CHAPTER V

ISOTOPE EFFECT AND VAN HOVE SINGULARITY IN HIGH-T_c SUPERCONDUCTORS

In the previous chapter the small oxygen isotope effect in various copper oxides has been considered as an important piece of evidence for non-phononic superconductivity. However, many of the superconducting state characteristics show that the new copper oxides are BCS - like superconductors. These are NMR [87] and photoelectron spectrocopy [88]. In this chapter, we wish to show that the absence of the oxygen isotope effect can be understood in terms of a van Hove singularity in the density of states $N(\xi)$, near the Fermi energy E_F along with a conventional BCS phonon-mediated pairing which is also responsible for the high temperature superconductivity.

The van Hove singularity in the density of states $N(\xi)$ has been proposed in the past as a T_c- enchancement mechanism for the conventional A15 superconductors [89-91] and more recently for the high-T_c copper oxides [92-95]. Although one-electron band structure calculations [96, 97] have shown that the copper oxides are characterized by some two dimensional Fermi surface nesting, the idea of achieving high-T_c superconductivity with the aid of the sharp peak in $N(\xi)$ at E_F along with phonon-mediated pairing has not been widely accepted. The main objections to this high-T_c mechanism mostly stem from the fact that there is no convincing direct experimental evidence since $N(\xi)$ can be easily smeared out by disorder due to doping and various structural defects.

Furthermore, the effectiveness of a van Hove singularity in enhancing T_c is supposed to diminish greatly if there is a slight shift of E_F from the singularity. This high-T_c mechanism has been resurrected recently on the basis of the recent

observation of the linear correlation of T_c with the resistivity along the copper-oxide plane, $\rho_{//}(T)$. It is suggested that the proximity of the van Hove singularity to the Fermi level determines the dependence of $\rho_{//}(T)$, i.e., $\rho_{//}(T)$ is proportional to T or T^2, and the magnitude of T_c. It is estimated that, even with a modest electron-phonon interaction ($\lambda < 1$), a T_c of the order of 100 K can be achieved with the aid of a logarithmic singularity in $N(\xi)$ near E_F. It is known that, although T_c can be limited by various $N(\xi)$ broadening effects on the scale of $k_B T_c$, these effects are not sufficient to invalidate this high-T_c mechanism for copper oxide superconductors. A recent slave-boson mean-field band-structure calculation [98] for CuO_2 layered systems indicates that the Fermi level is pinned very closely to the nearly logarithmic van Hove singularity. Tsuei et. al.[30], show that the combination of a two dimensional van Hove singularity in the hole density of states, combined with a BCS phonon-mediated pairing, can lead to a substantial oxygen isotope effect exponent, even exceeding 1/2 in the $La_{2-x}Sr_xCuO_4$ system [23] and show that α should be small near the peak in T_c as a function of hole concentration and increase away from the T_c peak.

In the tight-binding calculation for the energy band, the density of state shows logarithmic singularity at the Fermi level [99] . To see this role of a two dimensional van Hove singularity in determining the magnitude of T_c and the isotope effect, we assume a logarithmic density of states

$$N(\xi) = N(0) \ln [E_F / (\xi - E_F)] \qquad (5.1)$$

where N(0) is the density of states at the absolute temperature.

Following the work of Daemen and Overhuaser [29]which the effect of the short-range interactions has been studied in the high-T_c superconductors. To see this short-range effect we use a simple square-well model first introduced by Bogoliubov, Tolmachev and Shirkov[100]. The scattering matrix element appearing in the BCS gap equation is given by

$$V_{kk'} = \begin{cases} -(V_1 + V_2) & \text{if} \quad |\xi_k| < \hbar\omega_D \quad \text{and} \quad |\xi_{k'}| < \hbar\omega_D \\ -V_2 & \text{unless} \quad |\xi_k| > E_c \quad \text{or} \quad |\xi_{k'}| > E_c \end{cases} \tag{5.2}$$

where V_1 is the phonon-mediated interaction, and V_2 is a short-range interaction such as Coulomb repulsion or an attractive interaction . E_c is the energy cut off for V_2, ξ_k is the electron energy measured with respect to the Fermi energy and ω_D is the Debye cut off frequency.

