CHAPTER II

EDDY CURRENT IN DISKS

Driving Forces and Torque

A large thin metal disk, as in Fig. 2.1 is pierced by fluxes ϕ_J and ϕ_K. These are the r.m.s. values of a-c fluxes, in quadrature. Such fluxes will be called a flux pair. Each is a uniform flux within a circle. The distance between circle centers J and K is a centimetres. The fluxes have a sine wave variation at frequency f. The circles do not overlap.

The variation of ϕ_K sets up concentric eddies in quadrature with ϕ_K. Eddy current maximum occurs when ϕ_J is maximum, ϕ_K eddies act on ϕ_J to produce a sine squared wave of force. It will be shown that the average force, F_{KJ} in dynes, acting along Z_K or s is

$$F_{KJ} \quad = \quad G/S \qquad (2.1)$$

where G is developed further along.

The force F_{KJ} is actually a resultant force made up of an infinitude of forces, each due to an element of current reacting with the flux in the J circle. The problem of finding the resultant can be avoided by resorting to familiar analogous situation.

In the analogy, the two circles become sections of long parallel conductors, normal to the paper. Conductor currents, I_J and I_K, are distributed uniformly. I_K sets up a concentric magnetic field. It long since has been shown that

(a) Outside its conductor, the field due to I_K is the same as if I_K were centered (concentrated in a wire of infinitesimal section at K)

(b) The force due to the distributed I_J reacting with the flux of I_K is the same as it would be if I_J were centered at J.

(c) The force per unit length for such a combination is proportional to $(I_J \; I_K)/s$

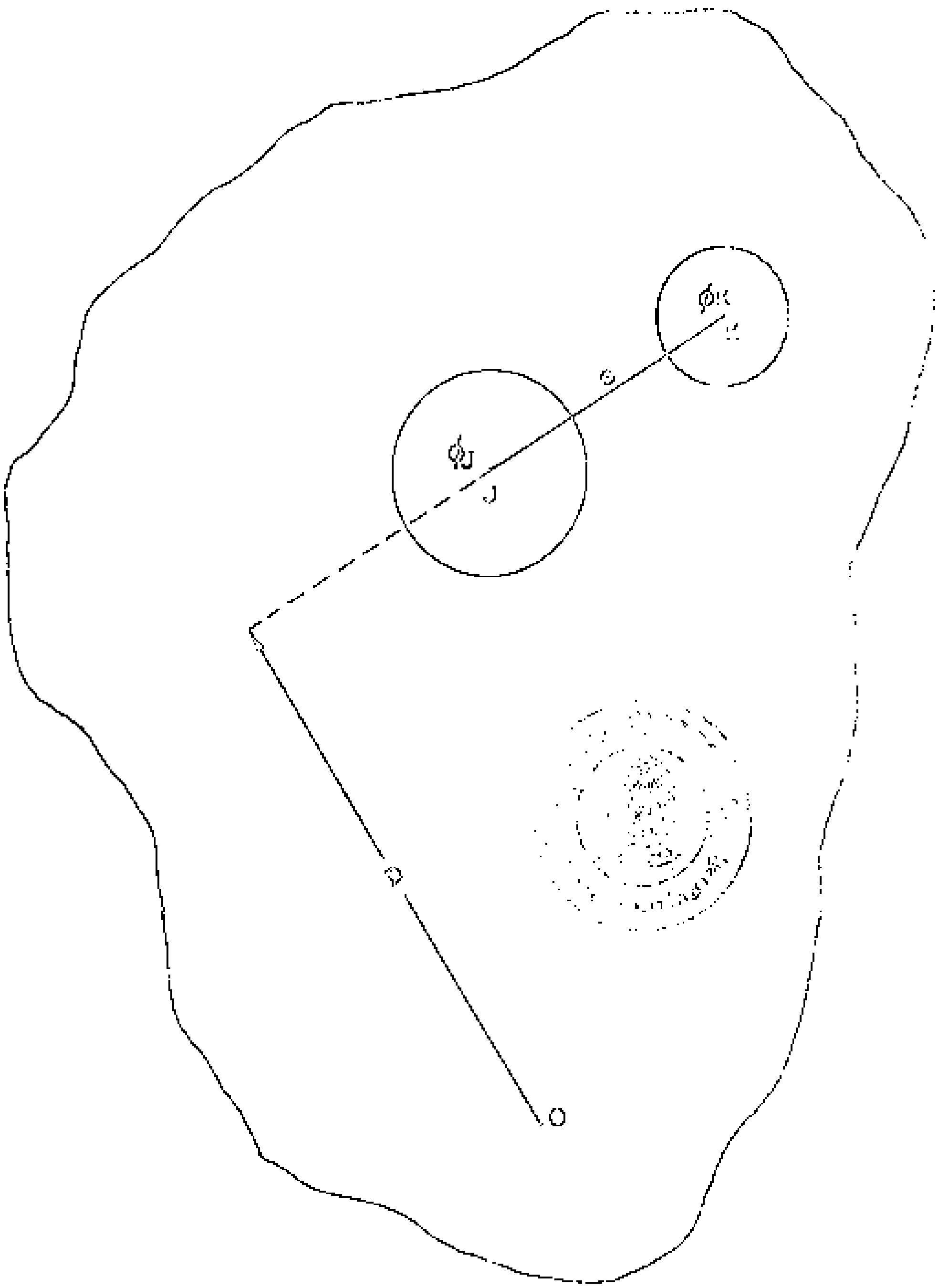

FIG 2.1 LARGE DISK AND FLUX PAIR

From the analogy, it is clear that in our disk case, ϕ_K can be centered at K, becoming a pencil of flux; ϕ_J can be centered at J, and $F_{KJ} \propto \phi_J \phi_K / S$

Development of G.

In Fig. 2.2, ϕ_K has been centered, thereby becoming a flux pencil. In centering ϕ_J, we have ds, at J. The force originating in this square is F_{KJ}, and it now can be developed fully.

The r.m.s. voltage E developed by ϕ_K in the disk turn = 1 and acting around the current ring of ds width is given by the familiar equation

$$E = 2\pi f \phi_K \times 10^{-8} \qquad (2.2)$$

where E = r.m.s. value of induced voltage
ϕ_K = r.m.s. value of a-c flux

To find R, the resistance of the current ring,

ρ = disk resistivity, ohm per cm.
b = disk thickness, centimetres
$2\pi S$ = path length
bds = path section

$$R = \frac{\rho\, 2\pi S}{bds} \qquad (2.3)$$

To find the r.m.s. current of the ds by ds square

$$I = E/R$$

$$= \frac{2\pi f \phi_K}{\frac{\rho\, 2\pi S}{bds}} \times 10^{-8}$$

$$= \frac{fb\, \phi_K\, ds}{\rho\, S} \times 10^{-8} \qquad (2.4)$$

The force in dynes is given by the familiar form, BIL/10. In the ds by ds square, $B = \phi_J/(ds) \times (ds)$, I is given by the equation above and L is equal to ds. The force is then,

$$F_{KJ} = \left(\frac{\phi_J}{ds \cdot ds}\right) \times \left(\frac{fb\, \phi_K\, 10^{-8} ds}{\rho\, S}\right) \times (ds) \times \frac{1}{10}$$

