CHAPTER II

THEORY

2.1 Neutron Spectra in Reactor. (1)

The neutron spectra is generally defined as the neutron energy spectrum. Normally the neutron spectrum is reported as the number of neutrons per unit energy as a function of neutron energy, is represented by N(E) versus E. The neutron spectrum is divided into three regions.

1. Fast neutron region. This region deals with neutrons which have not been moderated. The fission neutrons exhibit a wide energy range, from zero to at least 15 Mev. The energy distributed is represented by the formula suggested by Watt as :

$$N(E) = \sinh (2E)^{0.5} \ e^{-E} \qquad \text{........(1)}$$

where N(E) = number of neutrons per unit energy

E = neutron energy in Mev.

The most accurate fission spectrum is shown in Fig.1

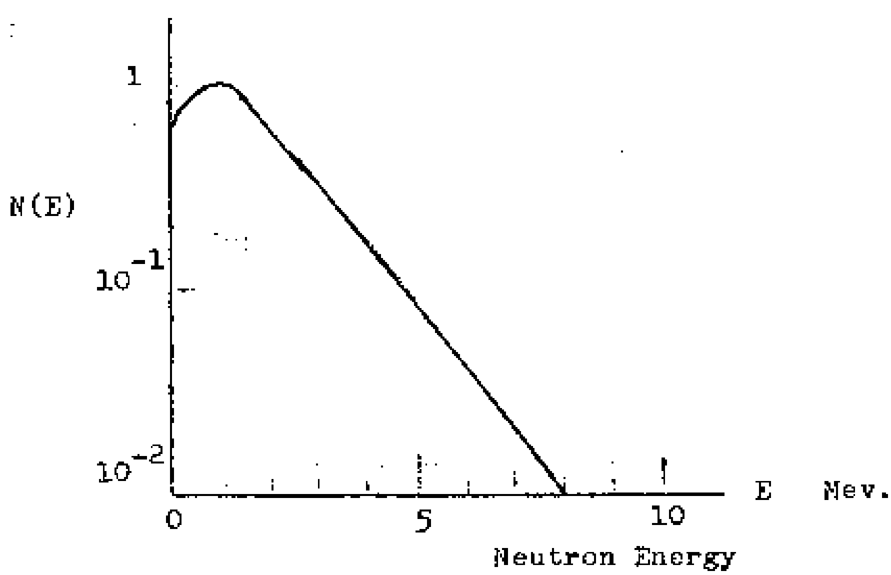

Fig.1 Fission Spectrum.

2. <u>Resonance Neutron Region.</u> As the neutrons from the fast region collide with moderator atoms, they are rapidly slowed down into the $\frac{1}{E}$ energy distribution region. Another name for this region is the resonance region, which ranging from about 1 Mev. down to lev. A convenient method of representing the spectrum of neutrons in the resonance region is by a relationship between neutron flux and neutron energy. A plot of $\frac{1}{E}$ spectrum is shown in Fig.2

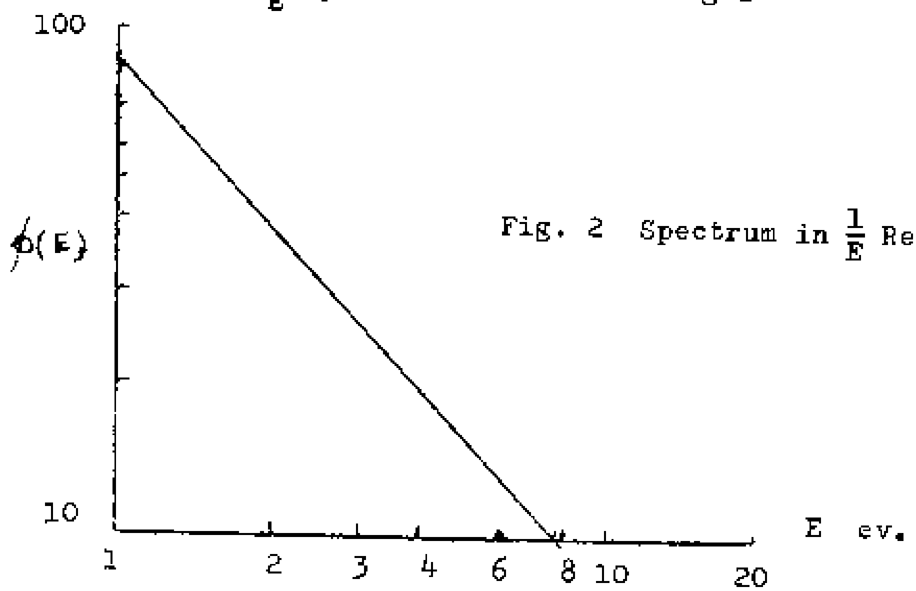

Fig. 2 Spectrum in $\frac{1}{E}$ Region

3. <u>Thermal Region.</u> After making elastic collisions with a moderator, the neutrons are slowed down. They will be slowed down until they have approximately the same average kinetic energy as the molecules of the moderator. The energy depends on the temperature of the medium, and so it is called the thermal neutron region. In this region, the energy exchange between the medium and the neutron is zero.

