II THEORY

General analysis of snap buckling of bowed strut by energy criterion.

A small curved, bowed strut subjected to a central lateral coocentrated load P as shown in figure 3. is analyzed.

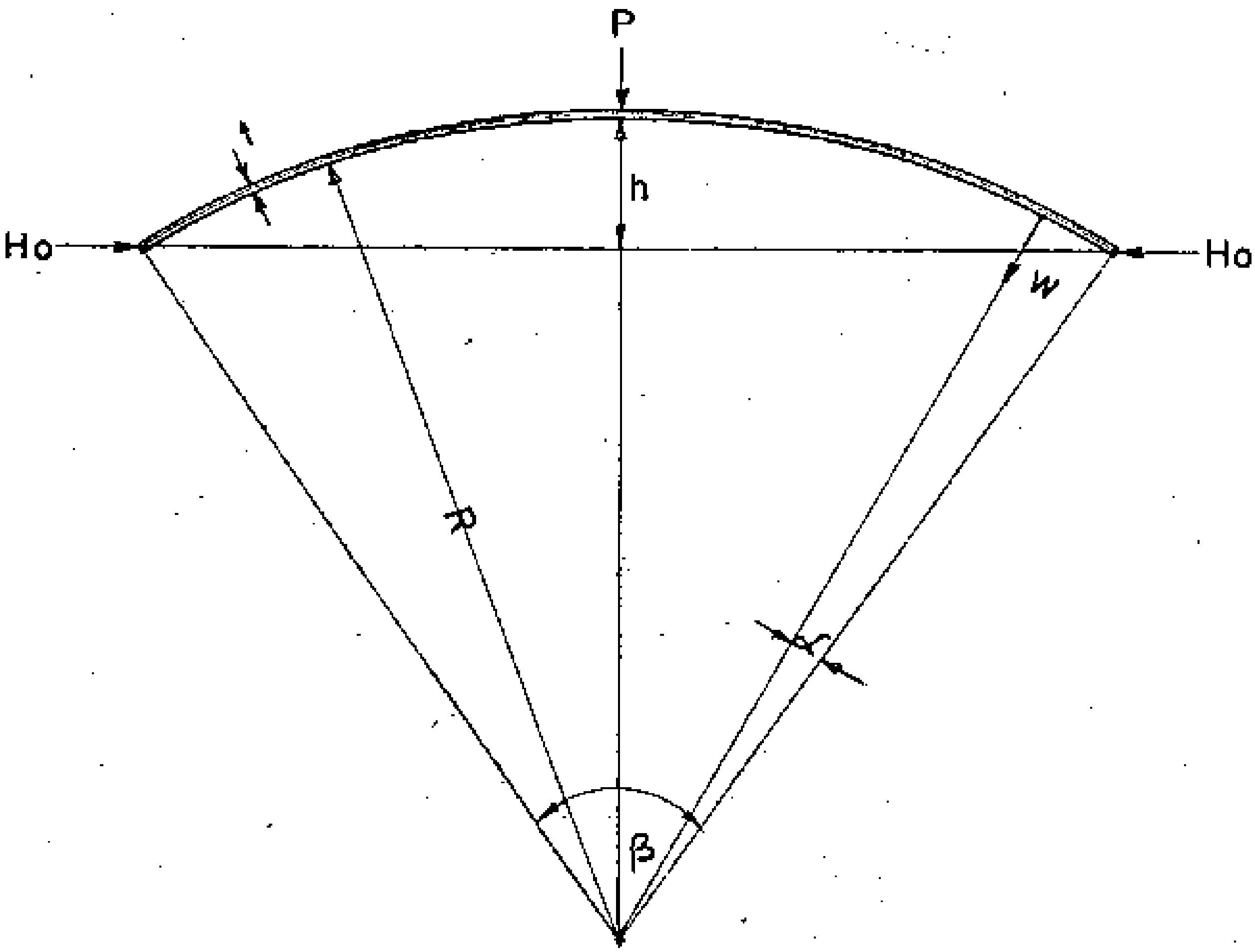


Figure 3. A small curved, bowed strut under a central lateral concentrated load.

From Journal of the Engineering Mechanics Division (3), the axial strain ϵ for thin curved beam is

$$\epsilon = - \frac{1}{R\beta} \int_0^{\beta} (W - \frac{1}{2R} W_\alpha^2) \, d\alpha \quad \text{..................(1)}$$

and K, the change in curvature, is

$$K = \frac{1}{R^2} W_{\alpha\alpha} \quad \text{.....................................(2)}$$

in which W is the radial displacement function, R is the redius of the curvature, α is the polar angle measured from one end of the bowed strut, and β is the included angle of the bowed strut. First and Second differentiation with respect to α are denoted by the subscripts α and $\alpha\alpha$ respectively.

