OPERATING CHARACTERISTICS

Resistance and Reactance

The length of the mean-turn of the winding l_t can easily be calculated from a sketch of the coils.

The annealed copper conductor

Resistance at 20°C	ohm-mm²/Km	=	17.241
Weight	Kg/mm²/Km	=	8.89
Temperature coefficient of resistance at 20°C per °C		α =	0.00393

$$R_t = R_{20}\left[1 + \alpha(t - 20)\right] . \qquad (20)$$

The total resistance of the winding

$$R = l_t N R_t \quad \text{ohm.} \qquad (21)$$

The effective resistances of the windings are often only slightly greater than their direct-current resistance.

The load losses include I^2R losses in the winding and stray losses due to stray fluxes in the winding, core clamps, etc.

To obtain the effective alternating-current resistance, the stray load-losses will be estimated equal to 10 per cent of the total I^2R losses.

Although, in a loaded transformer, magnetic leakage is important, it is usually very small in an iron-core reactor or in transformer at no load. Then the apparent reactance X_a practically equals the reactance due to the core flux.

$$X = \omega L \quad \text{ohms.} \qquad (22)$$

<u>Quality Factor</u>

In many applications of inductance coils, the ratio of inductive reactance ωL to apparent resistance R_a should be as large as possible. This ratio is commonly denoted by the symbol Q and may be thought of as a quality factor. In a high-Q coil, relatively little loss is associated with the desired inductance. The Q of the reactor is

$$Q = \frac{X}{R_a} = \frac{\omega L}{R_a} = \omega T \qquad (23)$$

where

$$T = \frac{L}{R_a} = \text{the time constant.} \qquad (24)$$

In spite of the increase in apparent resistance due to core loss and the decrease in apparent inductance due to the screening effect of the eddy currents, the ratio $\omega L/R_a$ can be made larger with an iron core than with an air core. To achieve this condition the maximum flux density must be kept small, and thin laminations of high resistivity or powdered cores must be used to keep the core losses and the screening effect of the eddy currents small.

<u>General Relations</u>

Assume that the following values have been observed on the reactor. Any consistent unrationalized system of units may be used.

A_c = cross-sectional area of magnetic material at any convenient section of the core,

l_c = mean length of flux path,

N = number of turns in the winding,

R = resistance of the winding,

V = rms value of the applied voltage,

I = rms value of the current,

P = average power absorbed by the reactor.

From these measured values, the following quantities can be computed:

Z_a = apparent impedance of the reactor

$$= \frac{V}{I} \tag{25}$$

$\cos\theta_a$ = apparent power factor

$$= \frac{P}{V\,I} \tag{26}$$

R_a = apparent resistance

$$= Z_a \cos\theta_a = \frac{P}{I^2} \tag{27}$$

X_a = apparent reactance

$$= Z_a \sin\theta_a = \sqrt{Z_a^2 - R_a^2} \tag{28}$$

L_a = apparent inductance

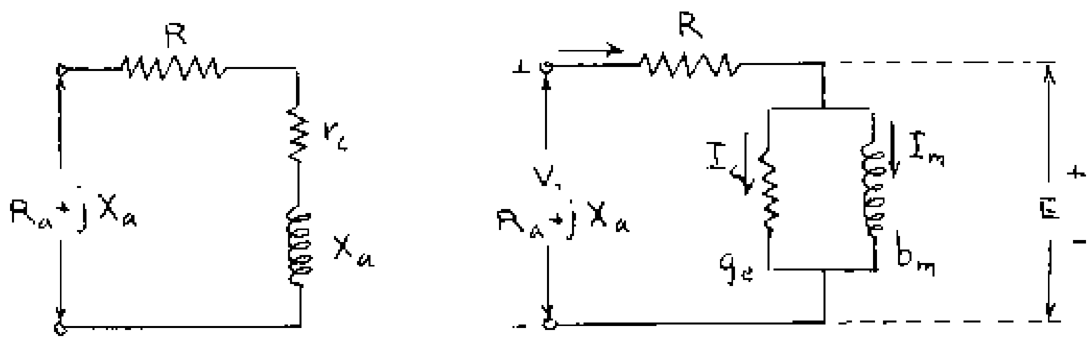
$$= \frac{X_a}{\omega} \tag{29}$$

Q_a = quality factor

$$\frac{X_a}{R_a} = \frac{\omega L_a}{R_a} = \omega T_a. \tag{30}$$

The reactor can be represented by an equivalent circuit comprising the winding resistance in combination with either a series or a parallel arrangement of a resistance and an inductance, as shown in Fig. 1. In these circuits,

E = rms value of the voltage induced by the flux.



(a) (b)

Fig. 1. Equivalent circuits for an iron-core reactor

Fig.(a) represents the series arrangement,
Fig.(b) represents the parallel arrangement.

The vector diagram is shown in Fig.2, from which, vectorially

$$E = V - I R \qquad (31)$$

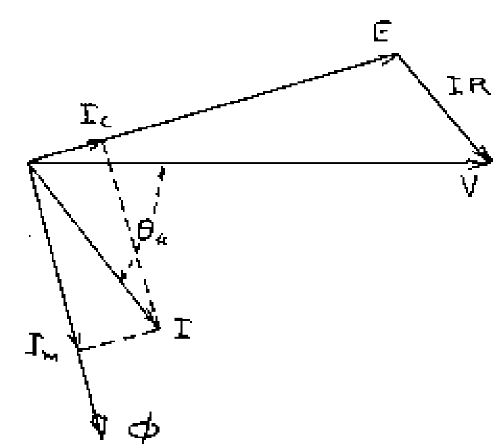


Fig. 2. Vector diagram for an iron-core reactor.

The core loss can be determined from the measured power input; thus

$$P_c = \text{core loss}$$
$$= P - I^2R \, . \qquad (32)$$

The current can now be resolved into its core-loss and magnetizing components, and the parameters of the equivalent circuit of Fig. 1b can be determined as follows:

$$I_c = \text{core-loss component of the current}$$
$$= \frac{P_c}{E} \qquad (33)$$

$$G_c = \text{core-loss conductance}$$
$$= \frac{I_c}{E} = \frac{P_c}{E^2} \qquad (34)$$

$$I_m = \text{magnetizing component of the current}$$
$$= \sqrt{I^2 - I_c^2} \qquad (35)$$

$$b_m = \text{magnetizing susceptance}$$
$$= -\frac{I_m}{E} \, . \qquad (36)$$

The negative sign in Eq.36 indicates that b_m is an inductive susceptance. In Fig. 1b, the vector admittance of the parallel circuit which represents the effects of the core as viewed from the winding,— called the vector admittance of the core,— is

$$Y_\varphi = \text{vector admittance of the core}$$
$$= G_c + j\, b_m. \qquad (37)$$

The relations between the parameters of the equivalent circuits, Fig. 1a and 1b, are:

$$Z_\varphi = \text{vector impedance of the core}$$
$$= \frac{1}{Y_\varphi} \qquad (38)$$

r_c = equivalent series resistance of core loss

= real part of Z_φ

$$= \frac{g_c}{g_c^2 + b_c^2} \quad (39)$$

Note that r_c does not equal $1/g_c$.

x_c = equivalent series reactance

= imaginary part of Z_φ

$$= \frac{-b_c}{g_c^2 + b_c^2} \quad (40)$$

The magnetic conditions in the core are as follows:

λ = rms value of the alternating flux linkage

$$= \frac{E}{\omega} \quad (41)$$

ϕ = rms value of the alternating flux

$$= \frac{\lambda}{n} \quad (42)$$

B = rms value of the flux density at the cross-section A_c

$$= \frac{\phi}{A_c} \quad (43)$$

F = rms value of the magnetomotive force

$$= 4\pi n I_m \quad (44)$$

H = rms magnetomotive force per unit length

$$= \frac{F}{l_c} \quad (45)$$

Equations 42 and 43 assume that all the flux is confined into the core and that the flux density is uniform over the cross-section A_c.

