CHAPTER III

NUMERICAL SOLUTION SCHEME

3.1 <u>Numerical Inversion of Laplace-Hankel Integral Transforms</u>

From the chapter II, the solution of eqn(2.42) yields the Laplace-Hankel transforms of displacements and pore pressure at layer interfaces for discrete values of ξ and s . Thereafter, the required influence functions are determined by numerically evaluating the integrals appearing in eqn(2.14). The integral with respect to ξ in eqn (2.14) is evaluated by using the numerical integration scheme, namely the rectangular rule. It is found that the numerical integration is stable for very large value of ξ_L where ξ_L is defined as the upper limit of the integration with respect to ξ in eqn(2.14). It is also noted that the Laplace inversion can be carried out very accurately for poroelasticity problems (Rajapakse and Senjuntichai,1993 ; Detournay and Cheng ,1988) by using the numerical Laplace inversion method proposed by Stehfest (1970). The formula due to Stehfest is given by

$$f(t) \approx \frac{\ln 2}{t}\sum_{n=1}^{N_e} c_n \bar{f}(n\frac{\ln 2}{t}) \tag{3.1}$$

where $\bar{f}$ denotes the Laplace transform of $f(t)$ and

$$c_n = (-1)^{n+N_e/2} \sum_{k=[(n+1)/2]}^{\min(n,N_e/2)} \frac{k^{N_e/2}(2k)!}{(N_e/2-k)!k!(k-1)!(n-k)!(2k-n)!} \tag{3.2}$$

and N_e is even . It is found that accurate time-domain solutions are obtained from eqn(3.1) with $N_e \geq 6$ for poroelasticity problems (Rajapakse and Senjuntichai,1993).

It is important to note that the Stehfest method is computationally quite demanding although it is accurate. A more simple and computationally efficient scheme for quasi-static problem is given by Schapery (1962) which can be expressed as

$$f(t) \approx [s\bar{f}]_{s=0.5/t} \tag{3.3}$$

where $\bar{f}$ denotes the Laplace transforms of $f(t)$ and s is Laplace transform parameter. In this thesis , the Laplace inversion is carried out by using Schapery scheme in order to determine quasi-static response of a circular elastic plate.

3.2 Numerical Solution Procedure

A computer program has been developed based on the procedures described previously to investigate the flexural behaviour of a plate on a multi-layered poroelastic half-space. The tasks performed by the computer code are described as shown Fig.3.1 . This can be summarized as follows : (1) discretizing a plate into M ring elements ; (2) determining the influence functions that are required to establish the flexibility equation corresponding to a vertical displacement due to unit vertical pressure acting over an annular region by solving the global stiffness equation , eqn(2.42) ; (3) assembling the flexibility equation , eqn(2.70) and then solving for $\bar{T}_z(r,s)$; (4) determining the strain energy of a plate and a multi-layered poroelastic half-space from eqns(2.53) and (2.71) , respectively ; (5) minimizing the total potential energy functional to form the simultaneous linear equation system, eqn(2.77) ; (7) solving the simultaneous linear equations to obtain the generalized coordinates,$\bar{\alpha}_n$ and then applying the Laplace inversion scheme given by eqn(3.1) or (3.3) to transform the generalized coordinates to time domain, α_n . Finally , the flexural behaviour of the plate can be obtained by back substituting α_n into eqn(2.43) and eqns(2.48) and (2.49) .

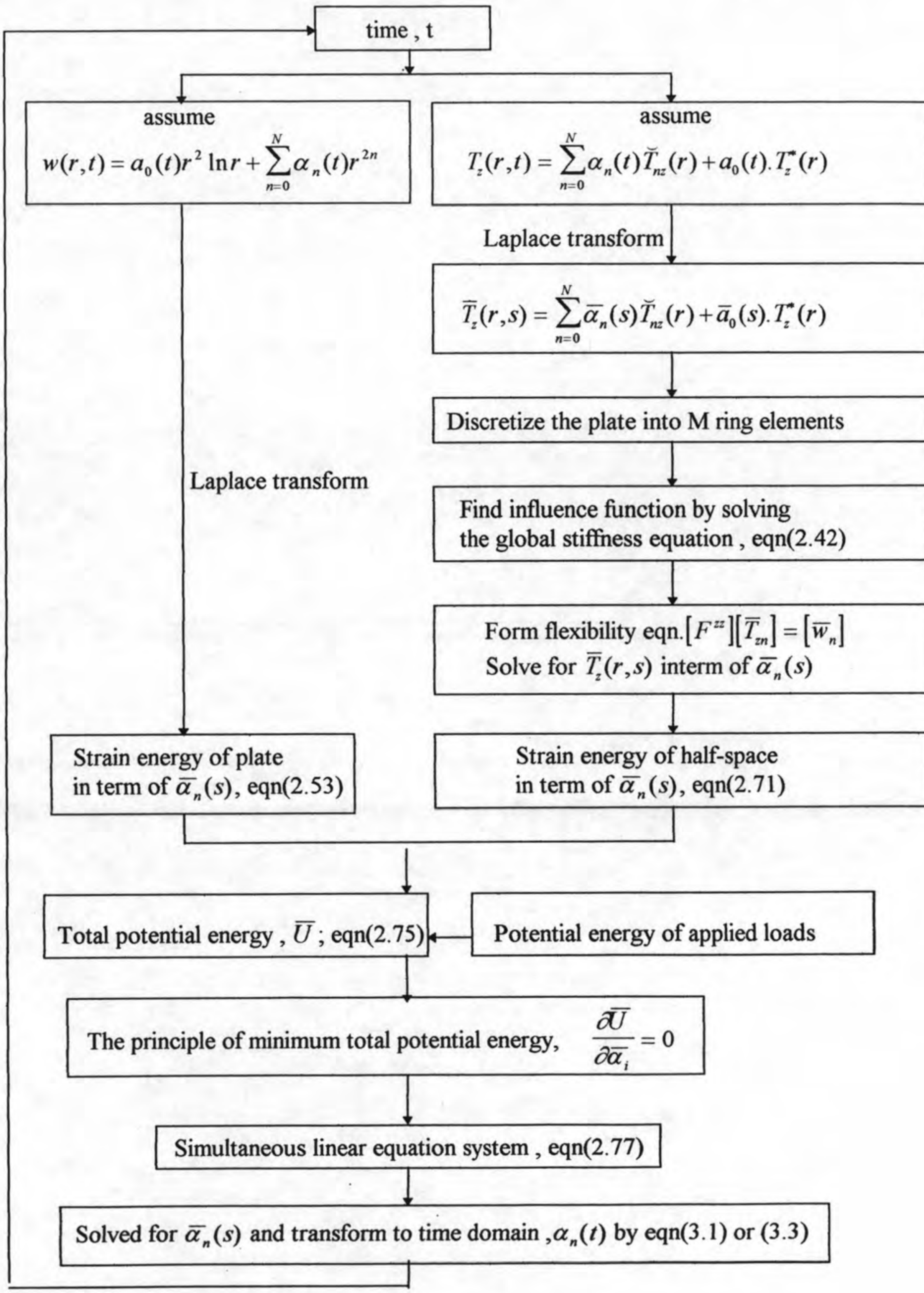

Figure 3.1 : Numerical Solution Procedure