The kinetic energy of the neutrons of this region will be distributed statistically according to the Maxwell-Boltzmann distribution law, which is derived from the kinetic theory of gases. The Maxwell-Boltzmann distribution of velocity for a total of n_o neutron per unit volume is given by

$$dn = n(v)\,dv$$

$$= \frac{4\, n_o \pi}{\left(\frac{2\pi kT}{m}\right)^{\frac{3}{2}}}\, v^2 \exp\left(-\frac{\frac{1}{2}mv^2}{kT}\right) dv \quad \ldots\ldots(2)$$

where n_o = total neutron density

k = Boltzmann's constant

T = neutron temperature (absolute)

E = $kT = \frac{1}{2} mv^2$

The energy distribution can be written as:

$$dn = n(E)\,dE$$

$$= \frac{2\pi n_o}{(\pi kT)^{\frac{3}{2}}}\, E^{\frac{1}{2}} \exp\left(-\frac{E}{kT}\right) dE \quad \ldots\ldots(3)$$

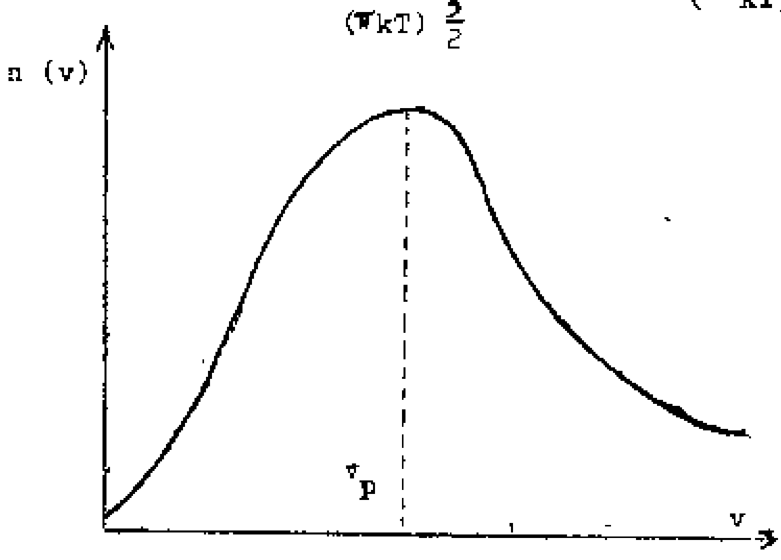

Fig. 3 Maxwell Spectrum in Thermal Region

The peak of the curve occurs at the most probable velocity v_p. The value of v_p can be found by taking the derivative of the neutron velocity distribution with respect to v, letting it equal to zero and solving for v.

The most probable velocity $v_p = \left(\frac{2kT}{m}\right)^{\frac{1}{2}}$
$= (1.648 \times 10^8 T)^{\frac{1}{2}}$

At room temperature 20°C, v_p will be 2200 m/sec, corresponds to an energy of 0.025 ev. The neutron average velocity can be obtained from

$$\bar{v} = \frac{\int_0^\infty n(v)\, v dv}{\int_0^\infty n(v)\, dv}$$

$$= \frac{2}{\sqrt{\pi}} v_p$$

2.2 Variation of Cross Section with Neutron Energy. (2)(3)

The neutron cross section depends not only on the nature of the target nucleus, but also on the neutron energy, they vary from one isotope to another of the same element. The neutron absorption cross section of a nuclide can be divided into three regions.

1. Low energy region. For neutron energies below 0.5ev. which includes the thermal range, the absorption cross section decreases steadily with increasing neutron energy. The absorption cross section is inversely proportional to the square root of the neutron energy. This is the $\frac{1}{v}$ region.