To discuss the effect of the van Hove singularity and the double two square-well model interaction on the transition temperature T_c and the isotope effect exponent, it will suffice to recall the standard BCS gap equation

$$\Delta_k(T) = -\frac{1}{2}\sum_{k'} V_{kk'} \frac{\Delta_{k'}(T)}{\sqrt{\xi_{k'}^2 + \Delta_{k'}^2(T)}} \tanh \frac{\sqrt{\xi_{k'}^2 + \Delta_{k'}^2(T)}}{2k_BT} \tag{5.3}.$$

The double two square-well model of the scattering matrix interaction in Eq.(5.2) gives the gap equation in the form

$$\Delta_k(T) = \begin{cases} \Delta_1(T) & \text{if } |\xi_k| < \hbar\omega_D \text{ and } |\xi_{k'}| < \hbar\omega_D \\ \Delta_2(T) & \text{unless } |\xi_k| > E_c \text{ or } |\xi_{k'}| > E_c \end{cases} \quad (5.4).$$

For the scattering matrix elements in Eq.(5.2), the first case of electron energy ξ_k less than the Debye cut off energy gives the following gap equation, [using Eq. (5.3) and Eq.(5.4)] ,

$$\Delta_1 = -\frac{1}{2}\frac{1}{\Omega}\sum_{k'}^{|\xi_{k'}|<\hbar\omega_D} -(V_1 + V_2)\frac{\Delta_1(T)}{\sqrt{\xi_{k'}^2 + \Delta_1^2(T)}}\tanh\frac{\sqrt{\xi_{k'}^2 + \Delta_1^2(T)}}{2k_BT}$$

$$-\frac{1}{2}\frac{1}{\Omega}\sum_{k'}^{\hbar\omega_D < |\xi_{k'}| < E_c} -(V_2)\frac{\Delta_2(T)}{\sqrt{\xi_{k'}^2 + \Delta_2^2(T)}}\tanh\frac{\sqrt{\xi_{k'}^2 + \Delta_2^2(T)}}{2k_BT}, \quad (5.5)$$

and for above the Debye cut off energy, the BCS gap equation leads to

$$\Delta_2 = -\frac{1}{2}\frac{1}{\Omega}\sum_{k'}^{|\xi_{k'}|<\hbar\omega_D} -(V_2)\frac{\Delta_1(T)}{\sqrt{\xi_{k'}^2 + \Delta_1^2(T)}}\tanh\frac{\sqrt{\xi_{k'}^2 + \Delta_1^2(T)}}{2k_BT}$$

$$-\frac{1}{2}\frac{1}{\Omega}\sum_{k'}^{\hbar\omega_D < |\xi_{k'}| < E_c} -(V_2)\frac{\Delta_2(T)}{\sqrt{\xi_{k'}^2 + \Delta_2^2(T)}}\tanh\frac{\sqrt{\xi_{k'}^2 + \Delta_2^2(T)}}{2k_BT} \quad (5.6).$$

We define two parameters Σ_1 and Σ_2 as

$$\Sigma_1 = -\frac{1}{2}\frac{1}{\Omega}\sum_{0}^{\hbar\omega_D}\frac{1}{\sqrt{\xi_{k'}^2 + \Delta_1^2(T)}}\tanh\frac{\sqrt{\xi_{k'}^2 + \Delta_1^2(T)}}{2k_BT} \quad (5.7)$$

$$\Sigma_2 = -\frac{1}{2}\frac{1}{\Omega}\sum_{\hbar\omega_D}^{E_c} \frac{1}{\sqrt{\xi_{k'}^2 + \Delta_2^2(T)}} \tanh \frac{\sqrt{\xi_{k'}^2 + \Delta_2^2(T)}}{2k_BT} \quad (5.8).$$

The transformation of the summations into integration operators, then changes Eq.(5.7) and (5.8) to

$$\Sigma_1 = \int_0^{\hbar\omega_D} d\xi\, N(\xi) \frac{1}{\sqrt{\xi_{k'}^2 + \Delta_1^2(T)}} \tanh \frac{\sqrt{\xi_{k'}^2 + \Delta_1^2(T)}}{2k_BT} \quad (5.9)$$

and

$$\Sigma_2 = \int_{\hbar\omega_D}^{E_c} d\xi\, N(\xi) \frac{1}{\sqrt{\xi_{k'}^2 + \Delta_2^2(T)}} \tanh \frac{\sqrt{\xi_{k'}^2 + \Delta_2^2(T)}}{2k_BT} \quad (5.10)$$

From Eq. (5.5), (5.6), the non linear gap equations for Δ_1 and Δ_2 in terms of $\Sigma_1$and Σ_2 are

$$\Delta_1 = (V_1 + V_2)\,\Delta_1\,\Sigma_1 + V_2\,\Delta_2\,\Sigma_2 \quad (5.11)$$

$$\Delta_2 = V_2\,\Delta_1\,\Sigma_1 + V_2\,\Delta_2\,\Sigma_2 \quad (5.12)$$

where Σ_1 and Σ_2 are defined in (5.9) and (5.10)