The strain energy due to the axial deformation U_d, non-dimensionalized by division by the term of EtfR, is

$$U_d = \frac{H_o \beta}{Etf} \epsilon + \frac{1}{2} \beta \epsilon^2 \quad \text{...........................(3)}$$

in which H_o is the initial axial thrust built in the bowed strut before application of the lateral load P, t is the thickness and f is the width of the strut, and E is Young's modulus. The strain energy due to bending U_b in non-dimensional form, is

$$U_b = \frac{t^2}{24} \int_0^{\beta} K^2 \, d\alpha \quad \text{...................................(4)}$$

The workdone by the lateral load P in non-dimensional form U_p, is

$$U_p = - \frac{P}{EtfR} (W)_{\alpha = \frac{\beta}{2}} \quad \text{...............................(5)}$$

The total energy U_T in non-dimensional form is given by

$$U_T = U_d + U_b + U_p \qquad \dots\dots(6)$$

Substituting the values U_d, U_b and U_p from Eq. 3, 4 and 5 respectively into the energy expression Eq. 6. Then the total energy U_T, becomes

$$U_T = \frac{H_o \beta}{Etf} \epsilon + \frac{1}{2} \beta \epsilon^2 + \frac{t^2}{24} \int_0^\beta K^2 \, d\alpha - \frac{P}{EtfR} (W)_{\alpha = \frac{\beta}{2}} \qquad \dots\dots(7)$$

Substituting the values for ϵ and K from Eq. 1 and 2 into Eq. 7, gives the total energy U_T as a function of W only

$$U_T = \frac{1}{2R^2 \beta} \left[\int_0^\beta (W - \frac{1}{2R} W_\alpha^2) \, d\alpha \right]^2 - \frac{H_o}{EtfR} \left[\int_0^\beta (W - \frac{1}{2R} W_\alpha^2) \, d\alpha \right]$$

$$+ \frac{t}{24R^4} \int_0^\beta W_{\alpha\alpha}^2 \, d\alpha - \frac{P}{EtfR} (W)_{\alpha = \frac{\beta}{2}} \qquad \dots\dots(8)$$

<u>Approximate solution</u>: An approximate solution can be obtained by considering that the deflection W can be represented by only two terms throughout the loading history. Let

$$W = B_1 W_1(\xi) + B_2 W_2(\xi) \qquad \dots\dots(9)$$

in which W_1 is a symmetric and W_2 an antisymmetric function in ξ; ξ is the ratio α/β, B_1 and B_2 are the amplitudes of the two deflected shapes. Substituting this value of W into the energy expression and replacing α by $\beta\xi$, then

$$U_T = \frac{1}{2R^2\beta}\left[\beta B_1 C_1 + \beta B_2 C_2 - \frac{1}{2R\beta}(B_1^2 C_3 + 2B_1B_2C_4 + B_2^2 C_5)\right]^2$$

$$- \frac{H_o}{EtfR}\left[\beta B_1 C_1 + \beta B_2 C_2 - \frac{1}{2R\beta}(B_1^2 C_3 + 2B_1B_2C_4 + B_2^2 C_5)\right]$$

$$+ \frac{t^2}{24R^4\beta^3}(B_1^2 C_6 + 2B_1B_2C_7 + B_2^2 C_8) - \frac{P}{EtfR}(B_1C_9 + B_2C_{10}) \quad \ldots(10)$$

in which the constants C_i are given by the integrals:

$$C_1 = \int_0^1 W_1 \, d\xi \quad \ldots\ldots\ldots\ldots(11a)$$

$$C_2 = \int_0^1 W_2 \, d\xi \quad \ldots\ldots\ldots\ldots(11b)$$

$$C_3 = \int_0^1 W_{1\xi}^2 \, d\xi \quad \ldots\ldots\ldots\ldots(11c)$$

$$C_4 = \int_0^1 W_{1\xi} W_{2\xi} \, d\xi \quad \ldots\ldots\ldots\ldots(11d)$$

$$C_5 = \int_0^1 W_{2\xi}^2 \, d\xi \quad \ldots\ldots\ldots\ldots(11e)$$

$$C_6 = \int_0^1 W_{1\xi\xi}^2 \, d\xi \quad \ldots\ldots\ldots\ldots(11f)$$

$$C_7 = \int_0^1 W_{1\xi\xi} W_{2\xi\xi} \, d\xi \quad \ldots\ldots\ldots\ldots(11g)$$

$$C_8 = \int_0^1 W_{2\xi\xi}^2 \, d\xi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11h)$$

$$C_9 = \left[W_1 \right]_{\xi = \frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11i)$$

$$C_{10} = \left[W_2 \right]_{\xi = \frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11j)$$

Introducing the non-dimensional amplitudes,

$$b_1^* = \frac{B_1}{R\beta^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12a)$$

$$b_2^* = \frac{B_2}{R\beta^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12b)$$

and a geometric parameter

$$\lambda^* = \frac{R\beta^2}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

non-dimensional load and axial thrust

$$P^* = \frac{PR}{Et^2 f\beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14a)$$

$$H^* = \frac{H_{oR}}{Et^2 f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14b)$$