2. Resonance region. After the $\frac{1}{v}$ region, many elements, especially those of higher atomic weight, exhibit peaks called resonance peaks, where the neutron cross sections rise sharply to high values for certain neutron energies, and then fall again. These regions occur with neutrons of energy between 0.1 and 10 ev. Some elements, such as cadmium and rhodium have only one resonance peak ,others have more than one.

3. Fast neutron region. For neutrons of high energy, in the Mev range, the total cross sections are small for all nuclides, being less than 10 barns.

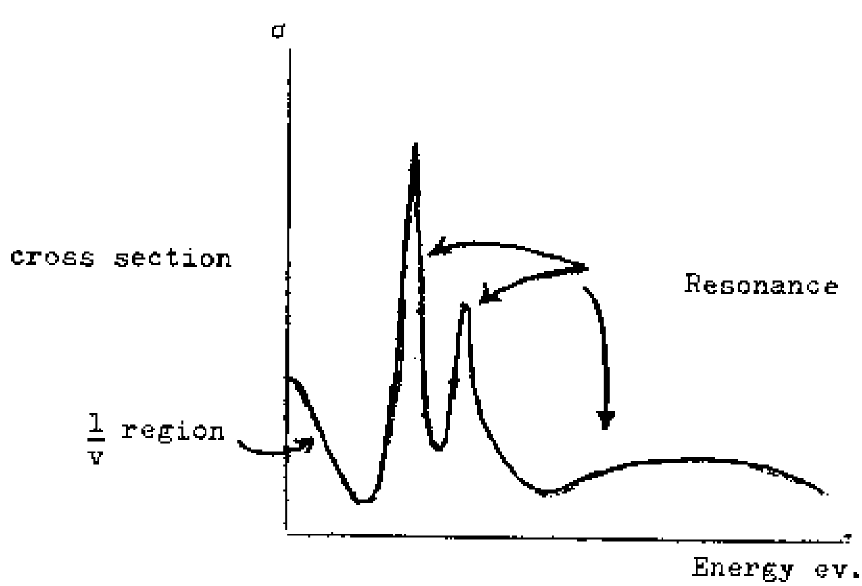

Fig.4 Variation of Neutron Cross Section with Energy for a Typical Nucleus

2.3 Flux Perturbation by Detecting Foils.(4)(5)(6)

Foils detectors have been used as instruments to measure the neutron flux. The observed activation of measuring foil in diffusing media must be corrected for the attenuation of the neutron flux in the foil and for the flux depression caused by the presence of the foil.

Consider a foil detector in a large volume of moderator in which thermal neutrons diffuse. Flux perturbation factor $\frac{\bar{\Phi}}{\phi}$ is defined as the ratio of the average perturbed flux to the average original flux in the volume of the foil. For foils of finite thickness, this ratio is less than unity, because of two effects.

1. The presence of a foil in the moderator will reduce the local flux. The foil absorbs neutrons which would have a chance to be scattered back by the surrounding medium. Consequently, this effect depends on the probability of absorption of the foil, and is referred to as flux depression. We define the ratio of the unperturbed flux to the flux at the foil surface as foil depression factor

2. The shielding of the inside of the foil by the outer layer. This effect depends on the absorption cross section and foil dimension. The foil may attenuate the flux internally, thus producing less activation density

internally than at the surface. Therefore, the ratio of the flux averaged through the volume of the foil to the incident flux averaged over the surface of the foil is less than unity. This ratio is defined as the self-shielding factor of the foil.

Neutron flux measurements made with absorbing foils require corrections for self-shielding. Since the neutron in a reactor is assumed to consist of resonance neutron and thermal neutron, the correction for self-shielding caused by the two components is called the resonance self-shielding factor, G_r, and the thermal self-shielding factor, G_{th}. The resonance self-shielding factor is obtained from Roe's report (6). The computed values of $1-G_r$ and G_r are plotted as function of Θ and τ

$$\Theta = \frac{4 E_o kT}{A \Gamma^2}$$

where E_o = energy of neutron at the center of the resonance

Γ = full width of the resonance at half maximum

A = atomic weight

kT = .025 ev.

τ = $N_a \sigma_o t$.

where N_a = number of atom/c.c

σ_o = peak cross section at the center of the resonance

t = thickness in cm.