$$= \frac{fb\, \phi_J\, \phi_K}{\rho\, S} \times 10^{-9} \qquad (2.5)$$

Let

$$G = \frac{fb\, \phi_J\, \phi_K}{\rho} \times 10^{-9} \qquad (2.6)$$

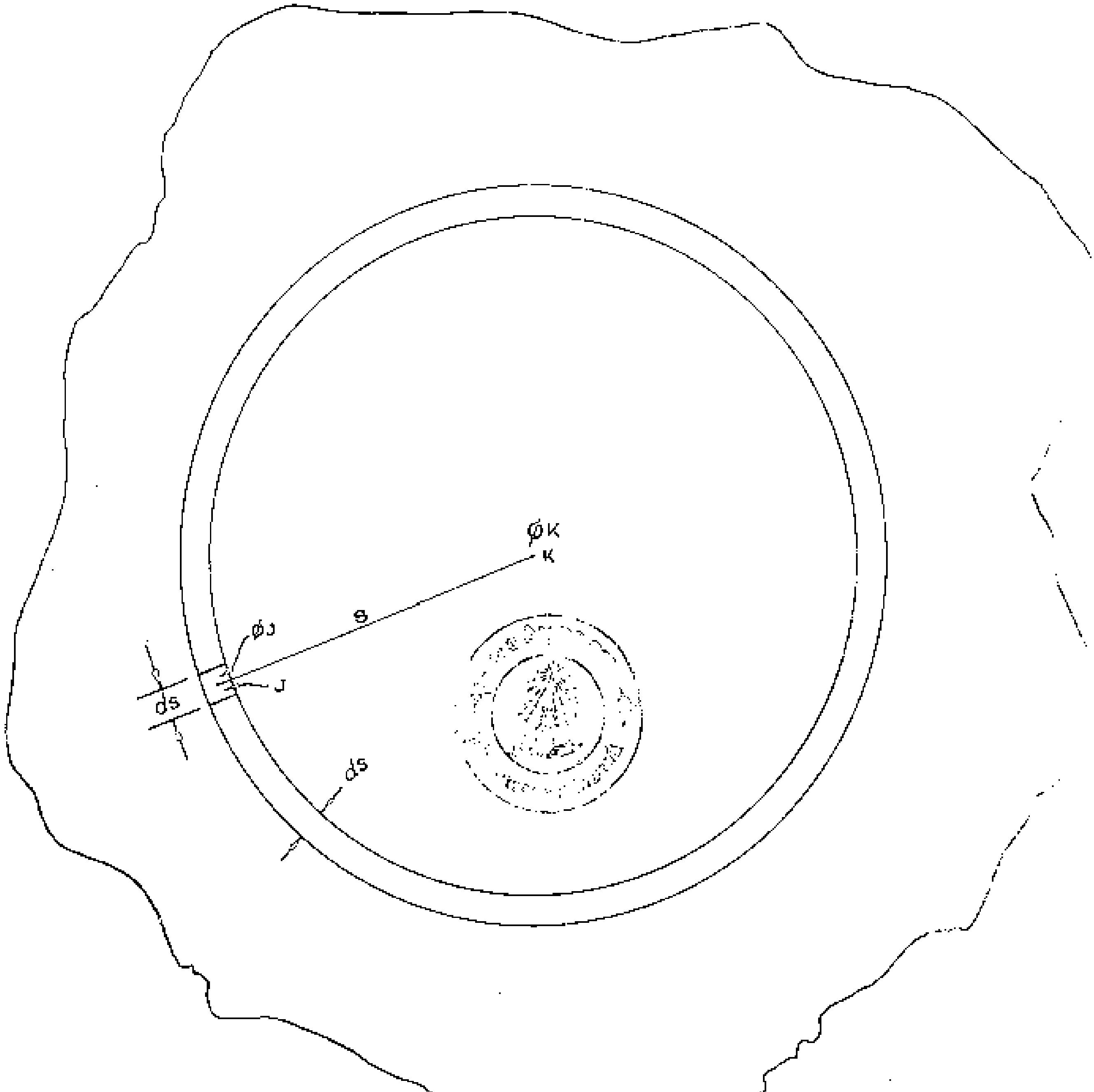

FIG 2.2 FORCE, CENTERED FLUXES

$$F_{KJ} = G/S \text{ dynes} \tag{2.7}$$

Because this force is the product of two effective values, and because the current and flux concerned are in phase, the force F_{KJ} is the time average of the actual force.

Torque

The torque T_{KJ} (Fig. 2.1) due to force F_{KJ} acting at lever arm D about some point O, in dyne - centimetres, is

$$\begin{aligned} T_{KJ} &= F_{KJ} D \\ &= G D/S \end{aligned} \tag{2.8}$$

Reciprocity Law and Torque of a Flux Pair

It is obvious that in addition to F_{KJ}, there is an equal force F_{JK}, due to J eddies acting on Φ_K. Thus there is a reciprocity law as to forces. The total force is the sum of the two. Thus, for a flux pair (either in nonoverlapping circles or as pencils) the total F_P is

$$F_P = 2\ G/S \tag{2.9}$$

As to torque, both forces act with lever arm D. Thus there is a torque reciprocity law. Then the total torque T_P for a flux pair is

$$T_P = 2\ G\ D/S \tag{2.10}$$

Circular Disk

If an a - c flux pencil at K, Fig. 2.3, pierced a circular disk of radius A centimetres, the eddy current occur in eccentric circles.

On a very large plate disk, Fig. 2.3, lay out the disk circle as shown. Extend OK. The image of the K - pencil of flux can be located at K', by theory long since developed. If OK = R

$$\begin{aligned} KK' &= (A^2 - R^2)/R \\ OK' &= KK' + R \\ &= \frac{A^2 - R^2}{R} + R \\ &= A^2/R \end{aligned} \tag{2.11}$$

