A DIRECT METHOD

FOR SOLVING SOME PROBLEMS IN THE CALCULUS OF VARIATIONS.

by

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ABSTRACT

In this thesis we study the variational problem of choosing $y(x)$ so that

$$I = \int_{x_o}^{x_n} F(x,y,y')\,dx$$

is a minimum, where $y(x_o) = y_o$, $y(x_n) = y_n$.

A direct method for solving this problem, is described in which the value of I is calculated along polygonal curves with variable vertices on the lines $x = x_0$, $x = x_1$, ... $x = x_n$ where $x_i = x_0 + i\,\Delta x$, $\Delta x = \frac{x_n - x_o}{n}$, having the ordinates y_o, y_1, y_2, ...y_n.

Starting from an arbitrary initial polygonal curve p^o the ordinates of the vertices are varied one by one so as to make I a minimum in each step.

By this method we get a sequence of polygonal curves and a corresponding monotonic decreasing sequence, I_o, I_1, I_2,........I_k......of value of I. The polygonal curves p^k converge to p^m which makes I a minimum. Then taking the limit as $n \longrightarrow \infty$ the polygonal curves p^m approach the smooth curve which is the required solution of the problem.

In the case of the problem of finding the shortest curve between two given points we can prove by the

direct method that the polygonal curve which make the distance I a minimum is the straight line joining the two points.

The method is next applied to the standard problem of finding the surface of revolution of minimum area, and to the Brachistochrone problem. We prove that the polygonal curves which make I a minimum approach the solution of Euler Equation

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0, \text{ as } n \longrightarrow \infty .$$

Numerical examples are worked out by the direct method described and the results are compared with the solution of the Euler Equation.

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