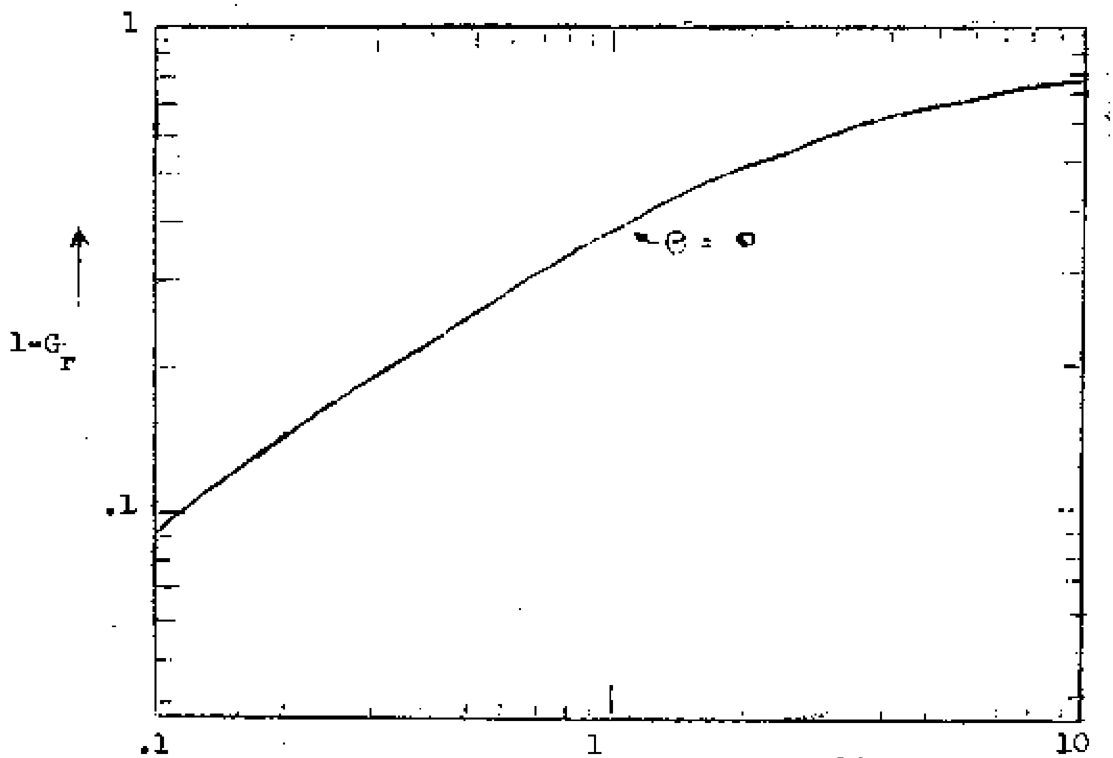

Fig.5 Self-Shielding Factor for Slabs $\tau \longrightarrow$

The **thermal** self-shielding factor can be calculated from the following expression (5) :

$$G_{th} = 1 - \frac{\Sigma t}{2} (.9228 - \ln \Sigma t)$$

where Σ = macroscopic absorption cross section

t = thickness in cm.

2.4 Westcott Effective Cross Section. (7)

The Neutron Spectrum is assumed to be made up of a thermal or Maxwellian component and an epithermal component. The reaction rate R, is equated to the product of the effective cross section $\hat{\sigma}$, and the conventional flux nv_o, where n is the neutron density in neutron/cm^3 and v_o is 2200 m/sec

$$R = \hat{\sigma} n v_o \qquad \text{..........(4)}$$

The effective cross section is defined by the relation

$$\hat{\sigma} = \sigma_o (g + rs) \qquad \text{..........(5)}$$

where σ_o is the monoenergetic 2200 m/sec cross section. The factor g and s are measures of the departure of cross section from $\frac{1}{v}$ law in the thermal and epithermal region respectively, for a $\frac{1}{v}$ absorber, $g = 1$ $s = o$ and r defines the proportion of epithermal neutron in the reactor spectrum. The effective cross section in a pure Maxwellian flux for which $r = o$, is $g\sigma_o$ The g and s factor can be used to deduce the epithermal ratio parameter **r** of a reactor spectrum from measured cadmium ratio.

2.5 <u>The Cadmium Ratio</u>. (1)(8)

Cadmium is commonly used in neutron activation experiments as an absorber for thermal neutrons. The reason for the use of cadmium is its high thermal cross section and an absorption resonance at 0.178 ev. The effective out off energy of cadmium filter is taken to be 0.4 ev.