Close to T_c , however, Δ_1 and Δ_2 are very small and Eqs.(5.11) and (5.12) reduce to the homogeneous system of linear equations, at T_c

$$\Sigma_1 = \int_0^{\hbar\omega_D} d\xi\, N(\xi) \frac{N(\xi)}{\xi} \tanh \frac{\xi}{2k_BT_c} \quad (5.13)$$

and

$$\Sigma_2 = \int_{\hbar\omega_D}^{E_c} d\xi\, N(\xi)\frac{N(\xi)}{\xi}\tanh\frac{\xi}{2k_BT_c} \tag{5.14}$$

which is the same as

$$\Sigma_2 = \int_0^{E_c} d\xi\, N(\xi)\frac{N(\xi)}{\xi}\tanh\frac{\xi}{2k_BT_c} - \int_0^{\hbar\omega_D} d\xi\, N(\xi)\frac{N(\xi)}{\xi}\tanh\frac{\xi}{2k_BT_c} \tag{5.15}$$

Using the van Hove singularity, the logarithmic density of states in Eq.(5.1), and defining the universal function F as

$$F(x) = \int_0^{x/2} dz\,\frac{\tanh(z)}{z}\,\ln\left[\frac{E_F}{2zk_BT_c}\right] \tag{5.16}$$

where $z = \xi / 2k_BT_c$, we have that

$$\Sigma_1 = N(0)\,F(\hbar\omega_D / k_BT_c) \tag{5.17}$$

and

$$\Sigma_2 = N(0)\,F(E_c / k_BT_c) - N(0)\,F(\hbar\omega_D / k_BT_c) \tag{5.18}$$

From Eq.(5.11) and (5.12)

$$\frac{\Delta_1}{\Delta_2} = \frac{V_1\Sigma_1}{1 - V_1\Sigma_1} \tag{5.19}$$

and

$$1 = (V_1 + V_2)\Sigma_1 + V_2(\Delta_1 / \Delta_2)\Sigma_2 \tag{5.20}$$

Then, substituting Eq. (5.19) into Eq. (5.20)

$$1 - \Sigma_1 V_1 - \Sigma_2 V_2 - \Sigma_1 V_2 + V_1 V_2 \Sigma_1 \Sigma_2 = 0 \qquad (5.21)$$

and from Eq. (5.18) ,

$$\Sigma_2 = N(0)\, F(E_c/k_B T_c) - \Sigma_1 \qquad (5.22)$$

Eq.(5.21) then changes to

$$\begin{aligned} &1 - \Sigma_1 V_1 - \Sigma_2 V_2 - V_2 [\, N(0)\, F(E_c / k_B T_c) - \Sigma_2 \,] \\ &+ V_1 V_2 \Sigma_1 [\, N(0)\, F(E_c / k_B T_c) - \Sigma_1 \,] = 0 \end{aligned} \qquad (5.23)$$

The strength parameter λ is defined by $\lambda = N(0)V_1$ and is the phonon coupling parameter. We also define the coupling strength σ defined by the relation $\sigma = N(0)V_2$. We can then write Eq.(5.23) in the form

$$1 - \lambda F\left(\frac{\hbar\omega_D}{k_B T_c}\right) - \sigma F\left(\frac{E_c}{k_B T_c}\right) - \lambda\sigma\left[F\left(\frac{\hbar\omega_D}{k_B T_c}\right)\right]^2 + \lambda\sigma F\left(\frac{\hbar\omega_D}{k_B T_c}\right) F\left(\frac{E_c}{k_B T_c}\right) = 0 \qquad (5.24)$$

We now start to calculate the isotope index exponent. The definition of α is

$$\alpha \equiv - \frac{M}{T_c} \frac{dT_c}{dM} \qquad (5.25)$$

We differentiate Eq.(5.24) with respect to M with the assumption that $\hbar\omega_D$ is proportional to $M^{-1/2}$ and the theoretical model that T_c is varying as $M^{-\alpha}$. We have that

$$\frac{dF\left(\frac{\hbar\omega_D}{k_BT_c}\right)}{dM} = \left[\hbar\omega_D \frac{d\,(1/k_BT_c)}{dM} + \frac{1}{k_BT_c}\frac{d\,(\hbar\omega_D)}{dM}\right] F'\left(\frac{\hbar\omega_D}{k_BT_c}\right) \tag{5.26}$$

$$= \frac{\hbar\omega_D}{k_BT_cM}\left(\frac{1}{2} - \alpha\right) F'\left(\frac{\hbar\omega_D}{k_BT_c}\right) \tag{5.27}$$

and

$$\frac{dF\left(\frac{E_c}{k_BT_c}\right)}{dM} = \left[\frac{E_c\alpha}{k_BT_cM}\right] F'\left(\frac{E_c}{k_BT_c}\right) \tag{5.28}$$

