THEORY

A number of formulae based on the theory of thick cylinders subjected to internal pressure are in use for the determination of the stresses in the hub of an interference fit. Among them are the formulae of Lamé, Clavarino and Birnie.

Lamé theory takes no account of lateral contraction as expressed by Poisson's ratio while the others do. His formula only go as far as giving internal stresses. Birnie's final form uses the stresses as given by Lamé or Baugher, but adds them together so as to obtain a criterion for failure according to maximum strain theory. His formula is for cylinders with open ends and is employed for the design of large guns. The Clavarino's final form is for a cylinder with internal pressure with closed ends. His criterion of failure is also on the basis of maximum strain theory, and while they have been used considerably in the design of hooped guns, they are not applicable to the usual case of a machine fit. Baugher (2) used the maximum shear theory instead of the maximum strain theory and his theory was widely accepted.

In a General Electric Review paper*, an explicit and easily followed demonstration of two fundamental principles applied to all thick cylinders, called Lamé's Laws are given.

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* "Increase of Bore of High Speed Wheels by Centrifugal Stresses," Trans ASME. 1912.

Lamé's Laws are:

1. The algebraic sum of the radial and tangential stresses at any radius has a constant value.

   It follows that the longitudinal strain is the same at all radii.

2. The difference between the radial and tangential stresses at any radius varies inversely as the square of the radius.

These laws are based on the convention that the tensile stresses are positive and compressive stresses negative.

However, the theory of Lamé, Birnie and Clavarino are all correct and in good agreement with that of Baugher*.

Another simple formula called Barlow's** formula:-

$$\sigma = \frac{P_i r_o}{t}$$

t = thickness σ = designed tensile stress at bore

This formula, similar to that of thin cylinder, use outer radius instead of mean radius. It gives values on the safe side and is useful only for quick estimation. In design purpose, it is uneconomical in material.

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** Deriviation of Barlow's formula can be found in "Mechanics Applied to Engineering", 8th Edition by Goodman J., Longmans, Green & Co. London, 1914 p.p. 421-423.

<u>Failing Under Combined Stresses Of Hub</u>

For interference fit assembly, Maximum Shear Stress theory* is generally used for ductile materials.

At hub bore

$$\tau_{max} = \tfrac{1}{2}(\sigma_t - \sigma_r)$$

but S_y = equivalent tensile stress = $2\tau_{max}$** = yield strength in simple tension.

$$\therefore \quad S_y = \frac{2D^2}{D^2 - d^2} P_i$$

For brittle material, Maximum Principal Stress theory is used*** when

$$\sigma_t = \sigma_{UTS.}$$

$$\sigma_{UTS.} = \frac{P_i(D^2 + d^2)}{D^2 - d^2} = P_i \cdot \frac{1 + (\frac{d}{D})^2}{1 - (\frac{d}{D})^2}$$

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* Ref. (1) and (2)

** Faires'Design of Machine Element' McMilland 1964 p. 229

*** See Ref. (1)

TABLE I*

Stresses and displacements of hub and shaft (Lamé Solution)