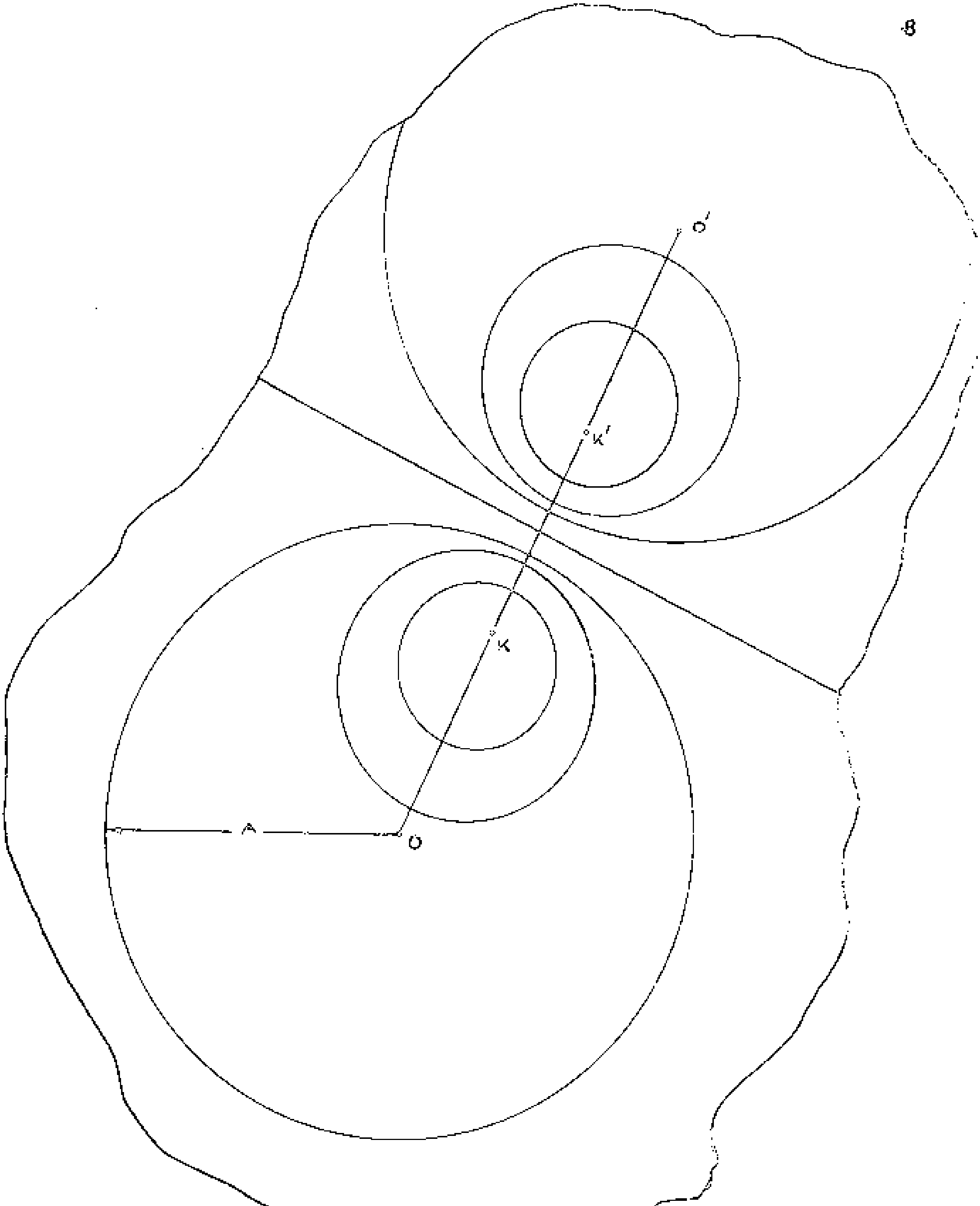

FIG.2.3 FLUX PENCIL, IMAGE, AND ECCENTRIC EDDY-CURRENT PATTERN.

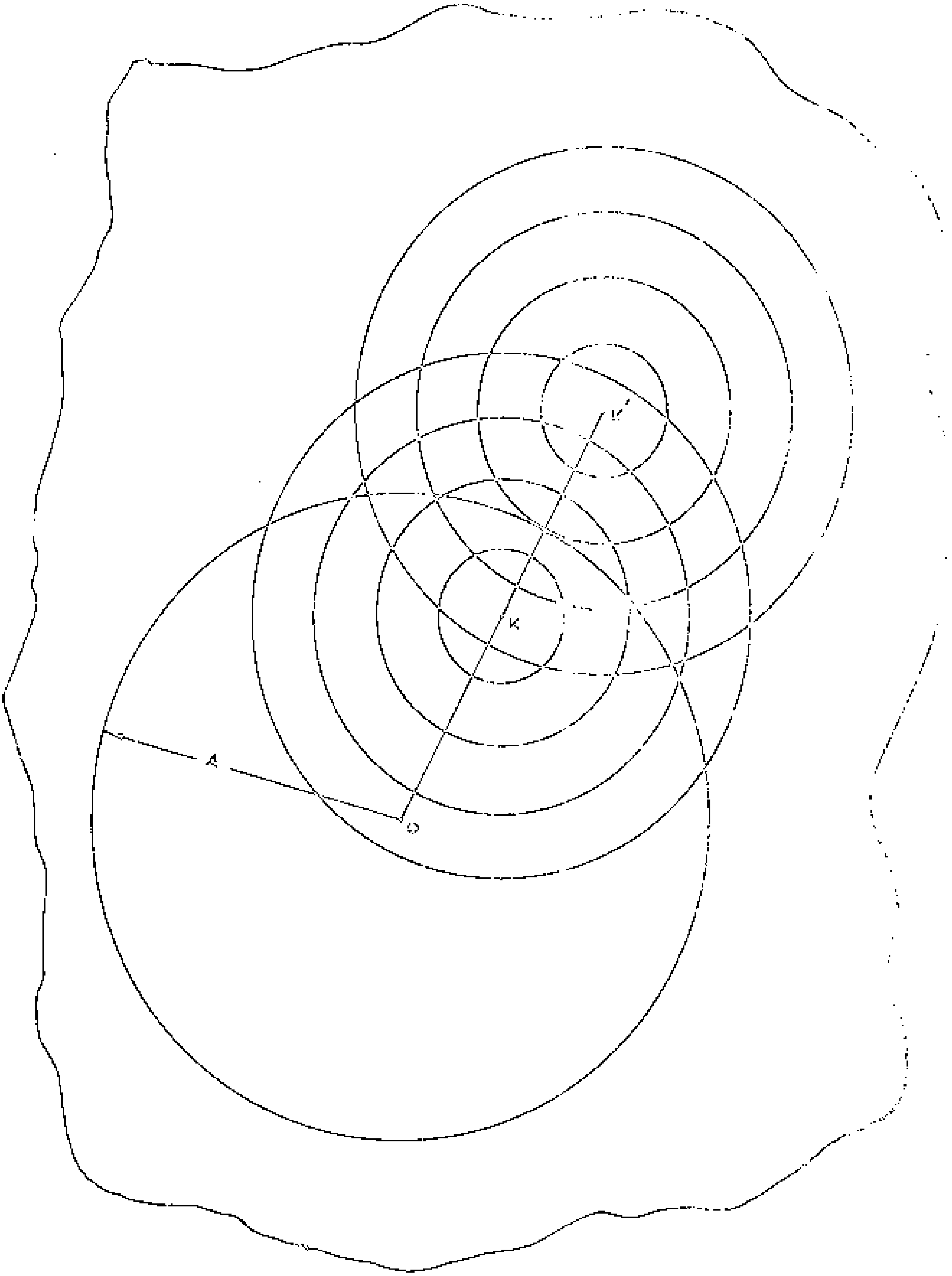

FIG. 2.4 CONCENTRIC EDDY CURRENTS CAUSED BY INDIVIDUAL FLUX PERIODS.

The image of K' pencil is equal to the K-pencil but is opposite in sign. Acting together, the pencil pair will set up the double eccentric eddy pattern, and within the disk circle, the eddies are identical with what the K-pencil alone would set up in the real disk.

Passing to Fig. 2.4 we avoid eccentricity troubles by dealing with the flux pencils individually. In the large disk, the K-pencil sets up component concentric eddies about itself and likewise, for the K'-pencil. (Note that this pair within itself creates no net force, the eddies of each are in quadrature with the flux of the other.)