We next define the function

$$G(z) = \int dz \frac{\tanh(z)\ln(E_F/2k_BT_c\,z)}{z} \tag{5.29}$$

Then, from Eq. (5.16) $$F(x) = G(x/2) - G(0) \tag{5.30}$$

with $$G(0) = 0 \tag{5.31}$$

$$F'(x) = \frac{d\,G(x/2)}{dx}$$

$$= \left[\frac{d\,G(z/2)}{dz}\right]_{z = x/2} \tag{5.32}.$$

The calculation is carried out by use of Eq.(5.29) in Eq. (5.32) . We have

$$F'(x) = \frac{1}{2}\frac{\tanh(x/2)\ln(E_F/2k_BT_c(x/2))}{x/2} \tag{5.33}$$

Thus Eqs. (5.27) and Eq. (5.28) can be written

$$\frac{d\,F\left(\frac{\hbar\omega_D}{k_BT_c}\right)}{dM} = -\frac{\hbar\omega_D}{k_BT_cM}\left(\frac{1}{2}-\alpha\right)\frac{1}{2}\tanh\left(\frac{\hbar\omega_D}{2k_BT_c}\right)\ln\left(\frac{E_F}{2k_BT_c}\frac{2k_BT_c}{\hbar\omega_D}\right)\frac{2k_BT_c}{\hbar\omega_D} \tag{5.34}$$

$$= -\frac{1}{M}\left(\frac{1}{2}-\alpha\right)\tanh\left(\frac{\hbar\omega_D}{2k_BT_c}\right)\ln\left(\frac{E_F}{\hbar\omega_D}\right) \tag{5.35}$$

and

$$\frac{d\,F\left(\frac{E_c}{k_BT_c}\right)}{dM} = \frac{\alpha}{M}\tanh\left(\frac{E_c}{2k_BT_c}\right)\ln\left(\frac{E_F}{E_c}\right) \tag{5.36}.$$

We also have

$$\frac{d\,F^2\left(\frac{\hbar\omega_D}{k_BT_c}\right)}{dM} = 2F\left(\frac{\hbar\omega_D}{k_BT_c}\right)F'\left(\frac{\hbar\omega_D}{k_BT_c}\right) \tag{5.37}$$

and

$$\frac{d\left[F\left(\frac{\hbar\omega_D}{k_BT_c}\right)F\left(\frac{E_c}{k_BT_c}\right)\right]}{dM} = F\left(\frac{\hbar\omega_D}{k_BT_c}\right)F'\left(\frac{E_c}{k_BT_c}\right) + F\left(\frac{E_c}{k_BT_c}\right)F'\left(\frac{\hbar\omega_D}{k_BT_c}\right) \tag{5.38}.$$

Substituting expressions (5.35), (5.36), (5.37) and(5.38) into Eq. (5.24) gives after the some algebra,

$$A\,\lambda\left(\frac{1}{2}-\alpha\right)\left[1+2\sigma F\left(\frac{\hbar\omega_D}{k_BT_c}\right)-\sigma F\left(\frac{E_c}{k_BT_c}\right)\right]+B\sigma\alpha\left[\lambda F\left(\frac{\hbar\omega_D}{k_BT_c}\right)-1\right] = 0 \tag{5.39}$$

where

$$A \equiv \tanh\left(\frac{\hbar\omega_D}{2k_BT_c}\right)\ln\left(\frac{E_F}{\hbar\omega_D}\right) \tag{5.40}$$

$$B \equiv \tanh\left(\frac{E_c}{2k_BT_c}\right)\ln\left(\frac{E_F}{E_c}\right) \qquad (5.41).$$

Then we have

$$\sigma F\left(\frac{E_c}{kT_c}\right) = \frac{1 - \lambda F\left(\frac{\hbar\omega_D}{k_BT_c}\right) - \lambda\sigma F^2\left(\frac{\hbar\omega_D}{k_BT_c}\right)}{1 - \lambda F\left(\frac{\hbar\omega_D}{k_BT_c}\right)} \qquad (5.42)$$

By using Eq.(5.42) in Eq.(5.39) , we get the following expression for α,

$$\alpha = \frac{1}{2}\left[1 + \left\{\frac{\tanh\left(\frac{E_c}{2k_BT_c}\right)\ln\left(\frac{E_F}{E_c}\right)}{\tanh\left(\frac{\hbar\omega_D}{2k_BT_c}\right)\ln\left(\frac{E_F}{\hbar\omega_D}\right)}\right\}\left\{\frac{\left[1 - \lambda F\left(\frac{\hbar\omega_D}{k_BT_c}\right)\right]^2}{1 - \left[1 - \lambda F\left(\frac{\hbar\omega_D}{k_BT_c}\right)\right]^2}\right\}\right]^{-1}, \qquad (5.43)$$

The expression for α that is shown in Eq. (5.43) is the same as that given by Deamen and Overhauser (Eq. 4.38) except for a different function F(x), defined in Eq. (5.16), resulting from the use of the logarithmic density of states $N(\xi)$ with a singularity at the Fermi level. A numerical calculation for α based on Eq. (5.43) to incorporate the effect of a logarithmic $N(\xi)$ and the double well model of the microsocopic interaction $V_{kk'}$, gives the results for Fig. 20. The calculated values for T_c and α are plotted as a function of λ.