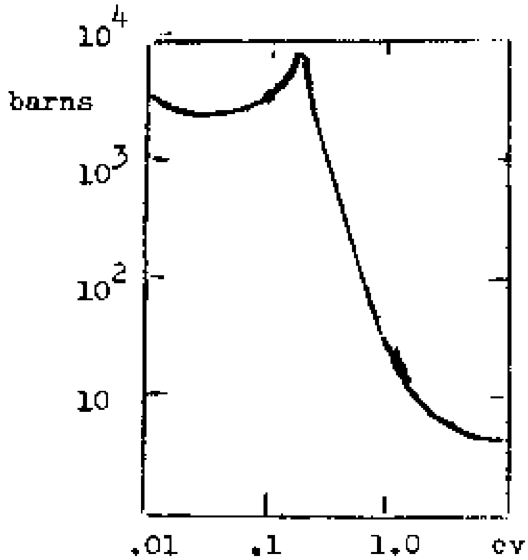

Fig.6 Total Cross Section of Cadmium as a function of Neutron Energy

The figure above shows the total absorption cross section of cadmium as a function of neutron energy in the range of energies from 0.1 to 10 ev. It becomes apparent from the curve that cadmium is opaque to thermal neutrons, but transparent to neutrons having energies greater than about 0.4 ev.

A convenient experimental method for the determination of the ratio of thermal to resonance flux is based on the cadmium ratio. The cadmium ratio is defined as the ratio of the reaction rate of a bare foil to the reaction rate of the cadmium-covered foil. The bare foil responds to the resonance plus the thermal flux, while the cadmium-covered foil to the resonance flux only, because the cadmium cover stops all the thermal neutrons but is transparent to those neutrons of energy above the cut off energy. The ratio of the thermal flux to the resonance flux is then given by

$$\frac{\text{thermal flux}}{\text{resonance flux}} = \text{CdR-1} \qquad \text{.........(6)}$$

Where CdR is the measured cadmium ratio. Thus the cadmium ratio (minus one) may be taken as a measure of how well the neutrons are thermalized the greater ratio, the greater the degree of thermalization

2.6 Westcott Epithermal Index.

There have been varieties of analytical methods to obtain informations concerning epithermal neutrons by measuring the cadmium ratio of the detecting foils. The result may be expressed as the ratio of epithermal flux per unit lethargy to the true thermal flux or as the Westcott epithermal index.

One method is simple and straight-forward. The expression is as follows (9):

$$\beta = \frac{\sigma}{\sigma_r(\text{CdR-1})} \qquad \text{........(7)}$$

where β stands for the ratio of the epithermal flux per unit lethargy to the true thermal flux. σ and σ_r are the thermal cross section and the resonance integral respectively. CdR represents the cadmium ratio which is, in general, experimentally determined.

It has to be realized that the foil must be practically infinitely thin. This condition is hardly met and corrections for self-shielding effect, especially for the epithermal neutrons, are necessary. In case of indium foil, the situation becomes more complicated due to the fact that cadmium is not an ideal filter with sharp cutoff resulting in some attenuation of the epithermal neutrons at the resonance region. Two other factors are, (a) part of the resonance peak being cut off by cadmium resulting in less epithermal activation and (b) the shift of cadmium cutoff with respect to cadmium thickness.

In Westcott's convention which has widely been followed with varieties of modifications. In the original form, the formula is as follows (10):

$$CdR = \frac{g + rs}{r(g + \frac{1}{K}\sqrt{\frac{T}{T_o}})} \quad \ldots\ldots\ldots\ldots(8)$$

Eq. (8) is applicable for infinitely thin foil. The definitions and tabulations of g, s, and K are given in various published literatures (10)(11)(12).

To include various correction factors into the equation, there are a number of modified versions as follows (12)(13) :

$$r\sqrt{\frac{T}{T_o}} = \frac{G_{th}}{(FCdR-1)\frac{S_o G_r}{g} + CdR\left(\frac{1}{K} - W\right)} \quad \ldots\ldots(9)$$

$$r\sqrt{\frac{T}{T_o}} = \frac{G_{th}}{(FCdR-1)\frac{S_o G_r}{g} + G_r\, CdR(\frac{1}{K} - W)} \quad \ldots\ldots\ldots(10)$$

where g, r, s are defined according to Westcott's notation as in 2.4. The epithermal index r , cannot be separated from the temperature term unless the temperature is independently determined, and $S_o = S\sqrt{\frac{T_o}{T}}$. F is the correction term for the attenuation of resonance neutrons by cadmium. The factor K is a coefficient calculated from the variation of the cadmium cross section with neutron energy and thickness of cadmium used in the irradiation. W is the correction for the part of the resonance peak shielded by cadmium. G_{th} is the correction for the perturbation of thermal flux by the detecting foil. And G_r is the correction for the self-shielding effect by the foil for the epithermal neutrons.