The foregoing is applied in Fig. 2.5 where a disk is acted on by flux pair. The flux ϕ_K can be centred first at K. The K-pencil image at K' is located next. The flux pair creates a total force as expressed in the equation:

$$F_P = 2G/S$$

and a total torque as given by the equation:

$$T_P = 2GD/S$$

Similarly, the flux pair ϕ_J and ϕ'_K yield another (smaller) force and torque, the force acting along S' at lever arm D'. Since the image flux is opposite to the K-pencil, the image torque opposes the direct torque, and the net torque is

$$T_P = 2G\left(\frac{D}{S} - \frac{D'}{S'}\right) \tag{2.12}$$

<u>General Torque Expression; Circular Disk and a Flux Pair</u>

To develop a general torque expression, use the additional constructions given in Fig. 2.5. Instead of lever arm D and D' lever arm C or OJ will be used. This calls for the nse, not of the whole forces along S and S', but of their horizontal components (taking C as vertical.) The components are H/S and H'/S', respectively, times two forces. The total torque can be written then

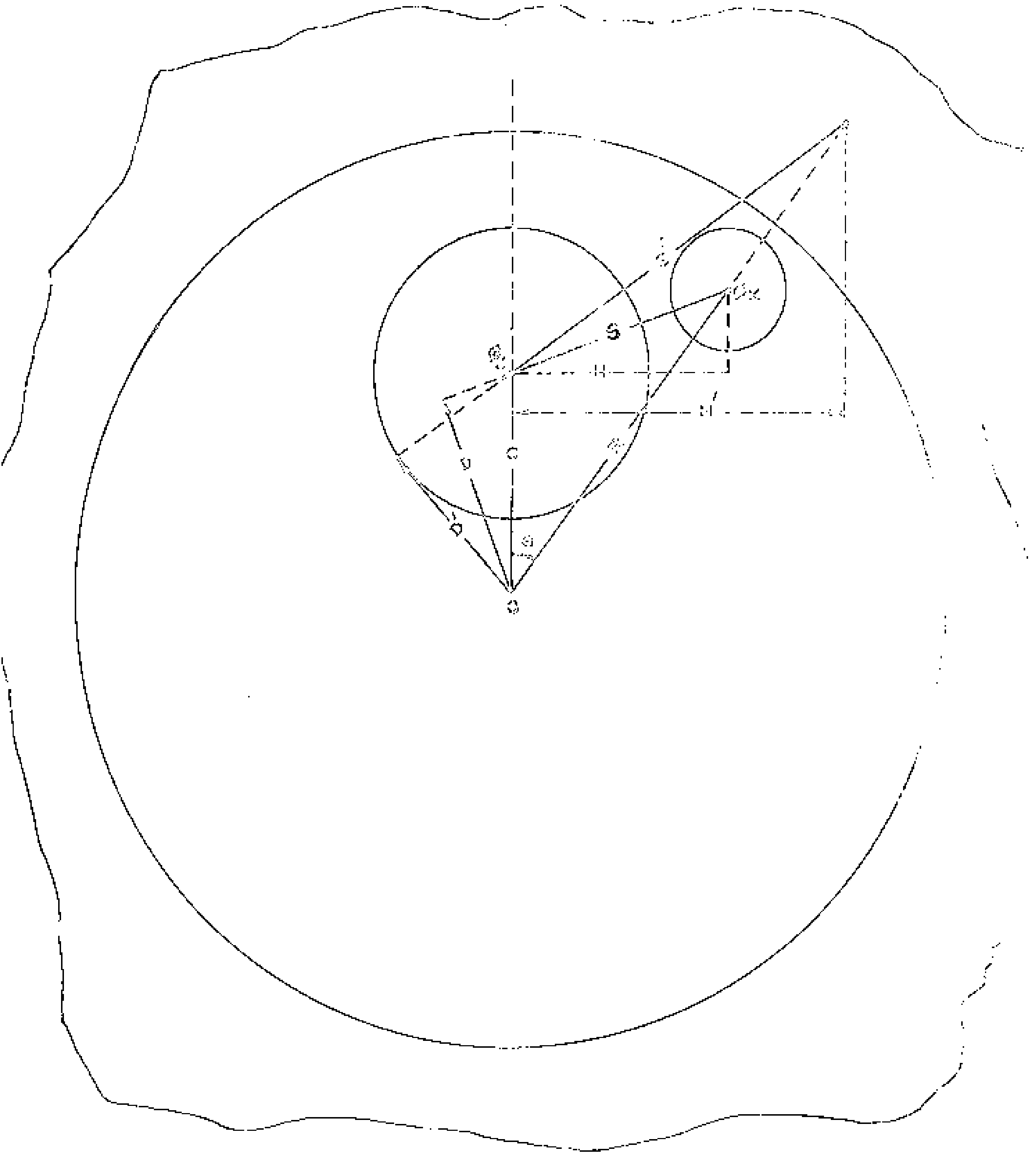

FIG. 2.5 TORQUE OF A FLUX PAIR

$$T_P = 2G\left(\frac{C}{S}\frac{H}{S} - \frac{C}{S'}\frac{H'}{S'}\right)$$

$$= 2GC\left(\frac{H}{S}2 - \frac{H'}{S'}2\right) \qquad (2.13)$$

In equation above H, H', S^2 and S'^2 can be replaced as follows, if Θ is the angle between C and R

$$H = R \sin\Theta$$

$$H' = \frac{A^2}{R}\sin\Theta$$

Using the cosine law of triangles

$$S^2 = C^2 + R^2 - 2\,CR\cos\Theta$$

and

$$S'^2 = C^2 + (A^2/R)^2 - 2C(A^2/R)\cos\Theta$$

$$T_P = 2GC\left[\frac{R\sin\Theta}{C^2+R^2-2CR\cos\Theta} - \frac{(A^2/R)\sin\Theta}{C^2+(A^2/R)^2-2C(A^2/R)\cos\Theta}\right] \qquad (2.14)$$

$$= 2G\left[\frac{\sin\Theta}{(C/R)+(R/C)-2\cos\Theta} - \frac{\sin\Theta}{(CR/A^2)+(A^2/CR)-2\cos\Theta}\right] \qquad (2.15)$$

$$= \frac{G\sin\Theta}{\frac{1}{2}\left(\frac{C}{R}+\frac{R}{C}\right)-\cos\Theta} - \frac{G\sin\Theta}{\frac{1}{2}\left(\frac{CR}{A^2}+\frac{A^2}{CR}\right)-\cos\Theta} \qquad (2.16)$$

If J and K are equidistant from O, C = R, and the torque reduces to

$$T_P = G\left[\frac{\sin\Theta}{1-\cos\Theta} - \frac{\sin\Theta}{\frac{1}{2}\left(\frac{C}{A}\right)^2+\frac{1}{2}\left(\frac{A}{C}\right)^2-\cos\Theta}\right] \qquad (2.17)$$

Variation of Disk Torque With Flux Pair Radius

Let the a - c driving fluxes of a circular disk consist of a flux pair at the same radius (C = R) either circle may be of any size. However, the circles may not overlap, nor may they fall partly outside the disk, if the equations are to hold. If Θ is taken constant at 30 degrees, for example, a curve of torque versus C for a disk of fixed A can be worked out. This curve, for G = 1 (for example, when f = 50 cps, b = 0.0915 cm, $\phi_K = \phi_J = 22.7$ maxwells, and $\rho = 2.83\text{x}10^{-6}$, ohm per cm, G is unity) is shown in Fig. 2.6 in terms of T_P versus C/A.[1] The trends of this curve are interesting.

As C approaches A, the images move in closer to the disk circle and have an increasing negative torque effect.

As the flux pair is made to retreat from the disk edge, the images move out and diminish in effect, and the net torque increases. But as C is reduced further, one is tempted to conclude that since low C means low lever arm, the torque would fall off again. But it must be remembered that under the conditions, S is proportional to C and that eddy current strength, for example, caused by ϕ_K through J, is inversely proportional to S. Therefore, torque will not fall off. Inspection of the last equation above shows that as C/A approaches zero, T_P approaches a constant value. The achievement of the greatest torque at the least possible radius may be somewhat surprising.