	Hub	Hollow Shaft	Solid Shaft
1. σ_t at any radius	$\frac{P_i d^2}{D^2-d^2}\left[\frac{D^2}{4r^2}+1\right]$	$-\frac{d^2}{(d^2-d_i^2)}\left[1+\frac{d_i}{4r^2}\right]P_i$	$-P_i$
2. σ_t at outside diameter	$\frac{2d^2}{D^2-d^2}P_i$	$-\left[\frac{d^2+d_i^2}{d^2-d_i^2}\right]P_i$	$-P_i$
3. σ_t at bore, inside diameter	$\left[\frac{D^2+d^2}{D^2-d^2}\right]P_i$	$-\left[\frac{2d^2}{d^2-d_i^2}\right]P_i$	
4. σ_r at any radius	$-\frac{d^2}{D^2-d^2}\left[\frac{D^2}{4r^2}-1\right]P_i$	$-\frac{d^2}{d^2-d_i^2}\left[1-\frac{d_i^2}{4r^2}\right]P_i$	$-P_i$
5. σ_r at outside diameter	0	$-P_i$	$-P_i$
6. σ_r at bore	$-P_i$	0	
7. δ at any radius	$\frac{(1-\mu)r+(1+\mu)\frac{D^2}{4r}}{\frac{D^2}{d^2}-1}\;\frac{P_i}{E}$	$-\frac{(1-\mu)r+(1+\mu)\frac{d_i^2}{4r}}{1-\left(\frac{d_i}{d}\right)^2}\;\frac{P_i}{E}$	$-(1-\mu)r\frac{P_i}{E}$
8. δ at bore	$\left[\frac{D^2+d^2}{D^2-d^2}+\mu\right]\frac{P_i d}{2E}$	$-\frac{d^2 d_i}{d^2-d_i^2}\;\frac{P_i}{E}$	
9. δ at outside diameter	$\frac{d^2 D}{D^2-d^2}\;\frac{P_i}{E}$	$\left[-\frac{d^2+d_i^2}{d^2-d_i^2}+\mu\right]\frac{P_i d}{E}$	$-(1-\mu)d\frac{P_i}{E}$
10. e_{max}. Same material. Ductile mat.		$\frac{S_y}{E}\;\frac{d^2(D^2-d_i^2)}{D^2(d^2-d_i^2)}$	$\frac{S_y}{E}$

* Ref. 1 p. A 185

TABLE II*

Pressure between mating surface (Lamé Solution)

Member	P
1. Hub & hollow shaft, Different materials	$\dfrac{e}{\frac{1}{E_h}\left[\frac{D^2+d^2}{D^2-d^2}+\mu_h\right]+\frac{1}{E_s}\left[\frac{d^2+d_i^2}{d^2-d_i^2}-\mu_s\right]}$
2. Hub & hollow shaft, Same materials	$\dfrac{(D^2-d^2)(d^2-d_i^2)Ee}{2d^2(D^2-d_i^2)}$
3. Hub & solid shaft, Different materials	$\dfrac{e}{\frac{1}{E_h}\left[\frac{D^2+d^2}{D^2-d^2}+\mu_h\right]+\frac{1-\mu_s}{E_s}}$
4. Hub & solid, Same material	$\dfrac{Ee}{2}\left[\dfrac{D^2-d^2}{D^2}\right]$

* Ref. 1. p. A 184

Lamé Solutions for solid shaft and hollow shaft with hub can be found in Table I. The Maximum Shear Stress theory of failure was assumed. This Table gives ideal solutions, assuming that there were neither out-of-roundness nor out-of-straightness effect, and that the shaft deforms uniformly under uniform pressure. Table II gives the unit pressure between the mating parts. This Table is also for ideal solutions and for elastic range only.

As the interference becomes greater, the hub inner surface yields. This yield boundary will penetrate deeper as the interference increases. In this elasto-plastic state, the Maximum Shear theory was assumed, that is

$$\sigma_t + \sigma_r = 2\tau_{max} \simeq S_y$$

It was also assumed that the material yields without changing the yield stress, i.e. $S_y = \frac{2D^2}{D^2 - 4\rho^2} P_x$ is constant.

The general solution for internal pressure to cause the elasto-plastic front penetrate to radius $= \rho$ is*

$$P' = -S_y \ln \frac{d}{2\rho} + \frac{S_y}{2} \frac{(D^2 - 4\rho^2)}{D^2} \qquad \text{(i)}$$

and for stresses:-

$$\sigma_t = S_y \ln \frac{r_i}{\rho} + \frac{S_y}{2} \frac{(D^2 + 4\rho^2)}{D^2} \qquad \text{(ii)}$$

$$\sigma_r = S_y \ln \frac{r_i}{\rho} - \frac{S_y}{2} \frac{(D^2 - 4\rho^2)}{D^2} \qquad \text{(iii)}$$

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* Ref. 7 p.p. 388 - 391

The general equation for radial deflection* of elasto-plastic is

$$U_{ri} = \frac{S_y \rho^2}{2\,E_{ri}} \left(1 + \mu\right) + \left[\frac{\ell}{r_o}\right]^2 (1 - \mu) \qquad \text{(iv)}$$

