CHAPTER VI

DISCUSSION AND CONCLUSION

In chapter I, the first quantum correction to the partition function of free particles system was reviewed showing that inter-potential can be produced by itself. This is a statistical effect for several systems. Bosons would like to have the same quantum numbers because of their attractive inter-potential, and fermions could not be in the same state because of their repulsive inter-potential. It has been shown that the development of the theoretical explanation of the Bose condensation is close to the superflidity of liquid helium. In this development it is recognised that Landau, the first to carry the idea of quasi-particles related to this problem, suggested the necessity of considering the collective elementary excitations rather than individual molecules in his classic papers in 1941, 1947[9-10]. Further, Feynman [11-14] developed the Landau suggestion for quantum liquid, superfluidity. For his calculations he introduced the new method of quantum statistical mechanics based on his quantum formalism path integrations. A few years before research, Bogoliubov [15] used and developed the idea of Landau for the problem of superfluidity in molecular Bose gas. In his work, he used the method of second quantization together with his (now) famous approximately procedure. Finally, he found a description of superfluidity with connections to excitation energy. Using the experimental method of high momentum neutron scattering on liquid helium [16], it is now known that the excitation spectrums of the Bose condensation has two branches.

In chapter II, the relationship between the Feynman propagators and the quantum partition functions was shown. This is very important because the study aimed to show the use of this method of functional path integration.

In chapter III, the ansatz interaction between particles was introduced, suggestiong a function in terms of the combination of exponential over the power laws, that is, $\left(\frac{e^{-\alpha r}}{(\beta r)^2}-\frac{e^{-\alpha r}}{(\beta r)}\right)$. It was shown that this function contains two aspects of interesting behaviour as did the original Lennard-Jones function: strong, short-range repulsive interaction and weak, long-range attractive interaction. The problem in this chapter was how to use this interaction in the momentum space and for non-uniform media. However, it was suggest that a method of transformation by some point in the approximation process is not good enough. However, most results push the claim that it is good enough.

In chapter IV, the Hamiltonian action was computed, using the Hamiltonian by the method of second quantization. This Hamiltonian method was used in the next calculation of action by the method of functional integration. A further point discussed is the broken symmetry of the Hamiltonian. By this procress, an important constrain about the conservation of the number of particles was undertaken, leading to the semi-classical integrals of motion which separate the particles into two types: the classical condensate particles and the quantum excited (over condensate) particles. The ability to separate provided a way of treating the particles in ground state (condensate state) similar to the classical particles; and, then, the creation annihilation operators of the k zero particles as the ordinary numbers can be approximated.

In chapter V, the partition function from the action in chapter IV was calculated. All terms of the calculation to the components of the matrices were moved in an effort to calculate the partition function by matrix algebra. Finally, a necessity of approximation was introduced because the system lies in the low temperature regime, the adiabatic approximation. It has been shown that the partition function will be infinite at some point in the phase space and by definition of excitation energy can be defined as the excitation spectrum of our system as a function in divided parts of the partition function. The excitation energy depends upon the momentum $\vec{k}$, the density of ground state particles ρ_0, the interaction between the condensate particles $g(0,0)$ the interaction between the over-condensate particles $g(\vec{k},\vec{k})$, and the interaction between the condensate and over-condensate particles $g(\vec{k},0)$. These spectrums have two dominant behaviours in different areas in phase space. The first spectrum dominate in the area of slow momentum and high density of ground state particles where as the second spectrum dominate in the area of rapid exists in the density of the ground state particles which lower than the existed density of the first branch. This result brings about to the interpretation of the phonon-roton and the maxon-roton spectrums respectively. What we have seen from this result is that the first branch spectrum is the spectrum of the system that leads to the Bose condensation and that the second branch spectrum is the spectrum of the system which passes from the Bose condensate region to the higher momentum area.

The energy shift of the second branch is an interesting point also. Here we would like to suggest that this energy shift is probably arisen from the separation of the ground state from the remain excited state. It is revealed that the additional $g(\bar{k},0)$ in the linear terms through H_A has the effect on the energy shift and is the cause of the energy shift also.