Disk Torque Due to Irregular Fluxes

Disk driving torque can arise due to two fluxes in quadrature, neither flux being contained within a circular area and both having irregular density distribution. Moreover, the fluxes may overlap greatly.

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[1] A.D. Moore, "Eddy Current in Disks." AIEE Transactions (on power apparatus and system) Vol. 66, 1947, p. 4

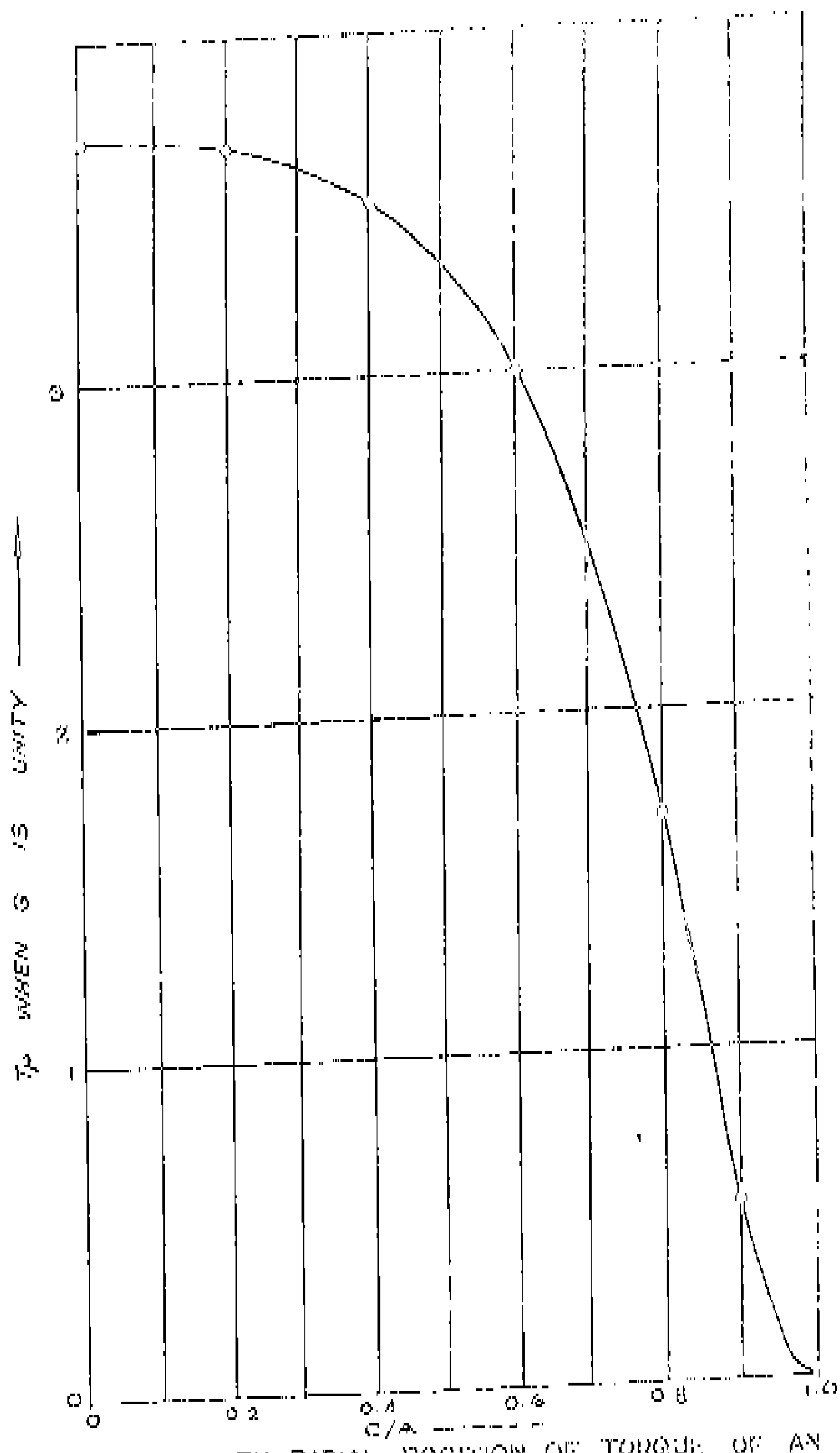

FIG 2.6 VARIATION WITH RADIAL POSITION OF TORQUE OF AN EQUAL RADIUS FLUX PAIR, G BEING UNITY.

The area where a flux pierces the disk can be divided into a number of rectangular areas, each containing a known amount of flux. The other flux is treated likewise. In each little area, the flux of that area is centered arbitrarily; it becomes a flux pencil located somewhere within the area. If the two fluxes are ϕ_J and ϕ_K, every J - pencils eddies must be allowed to react with every K - pencil, also (for the image) every J' - pencil must be allowed to react with every K - pencil, and a summation must be arrived at somehow. The amount of work involved in the solution could be great, if carried out in terms of equation (2.14), or (2.15).

Instead of using these general forms, it is much faster to retreat to the form of torque as given in equation (2.10). The case can be drawn to scale, D, S, D' and S' values can be measured directly, and the computing can be done by using a slide rule. Even so, the fairly accurate treatment would call for many measurements and a good deal of slide rule work.

Braking Forces and Torque

An electromagnet or a permanent magnet puts a flux, uniformly distributed and of density B, through a circular rotating disk, within the area shown in Fig. 2.7. The area has any size and shape. There may be more than one area, each with its own density.

The description of the Foucault currents caused by rotation, and the finding of resulting forces and torques heretofore have not been attacked successfully for snch general cases. Solutions can be achieved by means of what is believed to be a new concept, by the use of some of the a - c developments already described and of some development yet to be mentioned.

Flux Band and Pencil - Pair Concept.

The flux can be divided into bands, each of constant radius and narrow width, such as band JK in Fig. 2.7. Radial elements of metal sweep through the flux of the band and set up a constant electromotive force aronnd any such paths as path 1 and path 2.