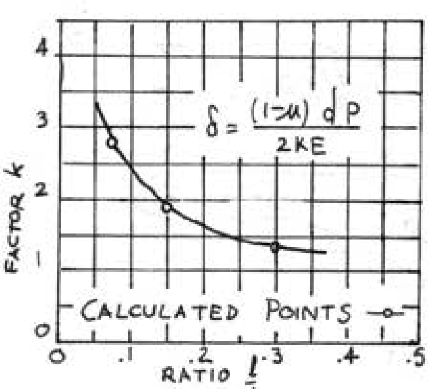
When a hub of short axial length is shrunk onto a relatively long shaft, the stress distribution and pressure will not be uniformed as assumed. Several approaches have been made to find the nearest solution for the actual pressure and stress distribution but most of them were too complicated to be used in practical problem. M.V. Barton (3), in 1941, derived solutions in Fourrier Series form. Rankin (4) in 1944, derived into Fourrier integral form but worked out for numerical solutions and introduced a factor "K" for the average deformation of solid shaft:-

$$s = \frac{(1 - \mu)dP}{2\,KE} \qquad \text{(vi)}$$

The factor K is a constant, obtained from Rankin chart, with the condition of fig 2a

FIG. 1

RANKIN CHART



* See deriviation in appendix

In practical problem of this work, the value of K = 1.1 is approximately for $\frac{l}{d} = 2$, incase of fig 2b