Then the energy can be written as:

$$U_T^* = \frac{U_T \lambda^*}{\beta 5} = \frac{\lambda^*}{2}\left[b_1^* C_1 + b_2^* C_2 - \frac{1}{2}\left(b_1^{*2} C_3 + 2 b_1^* b_2^* C_4 + b_2^{*2} C_5\right)\right]^2$$

$$- H^*\left[b_1^* C_1 + b_2^* C_2 - \frac{1}{2}\left(b_1^{*2} C_3 + 2 b_1^* b_2^* C_4 + b_2^{*2} C_5\right)\right]$$

$$+ \frac{1}{24\lambda^*}\left(b_1^{*2} C_6 + 2 b_1^* b_2^* C_7 + b_2^{*2} C_8\right) - P^*\left(b_1^* C_9 + b_2^* C_{10}\right) \quad \ldots\ldots(15)$$

Because of the symmetry and anti-symmetry of W_1 and W_2, it can be seen immediately that $C_2 = C_4 = C_7 = C_{10} = 0$.

The total energy U_T^* therefore reduces to

$$U_T^* = \frac{\lambda^*}{2}\left[b_1^* C_1 - \frac{1}{2}\left(b_1^{*2} C_3 + b_2^{*2} C_5\right)\right]^2 - H^*\left[b_1^* C_1 - \frac{1}{2}\left(b_1^{*2} C_3 + b_2^{*2} C_5\right)\right]$$

$$+ \frac{1}{24\lambda^*}\left(b_1^{*2} C_6 + b_2^* C_8\right) - P^*\left(b_1^* C_9\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

For equilibruim $\frac{\partial U_T^*}{\partial b_1^*} = 0$ and $\frac{\partial U_T^*}{\partial b_2^*} = 0$, which yields the following two equations:

$$P^* C_9 = \left[\lambda^*\left(b_1^* C_1 - \frac{1}{2} b_1^{*2} C_3 - \frac{1}{2} b_2^{*2} C_5\right) - H^*\right]\left[C_1 - b_1^* C_3\right] + \frac{1}{12\lambda^*}\left(b_1^* C_6\right) \ldots(17)$$

$$b_2^* C_5 H^* - \lambda^* b_2^* C_5\left[b_1^* C_1 - \frac{1}{2}\left(b_1^* C_3 + b_2^{*2} C_5\right)\right] + \frac{1}{12\lambda^*} b_2^* C_8 = 0 \quad \ldots(18)$$

Eq. 18 has the solutions

$$b_2^* = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

$$b_2^* = \sqrt{\frac{2}{C_5}\left(b_1^* C_1 - \frac{1}{2} b_1^{*2} C_3 - \frac{H^*}{\lambda^*} - \frac{1}{12\lambda^{*2}} \cdot \frac{C_8}{C_5}\right)} \quad \ldots\ldots\ldots(20)$$

If the quantity under the root is negative, the only real solution is $b_2^* = 0$. The deformation will then be symmetrical. The condition that a real solution for b_2^* exists other than $b_2^* = 0$ is, therefore, that

$$b_1^* C_1 - \frac{1}{2} b_1^{*2} C_3 - \left(\frac{1}{12\lambda^{*2}} \cdot \frac{C_8}{C_5} + \frac{H^*}{\lambda^*}\right) \geqslant 0 \quad \ldots\ldots\ldots\ldots(21)$$

which, when solved for b_1^*, yields

$$b_1^* \geqslant \frac{C_1}{C_3} - \left[\left(\frac{C_1}{C_3}\right)^2 - \frac{2H^*}{\lambda^* C_3} - \frac{C_8}{6\lambda^{*2} C_3 C_5}\right]^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots(22a)$$

$$b_1^* \leqslant \frac{C_1}{C_3} + \left[\left(\frac{C_1}{C_3}\right)^2 - \frac{2H^*}{\lambda^* C_3} - \frac{C_8}{6\lambda^{*2} C_3 C_5}\right]^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots(22b)$$

For "b_1^*" real, these relations will be satisfied over a finite interval if the quantity in the root is greater than zero. The necessary condition, therefore, that the anti-symmetric component be non-zero is that

$$\lambda^* \geqslant \frac{1}{C_1}\left\{H^* \cdot \frac{C_3}{C_1} + \left[\left(H^* \cdot \frac{C_3}{C_1}\right)^2 + \frac{C_3 C_8}{6\, C_5}\right]^{\frac{1}{2}}\right\} \quad \ldots\ldots\ldots\ldots\ldots(23)$$

If Eq. 23 is not satisfied, the load-deflection Eq. is obtained by setting $b_3^* = 0$ in the equilibrium Eq. 17,

$$P^* = \frac{1}{12\lambda^*} \cdot \frac{C_6}{C_9} \cdot b_1^* + \frac{C_3}{C_9}\left[\frac{C_1}{C_3} - b_1^*\right]\left[\frac{\lambda^* b_1^* C_3}{2}\left(2\frac{C_1}{C_3} - b_1^*\right) - H^*\right] \quad \ldots.(24)$$

Such a curve is plotted in Fig. 4. It can be considered to be composed of a straight line due to the bending stress, and of a curve anti-symmetrical about $b_1^* = \frac{C_1}{C_3}$ due to the axial stress.