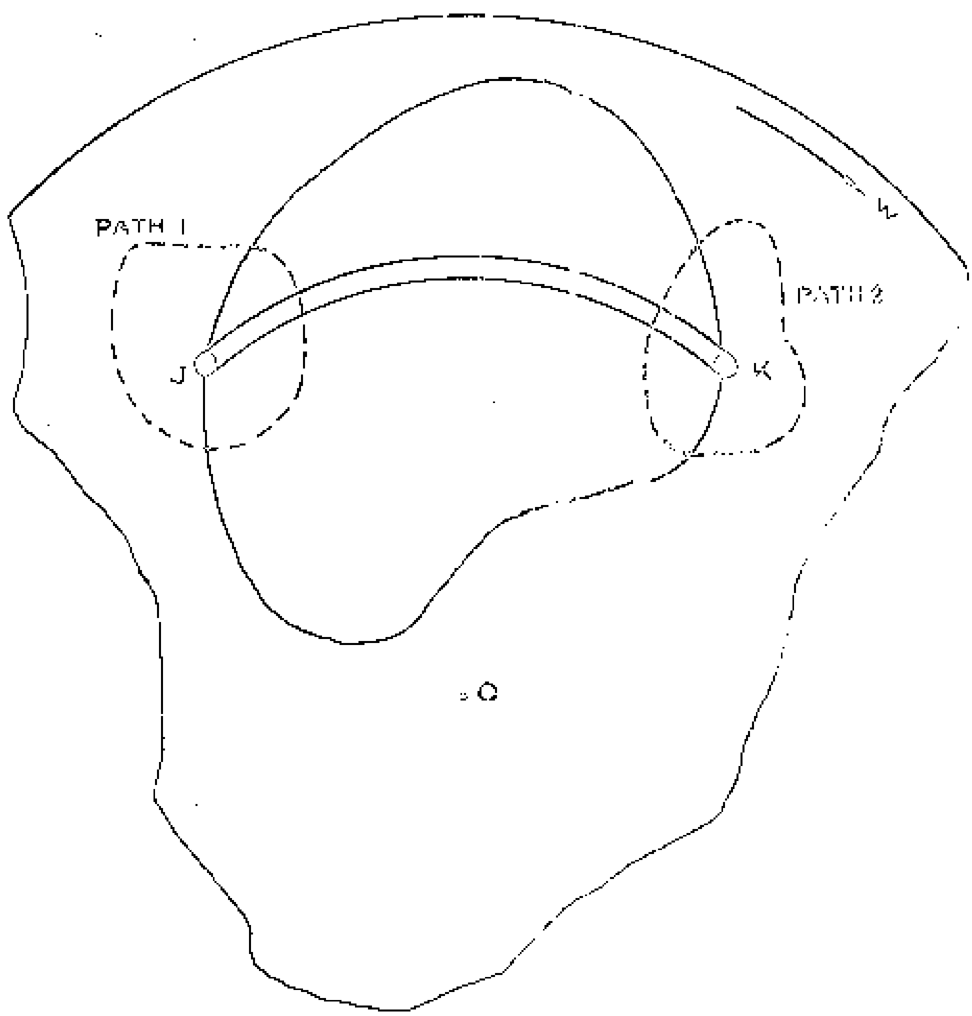

FIG 2.7 FLUX BAND AND PENCIL PAIR.

The concept referred to above is this; the same electromotive force can be set up around such paths in a fixed disk by means of pencils of flux piercing the disk at J and K, these being opposite flux pencils, and both increasing or decreasing at the same rate. Adoption of this concept makes it possible to define the current set up by the JK flux band and by all other such bands. The concept of fixed disk and pencil - pair is used only to define the current. The currents are allowed then to react with the actual flux in order to find forces and torques.

The adoption of the changing pencil pair makes it possible to adopt the image and conceutric current ideas already fully outlined in the a - c developments.

Force Due to a Pencil, and Flux in an Angle.

In Fig. 2. 8, there are two flux areas, area 1, and area 2, with densities B_1 and B_2. A narrow flux band JK at radius r in area 1 is replaced for current effects by flux pencils J and K changing through a fixed disk. As in previons developments, J has its (opposite) image pencil J' and K has its (opposite) image pencil K'.

There are four flnx pencils through a large disk, each setting up concentric eddiea about itself. The reaultant of these four current patterns would be identical within the disk circle, with the actual current set up in the rotating disk by the JK flux band. Such of these concentric currents as pass through the areas will react with B_1 and B_2 to produce forces.

Throughout, we may think either of the flux pencils as all constantly increasing or as decreasing. Herein, they will be treated as decreasing. With rotation as shown in Fig. 2. 8, and with downward B_1 flux, the electromative force due to flux band JK would be outward. Then the four pencils would have signs as shown in Fig. 2. 8 (b) and they would set np currents as shown. Theae currents would tend to prevent the (decreasing) change of the flux pencila.

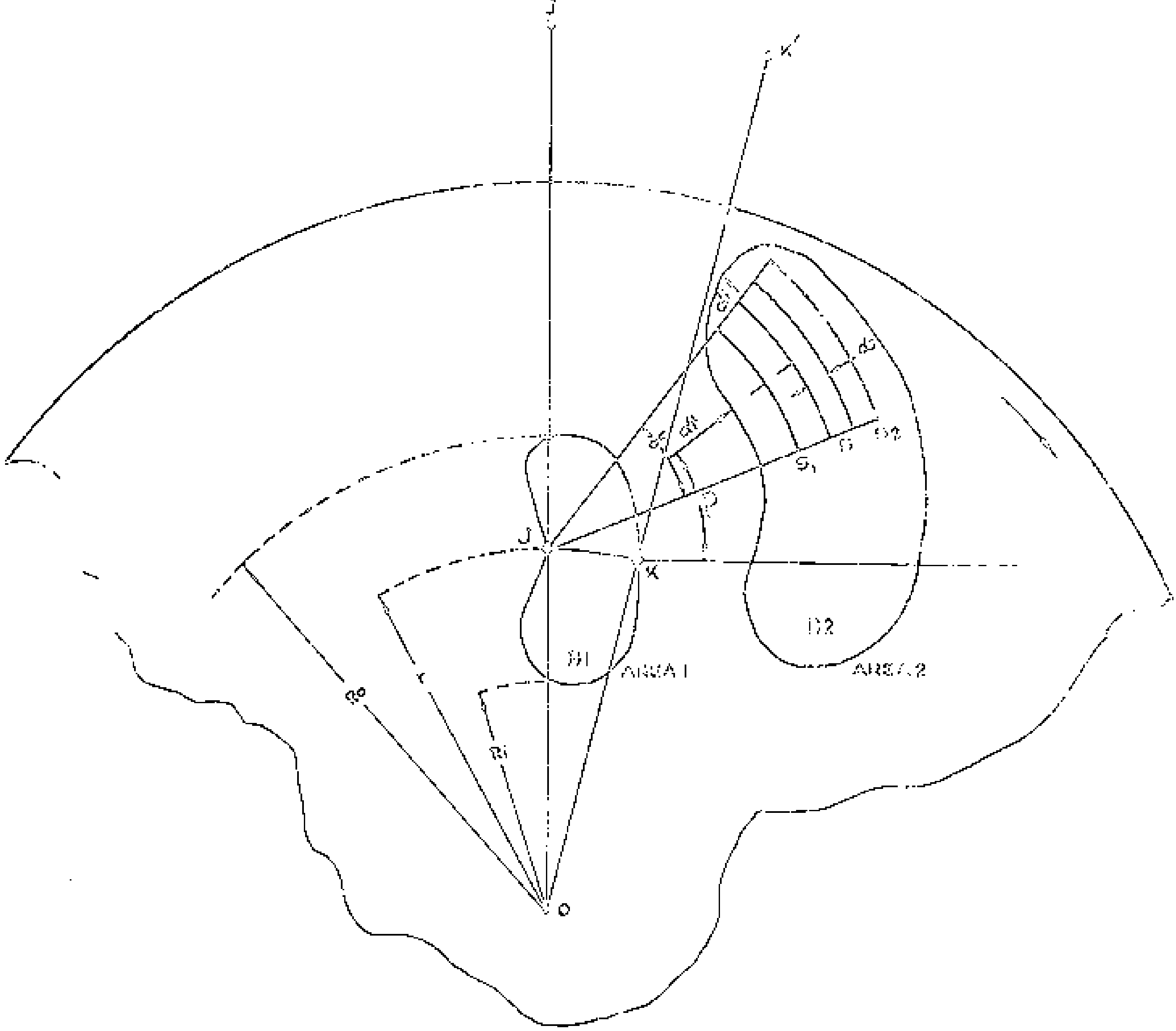

FIG 2.8 (a)