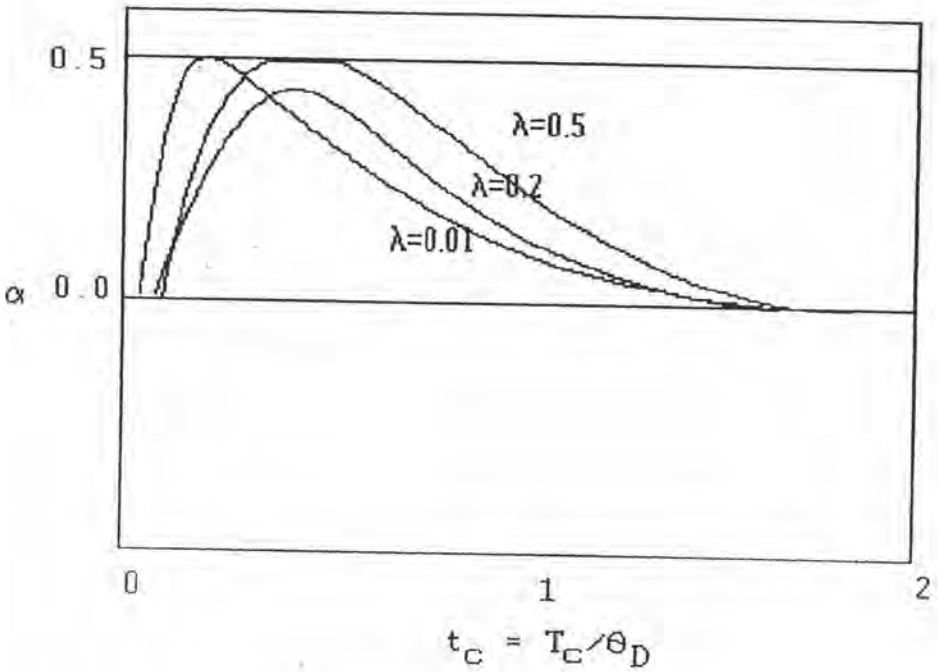

Fig. 20 The α and T_c are potted as function of λ.

Of particular interest is that α is dependent on λ . For all curves, α is less than 1/2 at low temperature and rises to 1/2 with incresing T_c. Then, for higher temperature , α tends to zero. These results roughly describe the experimental data described in the previous sections. Next we rewrite Eq.(5.43) in conventional form for finding an empirical electron-phonon coupling constant λ from the experimentally determined transition temperature T_c, the isotope effect exponent α and the Debye temperature θ_D,

$$\lambda = \frac{1}{F\left(\frac{\hbar\omega_D}{k_B T_c}\right)}\left[1-\left[1+\left\{\frac{1}{\left(\frac{1}{2\alpha}-1\right)}\frac{\tanh\left(\frac{\hbar\omega_D}{2k_B T_c}\right)\ln\left(\frac{E_F}{\hbar\omega_D}\right)}{\tanh\left(\frac{E_c}{2k_B T_c}\right)\ln\left(\frac{E_F}{E_c}\right)}\right\}\right]^{-\frac{1}{2}}\right] \qquad (5.44)$$

Here we use $E_c = 20\ k_B T_c$ and $E_F = 5800$ K, $\theta_D = 440$ K for the calculation. The experimental T_c and α are listed in Table 11 for $Ba_{1-x}K_xBiO_3$ ($0.37 < x < 0.5$), $La_{1.85}Sr_{0.15}CuO_4$, $YBa_2Cu_3O_{7-\delta}$, $Bi_2Sr_2CaCu_2O_8$, $Bi_2Sr_2Ca_2Cu_3O_{10}$ and in Table 12 we calculate the λ for some of the oxide superconductors with varying doping concentration , x , in $La_{2-x}Sr_xCuO_4$, $YBa_{2-x}La_xCu_3O_z$ ($z \cong 7$) and ($Y_{1-x}Pr_x$ $Ba_2Cu_3O_{7-\delta}$ together with the empirical electron-phonon coupling constant λ found using Eq. (5.44).