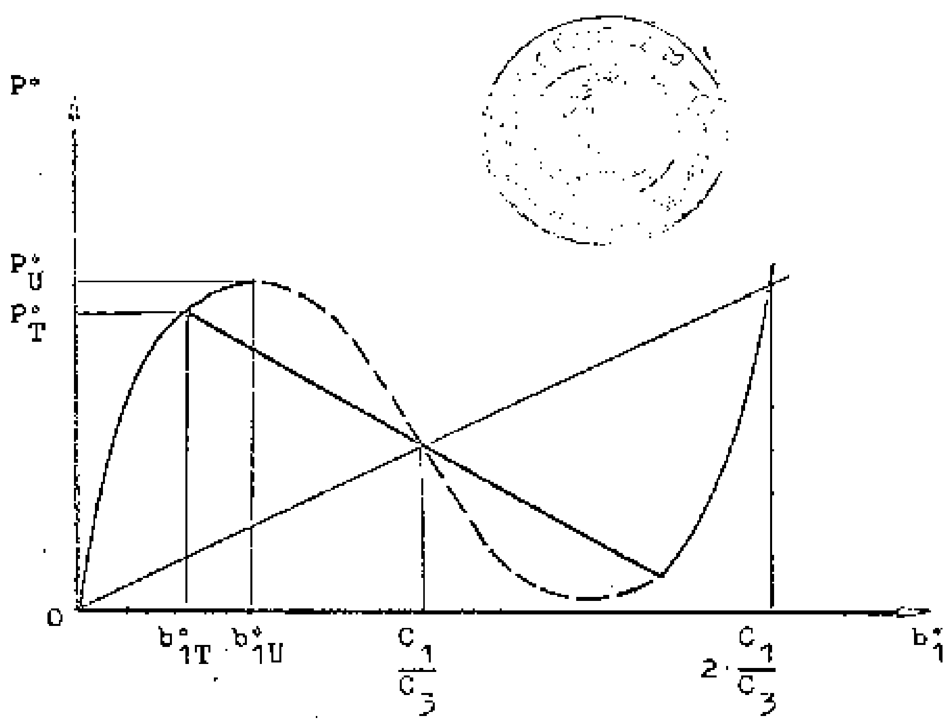


Figure 4. Load-deflection curve for bow strut showing anti-symmetrical transitional mode.

The range of stability can be found from the second variation of the energy. Substituting $b_2^* = 0$ into the total energy (Eq. 16) and differentiating twice with respect to b_1^* yields.

$$\frac{\partial^2 U_T^*}{\partial b_1^{*2}} = \lambda^*\left[(C_1 - b_1^* C_3)^2 - b_1^* C_1 C_3 + \frac{b_1^{*2} C_3^2}{2}\right] + \frac{C_6}{12\lambda^*} + H^* C_3 \quad \text{.......(25)}$$

For stability, this second variation must be greater or equal to zero. Solving for b_1^*, yields

$$b_1^* \leqslant \frac{C_1}{C_3} - \left[\frac{1}{3}\left(\frac{C_1}{C_3}\right)^2 - \frac{2H^*}{3\lambda^* C_3} - \frac{C_6}{18\,\lambda^{*2} C_3^2}\right]^{\frac{1}{2}} \quad \text{..............(26a)}$$

$$b_1^* \geqslant \frac{C_1}{C_3} + \left[\frac{1}{3}\left(\frac{C_1}{C_3}\right)^2 - \frac{2H^*}{3\lambda^* C_3} - \frac{C_6}{18\,\lambda^{*2} C_3^2}\right]^{\frac{1}{2}} \quad \text{.............(26b)}$$

The region of the load-deflection curve that is unstable vanishes when the quantity under the root in Eq. 26a and 26b becomes zero. An unstable region will therefore exist for

$$\lambda^* > \frac{1}{C_1}\left\{H^* \cdot \frac{C_3}{C_1} + \left[\left(H^* \cdot \frac{C_3}{C_1}\right)^2 + \frac{C_6}{6}\right]^{\frac{1}{2}}\right\} \quad \text{.................(27)}$$

and instability will occur after the equality sign in Eq. 26a is satisfied. Substituting this value for b_1^* into the load-deflection relationship (Eq. 24) gives the upper buckling load for the symmetrical mode:

$$P_u^* = \frac{1}{12\lambda^*} \cdot \frac{C_1 C_6}{C_3 C_9} + \lambda^* \frac{C_3^2}{C_9}\left[\frac{1}{3}\left(\frac{C_1}{C_3}\right)^2 - \frac{2H^*}{3\lambda^* C_3} - \frac{C_6}{18\,\lambda^{*2} C_3^2}\right]^{\frac{3}{2}} \quad \text{.....(28)}$$