Effects of an Air Gap.

Consider the effects of changing the length of an air gap in the magnetic circuit of a specified reactor. The loss ratio of the reactor can be expressed as

$$\text{Loss ratio} = \frac{1}{Q_m} = \frac{\text{core loss} + \text{copper loss}}{\text{reactive volt-amperes}} \qquad (46)$$

$$= \frac{P_c + I^2R}{E\,I_m} \qquad (47)$$

$$= \frac{P_c + I_c^2 + I_m^2R}{E\,I_m}. \qquad (48)$$

where

P_c = core loss,

E = rms value of the induced voltage,

I = rms current and equals $\sqrt{I_c^2 + I_m^2}$,

I_c = rms core-loss component of the current,

I_m = rms magnetizing component of the current,

R = resistance of the winding.

Let the frequency and rms value of the flux be maintained constant (by adjustment of the applied voltage as the air gap is changed), and assume that the waveform of the flux is sinusoidal; also neglect the effects of changes in magnetic leakage caused by changes in air-gap length. With these assumptions, the rms value of the flux density in the core is constant. The core loss P_c and the induced voltage E are therefore constant. Since

$$I_c = \frac{P_c}{E}, \tag{49}$$

the core-loss current I_c also is constant. Hence, in Eq. 48 only the reactive magnetizing current I_m is affected by changes in the gap length. The magnetizing current must adjust itself to produce the same flux in spite of changes in reluctance due to changes in the air gap.

Equation 48 shows that the loss ratio is affected in two ways. The first effect of increasing the gap length is an improvement in the quality factor Q_c for the magnetic circuit. Thus the loss ratio of the magnetic circuit (including the air gap) is

$$\frac{1}{Q_c} = \frac{P_c}{E I_m}, \tag{50}$$

and therefore the increased magnetizing current results in a lower loss ratio; that is, an improved Q_c. This result should be expected, since core loss occurs when pulsating energy is stored in iron, but no loss occurs when it is stored in air. The way to increase Q_c is therefore to store more energy in air; in other words, to insert an air gap in the magnetic circuit or to increase the length of an existing one. On the other hand, the second effect of the increased magnetizing current is a reduction of the quality factor Q_w for the winding. The loss ratio for the winding is

$$\frac{1}{Q_w} = \frac{I^2 R}{E I_m} = \frac{I_c^2 R + I_m^2 R}{E I_m}, \tag{51}$$

and, since the magnetizing current I_m usually is considerably greatter than the core-loss current I_c, the copper loss is approximately proportional to the square of the magnetizing current. Thus when the magnetizing current is increased by a lengthening of the gap, the copper loss increases more than does the reactive power. Therefore the loss ratio for the winding is increased and the quality factor Q_w is reduced by an increase in gap length.

The loss ratio for the reactor is the sum of the loss ratios for the magnetic circuit and winding, one of which is reduced while the other is increased by the increase in the gap length. To what extent, then, should the magnetizing current be increased by the insertion of an air gap when a minimum over-all loss ratio is desired? From Eq. 84,

$$\text{Over-all loss ratio} = \frac{P_c + I_c^2 R}{E\, I_m} + \frac{R\, I_m}{E}, \tag{52}$$

and for constant frequency and flux density, I_m is the only variable when the gap length is changed. To determine the value of I_m that results in a minimum loss ratio, Eq. 52 is differentiated with respect to I_m and the derivative set equal to zero; thus

$$0 = -\frac{P_c + I_c^2 R}{E\, I_m^2} + \frac{R}{E} \tag{53}$$

or

$$I_m^2 R = P_c + I_c^2 R. \tag{54}$$

Therefore if the frequency and the value of the flux density are maintained constant, the minimum loss ratio is obtained when the gap length is adjusted so that the copper loss due to the magnetizing current equals the sum of the core loss and the copper loss due to the core-loss current.