Table 11 Empirical values of the electron-phonon coupling constant of $Ba_{1-x}K_xBiO_3$ ($0.37 < x < 0.5$), $La_{1.85}Sr_{0.15}CuO_4$, $YBa_2Cu_3O_{7-\delta}$ $Bi_2Sr_2CaCu_2O_8$, and $Bi_2Sr_2Ca_2Cu_3O_{10}$ found from the isotope effect exponent α, and T_c using Eq. (5.44).

Compounds	Percent(%) ^{18}O exchange	T_c (K)	α	λ
$Ba_{1-x}K_xBiO_3$ ($0.37 < x < 0.5$)	-	30	0.41	0.059
$La_{1.85}Sr_{0.15}CuO_4$	-	37	0.14	0.024
	-	35	0.31	0.048
	73	36	0.16	0.027
	68	35	0.22	0.036
	75	37	0.13	0.023
$YBa_2Cu_3O_{7-\delta}$	80	90	0.019	0.0063
	80	90	0.028	0.0092
	90	90	0.017	0.0056
	94	90	0.020	0.0068
	91	90	0.016	0.0053
	89	90	0.019	0.0062
	92	90	0.013	0.0048
	86	90	0.015	0.0044
	96.5	90	0.05	0.0160
$Bi_2Sr_2CaCu_2O_8$	-	75	0.034	0.0089
$Bi_2Sr_2Ca_2Cu_3O_{10}$	-	110	0.023	0.0088

Table 12 Empirical values of the electron-phonon coupling constant of $La_{2-x}Sr_xCuO_4$, $YBa_{2-x}La_xCu_3O_z$ ($z \equiv 7$) and $(Y_{1-x}Pr_x)Ba_2Cu_3O_{7-\delta}$ found from the isotope effect exponent α and T_c using Eq. (5.44).

Compounds	x	Percent(%) ^{18}O exchange	T_c (K)	α	λ
$La_{2-x}Sr_xCuO_4$	0.075	90	21.2	0.40	0.049
	0.113	88	29.7	0.64	they are not
		86	29.6	0.60	appliable for
		87	29.6	0.64	$\alpha > 1/2$
	0.150	87	37.8	0.08	0.151
		82	38.3	0.10	0.186
		-	35	0.16	0.271
		-	34.5 - 37	0.09 - 0.37	0.016 - 0.058
	0.188	84	34.4	0.06	0.110
	0.225	88	23	0.10	0.147
	0.263	86	8.0	0.12	0.116
$(Y_{1-x}Pr_x)Ba_2Cu_3O_{7-\delta}$					
	0.2	84.6	75.6	0.09	0.024
	0.3	79.0	60.4	0.15	0.034
	0.3	72.1	60.0	0.17	0.037
	0.3	83.2	54.3	0.23	0.046
	0.4	85.5	46.2	0.27	0.049
	0.5	85.9	30.6	0.45	0.066
$YBa_{2-x}La_xCu_3O_z$ ($z \equiv 7$)					
	0.0	97	92.3	0.025	0.0084
	0.1	97	91.9	0.039	0.0128
	0.2	97	77.3	0.140	0.037
	0.3	97	73	0.213	0.0515
	0.4	95	49.3	0.324	0.0596
	0.5	94	38.3	0.380	0.0611

For the anomalous isotope effect ($\alpha > 1/2$) in the $La_{2-x}Sr_xCuO_4$ system with $x = 0.113$ the observation of the α values are 0.64 and 0.60. Our model does not apply to the anomalous isotope effect .(These anomalous isotope shifts follow the result in the inverse square root term of the λ expression are minus, then we obtain the λ as the complex values with $0.083 + 0.024i$ for $\alpha = 0.64$ and $0.084 + 0.027i$ for $\alpha = 0.60$.)

In our model we can see that the α must be smaller than 1/2. Nevertheless, there are other works such as the calculation of the isotope effect which includes the effect of pair breaking [88], the studying of the low energy dependence in the electronic density of states in the oxygen isotope effect [89], and anharmonic phonon formation [90]. In these models the value of α may exceed the BCS limit of $\alpha > 1/2$.

Having found the empirical values of the electron-phonon coupling constant data for a number of the oxide superconductors, we can investigate these in terms of the dependence on the temperature, according to the Table 11 and 12. The coupling λ is the characteristic of an increasing transition temperature. By use Eq. 5.24, we now wish to show the dependence of the λ and the other electronic strength parameter σ, on the temperature in Fig 21 .