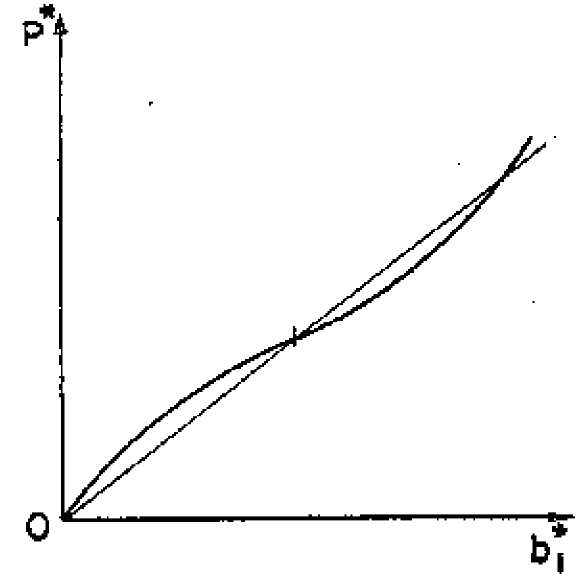
If the inequality (Eq. 23) is satisfied, b_2^* is not identically zero but is given by Eq. 20 in the interval given by Eqs. 22a and 22b. Substituting for b_2^* from Eq. 20 into Eq. 17 yields the load-deflection relationship in this interval:

$$P^* = \frac{1}{12\lambda^*}\cdot\frac{C_6}{C_9}\, b_1^* + \frac{1}{12\lambda^*}\frac{C_3\; C_8}{C_5\; C_9}\left(\frac{C_1}{C_3} - b_1^*\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(29)$$

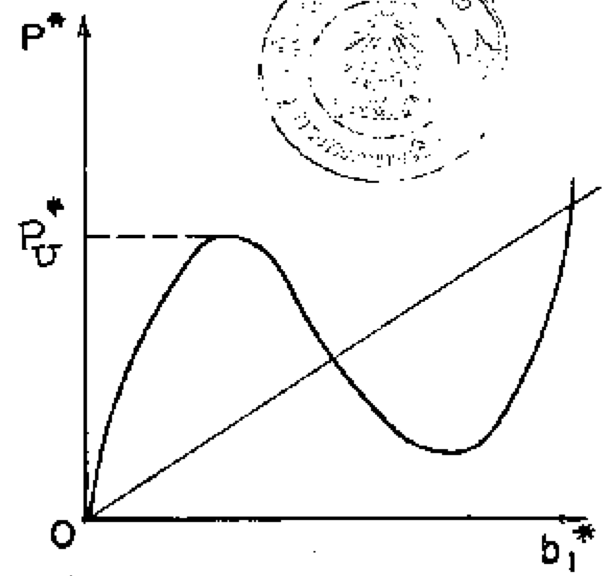
This is a straight line passing through $b_1^* = \frac{C_1}{C_3}$. The total load deflection curve will then be given by Eq. 24, except for the values of b_1^* in the interval (Eqs. 22a and 22b), in which it will be given by Eq. 29. This is plotted in Fig. 4. The value of b_1^* at which the nonsymmetrical transition mode enters, b_{1T}^*, is obtained from the equal sign in the inequality (Eq. 22a). However, buckling may already have occurred in the symmetrical mode before this value of b_1^* is reached. The condition for buckling in the nonsymmetrical mode must be $b_{1T}^* \leqslant b_{1U}^*$ in which b_{1U}^* is the deflection at the upper buckling load (Fig. 4). Substituting for b_{1U}^* and b_{1T}^* from Eqs. 26a and 22a on making both of these inequalities into equalities yields, for $b_{1T}^* \leqslant b_{1U}^*$

$$\lambda^* \geqslant \frac{1}{C_1}\left\{H^* \frac{C_3}{C_1} + \left[\left(H^* \frac{C_3}{C_1}\right)^2 + \frac{1}{12\, C_5}(3C_3C_8 - C_5C_6)\right]^{\frac{1}{2}}\right\} \quad \ldots\ldots\ldots(30)$$