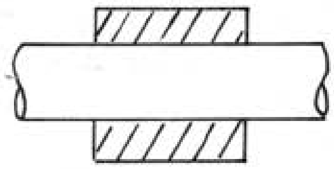


FIG. 2a

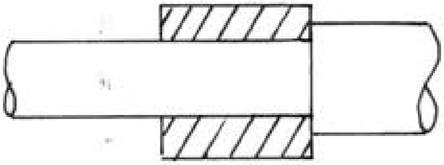


FIG. 2b

FIG. 3

MOVEMENT OF ELASTO-PLASTIC BOUNDARY IN HUB
CAUSED BY THE INCREASE IN INTERNAL PRESSURE.
MAXIMUM SHEAR STRESS THEORY

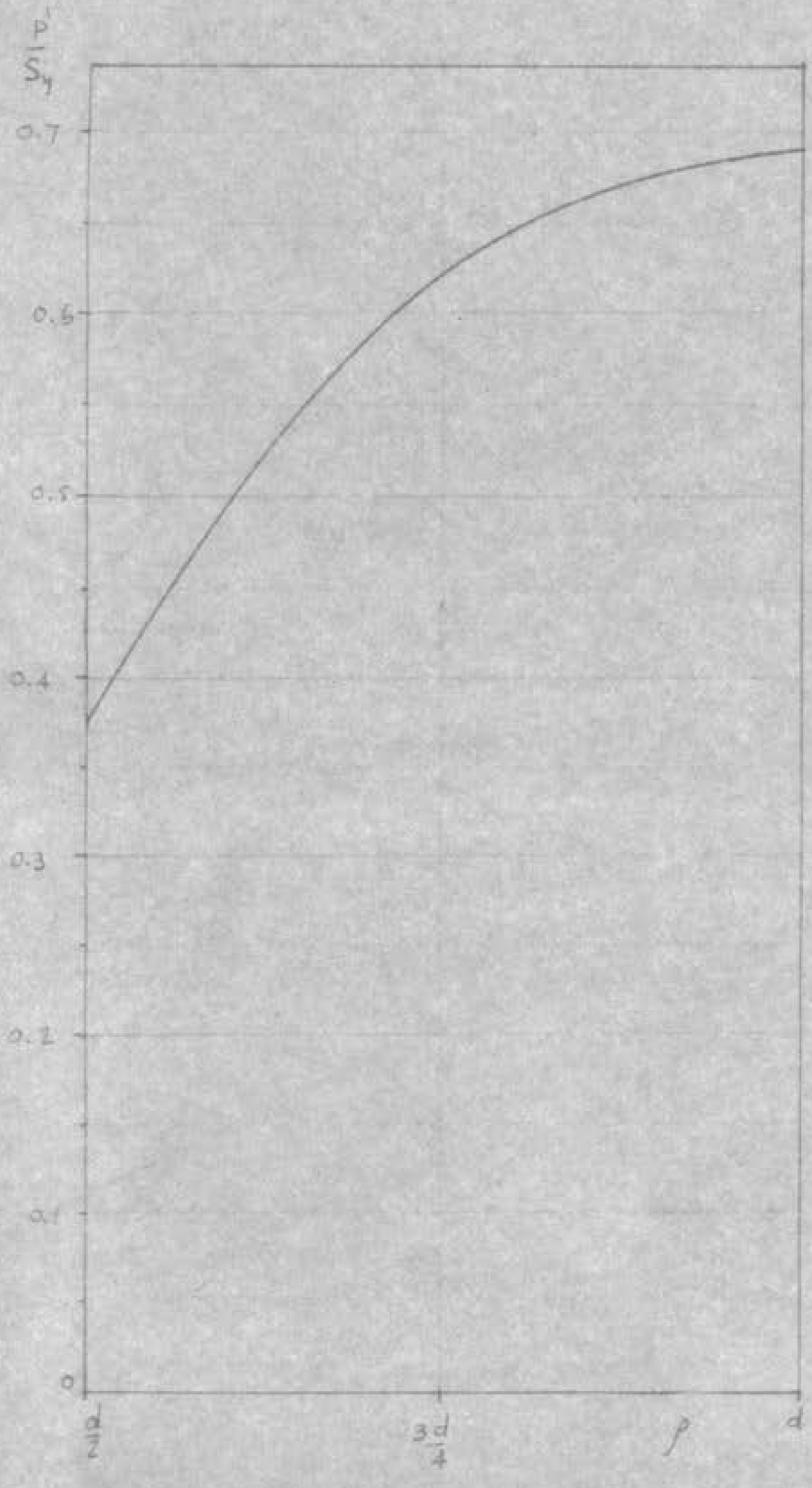
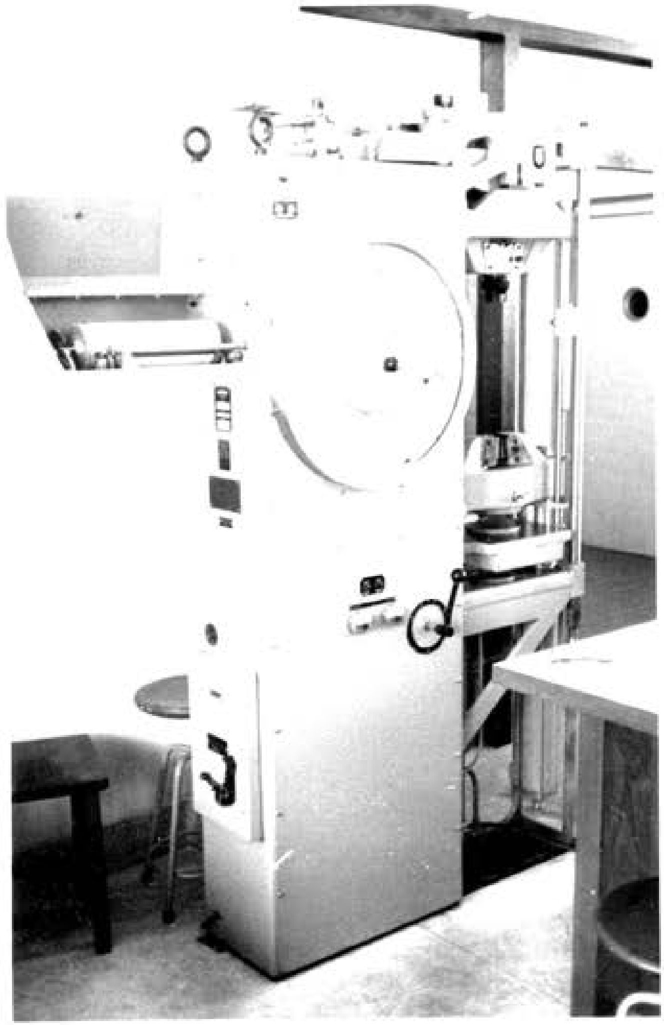


FIG. 4



THE AVERY UNIVERSAL TESTING MACHINE.

THE VERNIER MICROMETER AND THE ENGINEERING MICROSCOPE.