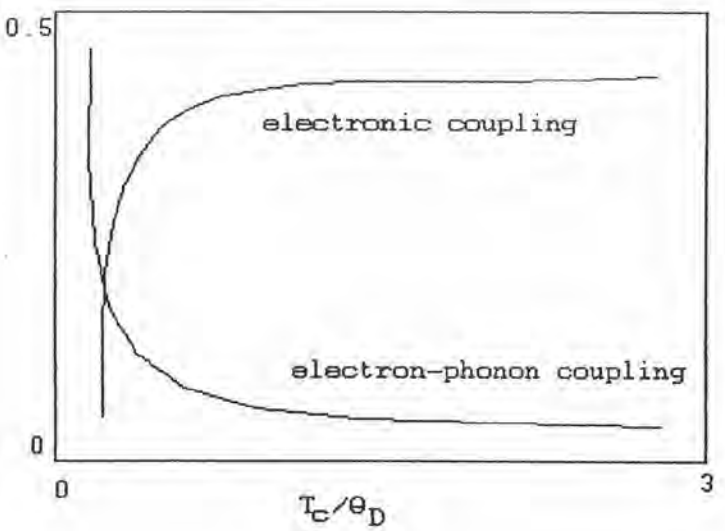

Fig. 21 The curves for σ and λ as the function of temperature.

The curve shows a strong dependence of the electronic coupling , σ, on the critical temperature compared with the electron-phonon coupling. We can see that at the higher critical temperatures, σ increases but λ is decreasing. In other words, some of the electronic mechanism may be relevant and dominant in the high temperature

oxide superconductors. This consideration suggests that the phonon contribution still plays a key role in the oxide superconductor, although minor and not as dominant as in the classical superconductors.

Furthermore, as in the classical superconductors, the oxides are described according into the isotope effect exponent, in two groups. The classical superconductors can be separated into the simple metallic superconductors, α with about 1/2 and the transition metallic superconductors, where α is ranged from 0 to 0.39 as shown in the Fig. 22.

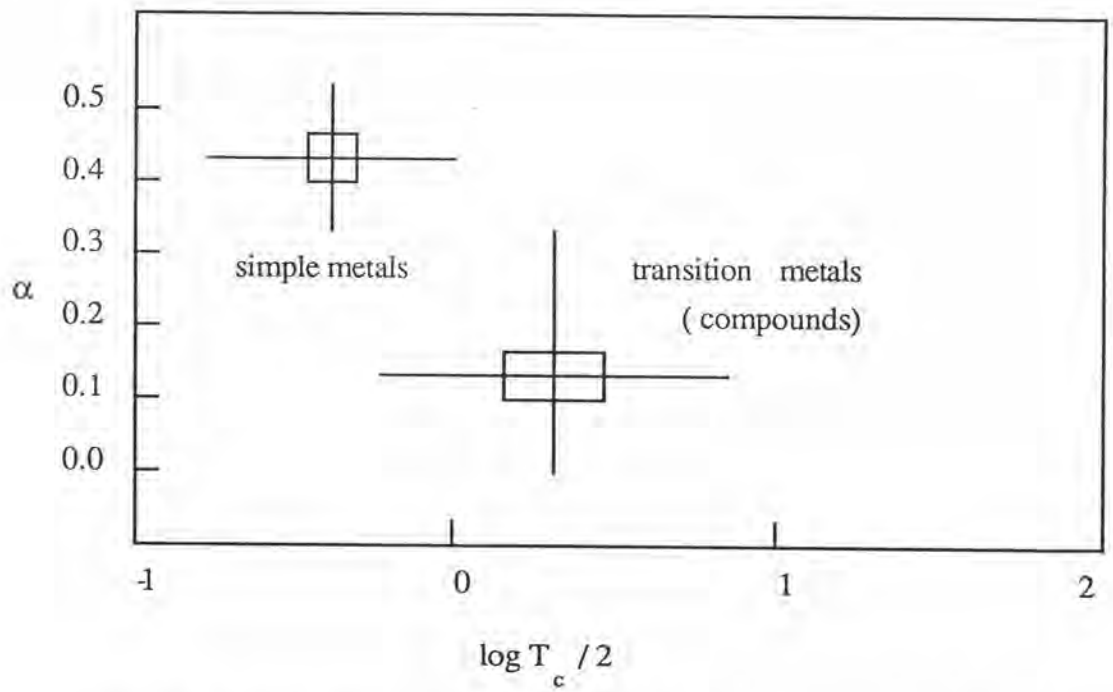

Fig.22 Isotope effect exponent α for simple and transition metallic with ranges shown for α and $\log(T_c / 2)$.