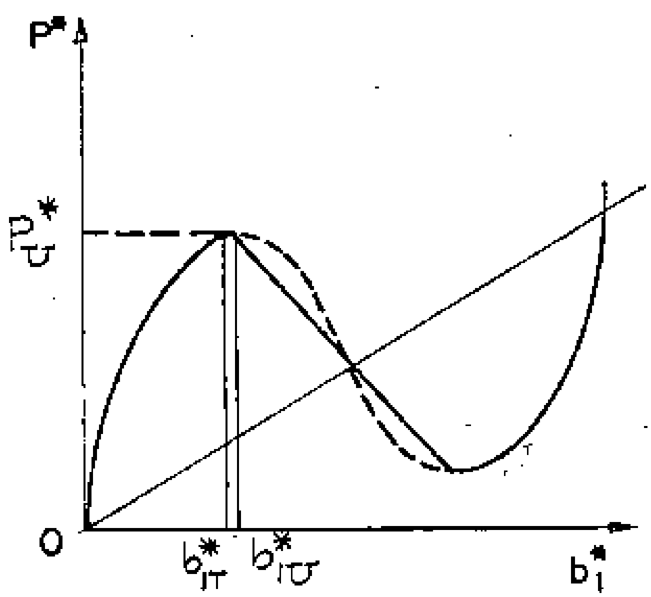
The behavior of the bowed strut for different values of λ^* is summarized in Fig. 5. The nonsymmetrical buckling load P_T^* is obtained by substituting for b_1^* from the equality condition of Eq. 22a into Eq. 24.



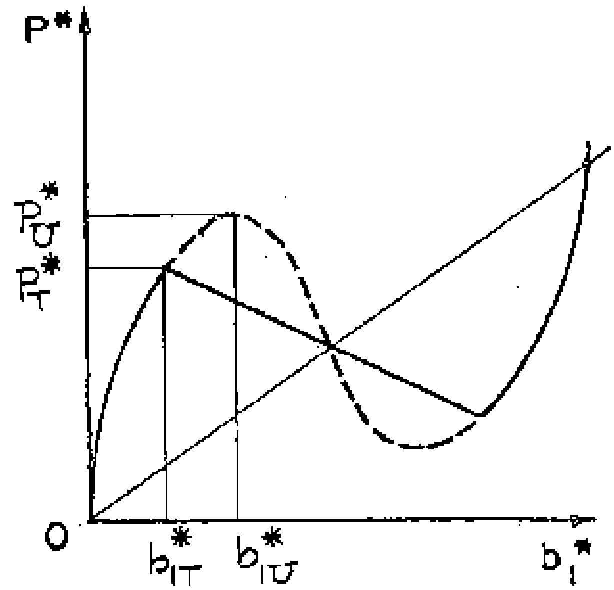
(a) No buckling



(b) Symmetrical buckling



(c) Symmetrical buckling, with transitional mode present in unstable region



(d) Transitional anti-symmetrical buckling

Figure 5. Variation of buckling modes with λ^*

$$P_T^* = \frac{1}{12\lambda^*}\,\frac{C_1\,C_6}{C_3\,C_9} + \frac{1}{12\lambda^*}\left[\frac{C_3\,C_8}{C_5\,C_9} - \frac{C_6}{C_9}\right]\left[\left(\frac{C_1}{C_3}\right)^2 - 2\,\frac{H^*}{\lambda^* C_3} - \frac{C_8}{6\,\lambda^{*2} C_3 C_5}\right]^{\frac{1}{2}} \quad \ldots(31)$$

Then both critical buckling loads P_U^* and P_T^* can be determined reasonably easily by evaluating the constants C_i.

<u>Numerical solutions</u>:

<u>Case 1</u> For a hinged bowed strut, the general displacement function that satisfies the boundary conditions is

$$W = B_1 \sin \pi \xi + B_2 \sin 2\pi \xi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(32)$$

This will also satisfy the orthogonality relationships. Then the constants (Eq. 11) can be readily evaluated,

$$C_1 = 0.63636 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(33a)$$

$$C_3 = 4.93875 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(33b)$$

$$C_5 = 19.75500 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(33c)$$

$$C_6 = 48.78250 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(33d)$$

$$C_8 = 780.52000 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(33e)$$

$$C_9 = 1.00000 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(33f)$$

Therefore the upper buckling load P_U^* of Eq. 28 with the necessary condition λ^* of Eq. 27 and Eq. 23, and the nonsymmetrical buckling load P_T^* of Eq. (31) with the necessary condition λ^* of Eq. 23 are rewritten as respectively:

$$P_U^* = \frac{0.52380}{\lambda^*} + 24.39125\ \lambda^*\left[0.00553 - 0.13498\cdot\frac{H}{\lambda^*} - \frac{0.11111}{\lambda^{*2}}\right]^{\frac{3}{2}} \quad \ldots(34)$$