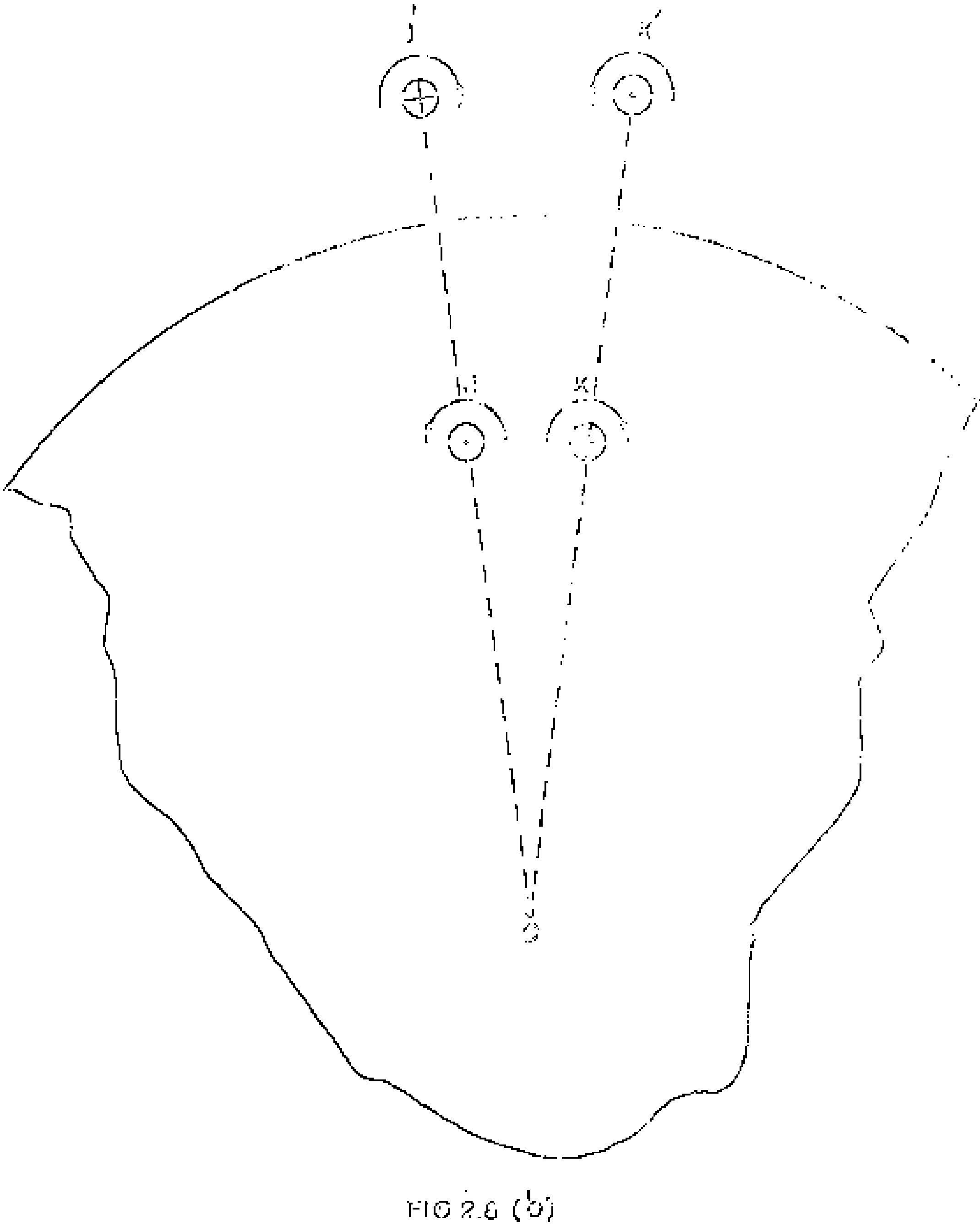

FIG 2.6 (b)

FIG. 2.8 J - PENCIL ACTING ON PART OF AREA 2, POLARISED .

The flux band electromotive force due to rotation, in volts per radial centimetre, is

$$E \quad = \quad wrB_1 \times 10^{-8} \qquad (2.18)$$

where E = e.m.f. due to rotation, in volt per radial centimatre

w = angular velocity in radians per second

B_I = d - c flux density

Ascribing this electromotive force to flux pencils in a fixed disk, we now find the currents element di of width ds, flowing concentrically about J at radius S from J, the resistance R offered to di is given by equation (2.3), then

$$di \quad = \quad E/R$$

$$= \quad \frac{b\ E\ ds}{\rho\ 2\pi S}$$

Consider the resultant dynes force df caused by the di current are within the angle α (see Fig. 2.8).

It is shown easily that the force due to the current arc is the same as if the current flowed along the chord. By symmetry, the force acts through J.

$$\text{Chord} \quad = \quad 2S \sin\frac{\alpha}{2}$$

$$F \quad = \quad (BIL)/10 \text{ dynes, in general}$$

$$df \quad = \quad 0.1B_2\ 2S \sin\frac{\alpha}{2}\ di$$

$$= \quad (0.2\ B_2 \sin\frac{\alpha}{2})\ S\ \frac{bEds}{\rho\ 2\pi S}$$

$$= \quad (0.1B_2 \sin\frac{\alpha}{2})\ \frac{bEds}{\pi\rho} \qquad (2.19)$$

Let F_α in dynes per centimetre of r designate the forces due to J - currents acting on B_2 flux contained within the area bounded by arcs of radius S_1 and S_2 and angle α .

$$F_\alpha = \int_{S_1}^{S_2} df$$

$$= \int_{S_1}^{S_2} (0.1\, B_2 \sin \frac{\alpha}{2}) \frac{bE}{\pi \rho}\, ds$$

$$= (0.1\, B_2 \sin \frac{\alpha}{2}) \frac{bE}{\pi \rho} (S_2 - S_1) \qquad (2.20)$$

If OJ (equal to radius r) is taken vertically, we need the horizontal component of F_α which, acting through J on lever arm r or OJ, produces torque about O. The horizontal component, $F_{\alpha H}$ is

$$F_{\alpha H} = \cos\beta\, F_\alpha$$

$$= (\sin \frac{\alpha}{2}) \left[(S_2 - S_1) \cos\beta \right] \frac{0.1\, bB_2 E}{\pi \rho}$$

$$= (\sin \frac{\alpha}{2}) \left[(S_2 - S_1) \cos\beta \right] \frac{0.1\, bB_2 wrB_1}{\pi \rho} \times 10^{-8}$$

$$= (\sin \frac{\alpha}{2}) \left[(S_2 - S_1) \cos\beta \right] \frac{bwrB_1 B_2}{\pi \rho} \times 10^{-9} \qquad (2.21)$$

To abbreviate, let

$$M = \sin \frac{\alpha}{2}$$

$$n = (S_2 - S_1) \cos\beta$$

$$w' = \frac{wrbB_1 B_2}{\pi \rho} \times 10^{-9}$$

Hence,

$$F_{\alpha H} = Mnw' \qquad (2.22)$$

<u>Integration of Horizontal Forces: The "Alpha" Chart</u>[2]

Some means of integration over the whole of area 2 must be found. For this, the "alpha" chart is devised as shown in Fig. 2.9 for