Crudely speaking, the general trend in the new oxide superconductors such as $Ba_{1-x}K_xBiO_3$ ($0.37 < x < 0.5$), $La_{1.85}Sr_{0.15}CuO_4$, $YBa_2Cu_3O_{7-\delta}$, $Bi_2Sr_2CaCu_2O_8$, $Bi_2Sr_2Ca_2Cu_3O_{10}$, and Tl-based compounds, is that α values range from 0.02 to 0.41

within the experimental error. These groups can be separated as shown in Fig. 23. Firstly the 30-K superconductors $Ba_{1-x}K_xBiO_3$ ($0.37 < x < 0.5$) and $La_{1.85}Sr_{0.15}CuO_4$ ($\alpha \cong 0.14$ - 0.41), and secondly the above 90-K superconductors $YBa_2Cu_3O_{7-\delta}$, $Bi_2Sr_2CaCu_2O_8$, $Bi_2Sr_2Ca_2Cu_3O_{10}$ ($\alpha \cong 0.02$ - 0.034).

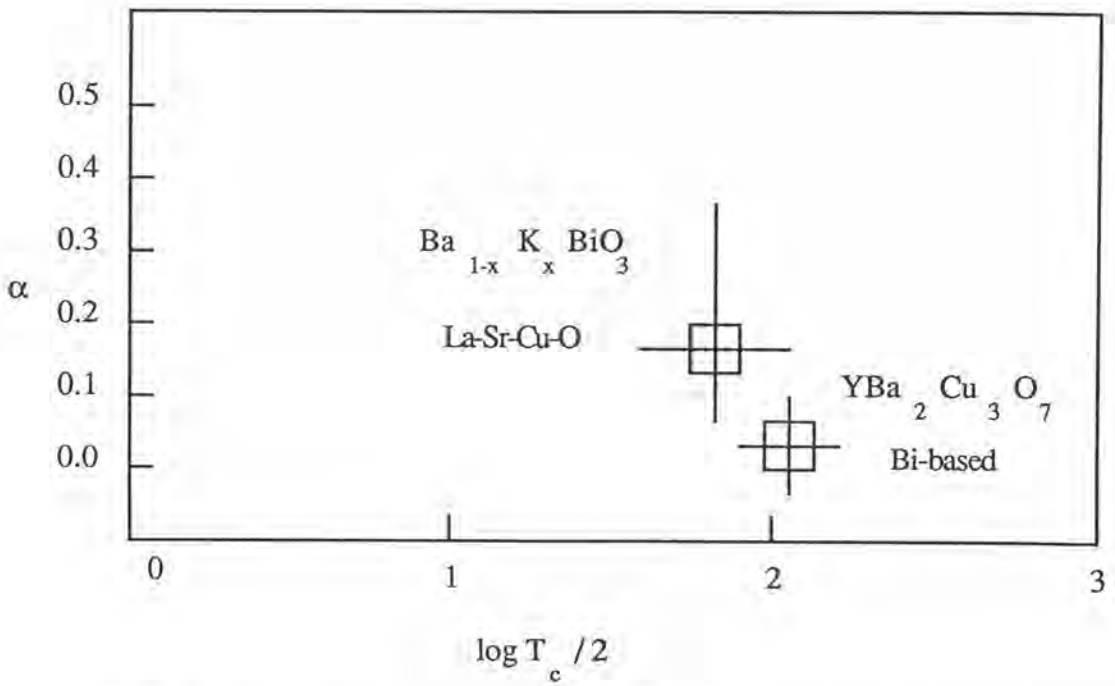

Fig.23 Isotope effect exponent α for the oxide superconductors with ranges shown for α and $\log(T_c / 2)$.

The values for α, as derived from Eq. (5.43), in which α depends on λ, exhibit a behavior that is qualitatively different for each group of the high-T_c superconductors. The dimensionless variable λ that is defined as the electronic density of states N(0) times the phonon mediated interaction V_1 is different but it should be emphasized that the effect of the novel electron-phonon mechanism is felt in any group of the superconducting materials. In other words, each group of superconductors can be separated according to the strength parameter λ. We now consider making used of the theoretical equation and the experimental results to extract the coupling constant λ, in

Table 11. we can obtain the empirical values of λ for each group of the new superconductors as can see in the Table 13.

Table 13. The λ values for each group of the 30-K and the above 90-K oxide superconductors .

Group I	Group II
The transition temperature is about 30 to 40K. These materials are $Ba_{1-x}K_xBiO_3$ ($0.37 < x < 0.5$), and $La_{1.85}Sr_{0.15}CuO_4$. The λ for this groups is from about 0.024 to 0.059.	The transition temperature is above 90 K. These materials are $YBa_2Cu_3O_{7-\delta}$, $Bi_2Sr_2CaCu_2O_8$ and $Bi_2Sr_2Ca_2Cu_3O_{10}$. The λ for this group is from about 0.0048 to 0.0160.