When

$$\lambda^* > \left[12.19595\ H^* + (148.74119\ H^{*2} + 20.07756)^{\frac{1}{2}}\right] \quad \ldots\ldots\ldots\ldots\ldots(35a)$$

$$\text{and } \lambda^* \leqslant \left[12.19595H^* + (148.74119\ H^{*2} + 80.31030)^{\frac{1}{2}}\right] \quad \ldots\ldots\ldots(35b)$$

$$P_T^* = \frac{0.5238}{\lambda^*} + \frac{12.19562}{\lambda^*}\left[0.01660 - 0.40496\ \frac{H^*}{\lambda^*} - \frac{1.33333}{\lambda^{*2}}\right]^{\frac{1}{2}} \quad \ldots(36)$$

When

$$\lambda^* \geqslant \left[12.19595\ H^* + (148.74119\ H^{*2} + 80.31030)^{\frac{1}{2}}\right] \quad \ldots\ldots\ldots\ldots\ldots(37)$$

<u>Case 2</u> For a clamped bowed strut, the general displacement function that satisfies the boundary conditions is

$$W = \frac{B_1}{2}(1-\cos 2\pi\xi) + \frac{B_2}{2}(1-\cos 4\pi\xi) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(38)$$

as previously:

$$C_1 = 0.50000 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(39a)$$

$$C_3 = 4.93875 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(39b)$$

$$C_5 = 19.75500 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(39c)$$

$$C_6 = 195.13000 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39d)$$

$$C_8 = 3122.08000 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39e)$$

$$C_9 = 1.00000 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39f)$$

$$P_U^* = \frac{1.64624}{\lambda^*} + 24.39125\,\lambda^*\left[0.00341 - 0.13498\,\frac{H^*}{\lambda^*} - \frac{0.44444}{\lambda^{*2}}\right]^{\frac{3}{2}} \quad \ldots (40)$$

When

$$\lambda^* > \left[19.755\,H^* + (390.26002\,H^{*2} + 130.08664)^{\frac{1}{2}}\right] \ldots\ldots\ldots\ldots (41a)$$

$$\text{and } \lambda^* \leqslant \left[19.755\,H^* + (390.26002\,H^{*2} + 520.34664)^{\frac{1}{2}}\right] \ldots\ldots\ldots\ldots (41b)$$

$$P_T^* = \frac{1.64624}{\lambda^*} + \frac{48.78250}{\lambda^*}\left[0.01024 - 0.40496\,\frac{H^*}{\lambda^*} - \frac{5.33333}{\lambda^{*2}}\right]^{\frac{1}{2}} \quad \ldots (42)$$

When

$$\lambda^* \geqslant \left[19.755\,H^* + (390.26002\,H^{*2} + 520.34664^{\frac{1}{2}}\right] \ldots\ldots\ldots\ldots (43)$$

<u>General analysis of the snap buckling load of a bowed strut by the classical criterion:</u> This different criterion has been applied to the small curved, hinged bowed strut by Fung and Kaplan who used two Fourier series to represent the initial shape Y_o and deflected shape y, after application of the lateral central concentrated load P, of the center line of bowed strut

$$Y_o = \sum_{m=1}^{\infty} A_m \sin \frac{m\pi x}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44a)$$

and $$Y = \sum_{m=1}^{\infty} B_m \sin \frac{m\pi x}{L} \qquad \text{.......................(44b)}$$

in which A_m and B_m are the amplitudes of the initial and deflected shape respectively, L is the span of the bowed strut. Assuming that the bowed strut is made of homogeneous material, of constant cross section and with small curvature so that Y_x^2 is negligible in comparison with 1; and that thickness of the strut is much smaller than the radius of curvature of the bowed strut. Then the usual beam theory gives

$$EI\,(y_{xx} - y_{oxx}) = M_b + H_o y_o - Hy \qquad \text{.......................(45)}$$

in which M_b and H are the bending moment and axial thrust built in the bowed strut due to the application of a lateral central concentrated load P. The bending moment M_b can be expressed in form of Fourier series

$$M_b = \frac{2PL}{\pi^2} \sum_{m=1}^{\infty} \frac{1}{m^2} \left(\sin \frac{m\pi}{2}\right) \sin \frac{m\pi x}{L} \qquad \text{................(46)}$$

and

$$H = H_o + \frac{ftE}{2L} \int_o^L (y_{ox}^2 - y_x^2)\, dx \qquad \text{.....................(47a)}$$

substituting y_o and y from Eq. 44 and Eq. 45 intn Eq. 47a then becomes

$$H = H_o + \frac{\pi^2 Eft}{4L^2} \sum_m m^2 (A_m^2 - B_m^2) \qquad \text{.....................(47b)}$$

Substituting these value for M_b, H, y_o and y into Eq. 45, then the equation of equilibrium can be obtained and expressed in terms of the Fourier Co-efficients

$$\frac{\pi^2 EI}{L^2} \sum m^2 (A_m - B_m) \sin \frac{m\pi x}{L} = \frac{2PL}{\pi^2} \sum \frac{1}{m^2} (\sin \frac{m\pi}{2}) \sin \frac{m\pi x}{L}$$

$$+ H_o \sum (A_m - B_m) \sin \frac{m\pi x}{L}$$

$$- \frac{\pi^2 Eft}{4L^2} \sum m^2 (A_m^2 - B_m^2) \sum B_m \sin \frac{m\pi x}{L} \quad ..(48)$$

