOEAs when the number of objective of an optimization problem is increased, and two proposed MOEAs – COGA-II and CCCOGA-II – are proposed to improve the performance of MOEAs for an optimization problem with many objectives, the employment of the objective increasing run with the proposed MOEAs can obtain the good solutions for all employed continuum topology optimization as described in Chapter V. By optimizing all considered objectives including these geometrically structural objectives, optimized topological solutions by the objective increasing run with the proposed MOEAs can be very useful for further detailed design such as shape optimization designs.

The objective increasing run may be useful for continuum topology optimization problems or other real-word optimization problems, because most real-world optimization problems have many design objectives.

6.1.2.3. MOEA Suitability

The employed continuum topology optimization problems – heat conduction, linear-elastic, and thermo-elastic problems – can be effectively solved by progressive refinement and objective increasing runs with the proposed MOEAs. Since derivative-based optimizers need an explicit mathematical formulation of decision variables of a topology optimization problem, it is however very difficult to identify the objective formulations of topology optimization problems. From the simulation results and weak points of derivative-based optimizers, it can be concluded that MOEAs are more suitable than derivative-based optimizers, to solve continuum topology optimization problems. Moreover, they can solve complex or non-linear continuum topology optimization problems, which are very difficult for the derivative-based optimizers. In addition, since an MOEA can obtain multiple best solutions or non-dominated solutions from an optimization run, it therefore gives various choices for a researcher as described by Figure 2.7.

6.2. Suggestions

The suggestions for future works are as follows.

6.2.1. The effectiveness of co-operative co-evolution for multi-objective optimization with linkage among decision variables especially for real-world continuum topology optimization problems should be further investigated. However, actually, a researcher may know the co-operative co-evolution is whether suitable for a particular continuum topology optimization problem or not by his experience.

6.2.2. Effect of division of the domain in first running stage for the progressive refinement run should be further studied. For a continuum topology optimization problem, if the divided domain in the first running stage is too coarse, poor solutions may be obtained for this running stage. Thereafter, good solutions may not be obtained in the final running stage. If this imperfection is solved by using finer divided domain in the first running stage, however, search space is much increased and then causes difficulty to an employed MOEA. In the other hand, it is not a necessary to divide domain into uniform elements. Some critical regions such as region near heat source in the heat conduction problem, and regions near applied force and elements contacting temperature surface in the thermo-elastic, very probably have more impact to a solution than other regions. The imperfection as previously stated may be efficiently solved by dividing domain into nonuniform elements in the first running stage in which high impact regions are finely divided, while other region is still coarsely divided. Although, search space of the first running stage by this improvement is increased still, it is not much increased as compared to the use of finer elements for the entire domain. It may not cause much difficulty to the employed MOEA; therefore, better solutions may be obtained in the final running stage.

6.2.3. Orders of increased objectives in the objective increasing run should be carefully identified. The initial optimized objectives should be more significant to a problem than the others. However, an optimization problem may be difficult to know significance of their design objective. To solve this difficulty, the good solutions of the problem may be obtained by runs of different orders of optimized objectives.

6.2.4. The proposed MOEAs, progressive refinement run, and objective increasing run will be used to solve complex continuum topology optimization problems.