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[2] Ibid, p. 8

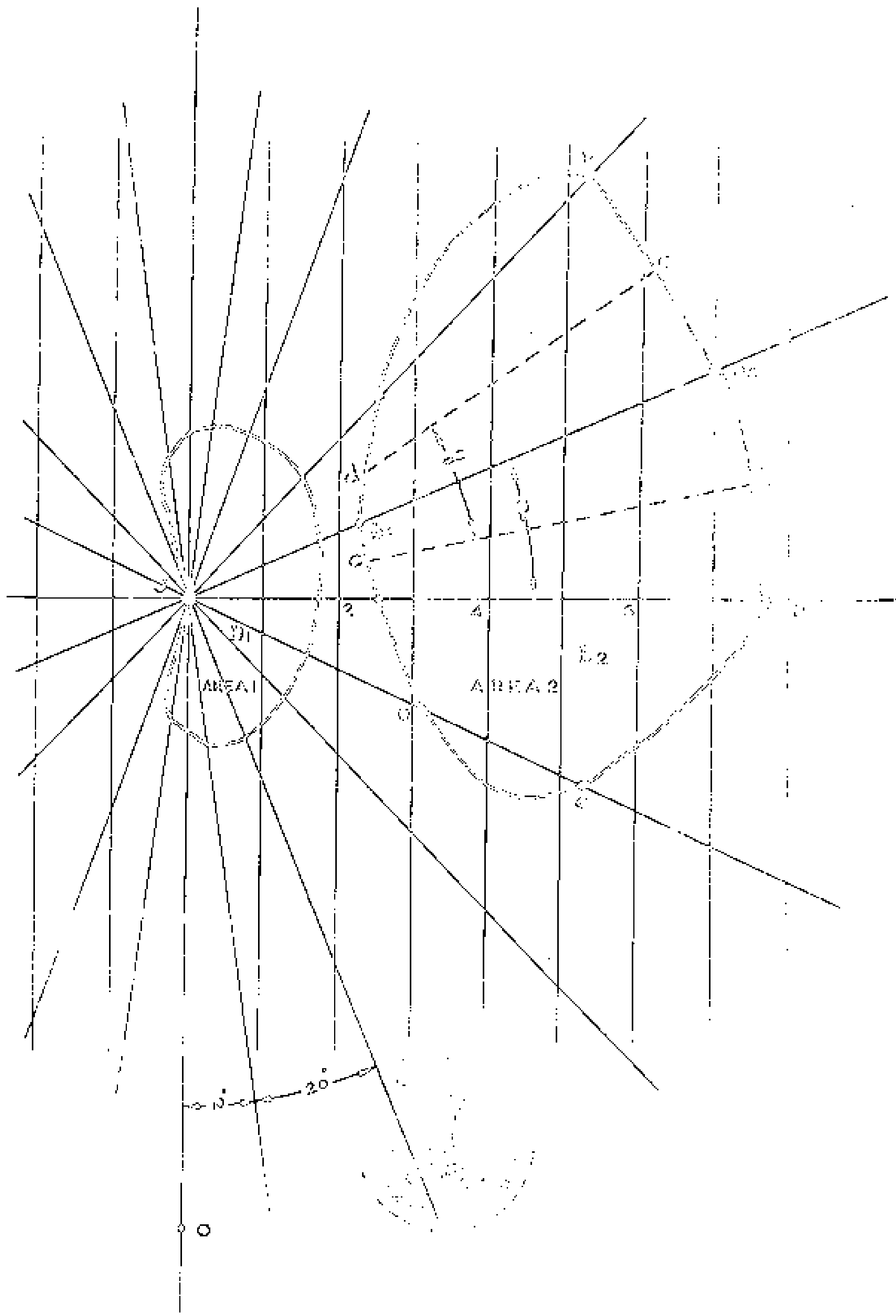

FIG. 2.9 ALPHA CHART FOR α = 20 DEGREES.

example, this example being drawn for α = 20 degrees, M = 0.1736. In the a, b, c, d area, it will be evedent that n, or $(S_2 - S_1) \cos\beta$, can be read directly from the vertical parallels spaced one centimetre apart, when the main line of the chart is lined up through disk centre O and chart apex is placed at pencil J. In this example, n = 6.9 - 2.4 = 4.5.

The total horizontal force F_H due to J - currents through ϕ_2 in area 2 would be (approximately) proportional to the sum of the several $F_{\alpha H}$ values

$$\text{Let} \quad N = \Sigma n \qquad (2.23)$$

In the example, start at point P, proceed clockwise around the perimeter of area 2, and read off successive values.

$$N = (5.2 + 6.9 + 7.7 + 5.3) - (3.1 + 2.5 + 2.4 + 2.5)$$

In general

$$F_H = MNw' \text{ dynes per centimetre of } r \qquad (2.24)$$

In practice, several "alpha" charts[3] are needed, with ranging from small to large. By selecting the right chart to use on a near or far area, the area can be sliced always in to enough sectors to insure high accuracy with minimum time spent at reading and summing up. The summing is done most easily by putting the readings directly onto the adding machine.

The less accurate readings are likely to be achieved where the radial lines soon leave after entering the area, as at ee in Fig. 2.9. In such cases, a correcting mental estimate is easily made. In general, the errors of this approximate chart method tend to cancel out, and high accuracy is achieved.

Torque: One Flux Interacting with another

If T_r in dyne - centimetres per centimetre of r is the torque due to F_H acting at lever arm r (or OJ), then

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[3] Ibid, p. 8

$$T_r = F_H\, r$$

$$= MNw' r$$

$$= \frac{MN\, w\, b\, B_1 B_2\, .\, r^2}{\pi \rho} \times 10^{-9} \qquad (2.25)$$

Let

$$W = \frac{w\, b\, B_1\, B_2}{\pi \rho} \times 10^{-9}$$

$$T_r = MNW\, r^2 \qquad (2.26)$$

Appropriately select several different radii from inner radius R_i to outer radius R_o, and find MNr^2 for each. This work is routinized and curried out in tabular form with the aid of "alpha" charts. W is common throughout and should be omitted until the tabulations are added up.

Plot a curve of MNr^2 versus r. Find the mean ordinate and call it T_A. Then, if T_J in dyne - centimetres is the torque due to all J - pencils on the J - side of area 1,

$$T_J = T_A\,(R_O - R_i)\,W. \qquad (2.27)$$

T_K is found likewise for all K - pencil on the K - side of area 1. The torque due to the image - pencil's currents requires a different treatment, because the forces act through the image. For the J' image,

$$T_r = (OJ')\, F_H$$

$$= MNw'\,(OJ') \qquad (2.28)$$

N is found by now centering the "alpha" chart apex at J'. As previously shown in equatiou (2.11)

$$OJ' = A^2 / r$$

Then $$T_r = \frac{MN\, wrb\, B_1 B_2}{\pi \rho} \times 10^{-9} \times \frac{A^2}{r}$$

$$T_r = \frac{MNA^2 \, w \, b \, B_1 \, B_2}{\pi \rho} \times 10^{-9}$$

$$= MNWA^2 \qquad (2.29)$$

For the same several radii from R_i to R_o as used before, MNA^2 is found and plotted. T_A is obtained, and

$$T_{J'} = T_A \, (R_o - R_i) \, W \qquad (2.30)$$

$T_{K'}$ is found likewise.

All four torques are combined then to get the net torque T_{12}, expressed positively as a braking torque, thus

$$T_{12} = T_K - T_J - T_{K'} + T_{J'} \qquad (2.31)$$

The routine above was presented in order to make the analysis easily understood. However, it requires plotting four curves. This can be avoided by combining, in the tabulation of data, the MNr^2 values (four iu all for each r used) and then plotting only one curve and finding only one T_A from it.

T_{12} is the torque due to all currents originated by area 1 actiug on the flux of area 2.