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THE APPLICATION OF WAVELETS IN FIELD THEORY

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พิมพ์ต้นฉบับบทคัดย่อวิทยานิพนธ์ภายในกรอบสีเขียวนี้เพียงแผ่นเดียว

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วิทยานิพนธ์นี้เป็นการศึกษาและทำซ้ำผลงานของเบสและเชฟเฟอร์เกี่ยวกับการประยุกต์เวฟเลทในทฤษฎีสนามในแบบจำลองเกาส์และแบบจำลองแลนดาว-กินซ์เบอร์ก ซึ่งสนามของแบบจำลองเกาส์ถูกกระจายในตัวแทนของเวฟเลท ด้วยวิธีการหาค่าพลังงานต่ำสุดในเกาส์เอนเซมเบิ้ล โดยพารามิเตอร์แปรผันคือความแรงของฟลัคทูเอชั่นของสัมประสิทธิ์เวฟเลท เราจะได้ความแรงของฟลัคทูเอชั่นของสัมประสิทธิ์เวฟเลทโดยการวิเคราะห์ ความแรงของฟลัคทูเอชั่นของสัมประสิทธิ์เวฟเลทและฟังก์ชันสหสัมพันธ์จะถูกคำนวณโดยการคำนวณเชิงตัวเลข ผลเป็นการยืนยันว่าเวฟเลทสามารถให้ข้อมูลของปรากฏการณ์วิกฤตโดยใช้จำนวนพารามิเตอร์แปรผันเพียงเล็กน้อย ในแบบจำลองแลนดาว-กินซ์เบอร์กโดยการประมาณเชิงวิเคราะห์ ได้แสดงให้เห็นว่าการเปลี่ยนเฟสสามารถถูกแสดงในปริภูมิเวฟเลทได้อย่างไรและเอกซ์โพเนนวิกฤตสามารถถูกคำนวณได้จากการกระจายเวฟเลทได้อย่างไร

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SOMCHAI RUANGPERMPOOL: THE APPLICATION OF WAVELETS IN FIELD THEORY

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In this thesis, it has been studied and repeated C. Best and A. Schafer's work on the application of wavelet to the Gaussian model and the Landau-Ginzburg model. The fields of the Gaussian model are expanded in wavelet representation and with the fluctuation strengths of wavelet coefficients as variational parameter, the free energy of the model in Gaussian ensemble is minimized analytically. We then numerically recalculate the fluctuation strengths of wavelet coefficients and thus the correlation function. The result confirms that wavelets provide a reasonable description of the critical phenomena with only a small number of variational parameters, which is the same as the applicaiton of wavelet in other fields.

In Landau-Ginzburg model, by analytic approximation, it is shown how the phase-transition is represented in wavelet space and how critical exponents can be computed from the wavelet expansion.

ภาควิชา ฟิสิกส์ ลายมือชื่อนิสิต ....

สาขาวิชา ฟิสิกส์ ลายมือชื่ออาจารย์ที่ปรึกษา ....

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INTRODUCTION AND SCOPE OF THESIS

The success of wavelet transform (Daubechies, 1988, Chui, 1992, M.B. Ruskai et al., 1992, Press et al., 1992, Meyer, 1993) in analyzing complex signals has indicated that they can be applied to field theories and other lattice systems. Their scaling properties and good time-frequency localization make them a practical tool in case that both the ordinary representation and the Fourier transform poorly represent the important features of the signal.

There are many applications of wavelet transform in theoretical models (Greiner et al., preprint HEPHY-PUB 568/93, Best and Schafer, hep-lat/9311031 and hep-lat/9402012, Halliday and Suranyi, hep-lat/9407010). Because wavelet analysis is very similar to the block spin or real space renormalization of statistical field theory, then it may be a possible candidate for renormalization group that could be more efficient both in term of how accurate physical process are depicted and how much numerical work is necessary to calculate the renormalization group flow. This thesis has reviewed Best and Schafer's work (hep-lat/9311031) which is so compact in their paper. The objective of this work is to investigate the descriptions of statistical field theory using Daubechies wavelet. In Chapter I, we review some useful concepts of wavelet theory and wavelet's development. In Chapter II, we review how to generate wavelet and use wavelet in one and two dimensions. The theories of phase transition and critical phenomena are reviewed in Chapter III and the Renormalization group theory are reviewed in Chapter IV. In Chapter VI, we review the definition and the method to study the statistical field theory. The numerical result of wavelet theory in Gaussian model, analytic approximate solution of Landau-Ginzburg model and renormalization group in wavelet space are represented in Chapter VI and the discussion and conclusion are presented in Chapter VII.