By equating the co-efficients of the corresponding terms in the right and left hand side, therefore a set of an infinite number of simultaneous equations is obtained

$$\frac{\pi^2 Eft}{4L^2} \left[\sum n^2 (A_n^2 - B_n^2) \right] B_m + \left(\frac{\pi^2 EI}{L^2} m^2 - H_o \right)(A_m - B_m) =$$

$$\frac{2PL}{\pi^2} \frac{1}{m^2} (\sin \frac{m\pi}{2}) \text{ (where } m = 1,2,3,4...) \quad ...(49)$$

To simplify the expressions, some non-dimensional terms have been introduced by Fung & Kaplan:

$$\lambda_m = 1.732 \frac{Am}{t} \quad ....(50a)$$

$$b_m = 1.732 \frac{Bm}{t} \quad ....(50b)$$

$$R = 1.732 \frac{PL^3}{\pi^4 EIt} \quad ....(50c)$$

$$S = \frac{H_o L^2}{\pi^2 EI} \quad \text{..................................(50d)}$$

Then the general equilibrium Eq. 49 becomes

$$b_m\left[\sum m^2\lambda_n^2 - \sum n^2 b_n^2 - (m^2 - S)\right] = \frac{2R}{m^2}\left(\sin\frac{m\pi}{2}\right) - \lambda_m(m^2 - S)$$

(where m = 1, 2, 3 , 4 ....) ....(51)

<u>A hinged sinusoidal bowed strut under a lateral central concentrated load.</u>

In order to get a simple solution, a sinusoidal hinged bowed strut subjected to a lateral central concentrated load P, as shown in Fig. 6, is considered

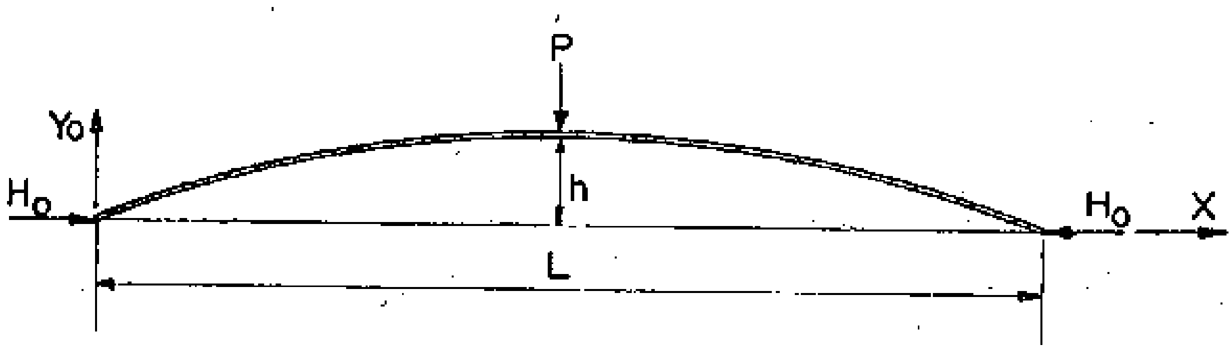


Figure 6. A lateral central concentrated load on a hinged sinusoidal bowed strut.

therefore $$Y_o = A_1 \sin\frac{\pi X}{L} \quad \text{........................(52)}$$

then $$\lambda_2 = \lambda_3 = \lambda_4 = \dots = 0 \quad \text{................(53)}$$

As previous section, the deflection curve is represented by two terms function again

$$Y = B_1 \sin \frac{\pi X}{L} + B_2 \sin \frac{2\pi X}{L} \quad \text{.....................(54)}$$

i.e. $b_3 = b_4 = b_5 = \ldots\ldots = 0$ ....(55)

From these particular cases, therefore both of critical buckling load R_T and R_U with the necessary conditions can be obtained from the equilibrium Eq. 51, yields

$$R_T = \frac{1}{2}\left[(1-S)\lambda_1 + 3(\lambda_1^2 + S-4)^{\frac{1}{2}}\right] \quad \text{...............(56)}$$

$$\text{for } \lambda_1 \geqslant (5.5 - S)^{\frac{1}{2}} \quad \text{...............(57)}$$

and

$$R_U = \frac{1}{2}\left[(1-S)\lambda_1 + 2\left(\frac{\lambda_1^2 + S-1}{3}\right)^{\frac{3}{2}}\right] \quad \text{...............(58)}$$

$$\text{for } (1-S)^{\frac{1}{2}} < \lambda_1 \leqslant (5.5-S)^{\frac{1}{2}} \quad \text{...............(59)}$$