

## CHAPTER III

### **A MULTI-LEVEL CAPACITED LOT SIZING PROBLEM WITH MULTI-WORKSTATION (MLCLSP-M)**

In a manufacturing, to find purchasing plan and production plan as well as to satisfy customer ordering due date (or to find lot size of purchasing and production order in each period) is one of the most challenging issues confronting management. The most important questions are how much lot size of the purchasing orders should be and how much lot size of the production orders should be. As a consequence, a major challenge faced by the manager is the optimal plan for any activity that affects their production. The major activities include purchasing planning, production planning and stock consideration. Many of the responses to this challenge can best be described as point solutions that are limited by integration boundaries. Lacking exploration of this broader holistic planning view has led to decreasing confidence on part of the agreed planning system with belief that it is allowing an unreal optimum solution or a poor plan. This belief has resulted in a necessary to examine model of lot-sizing for purchasing and production planning system with on-hand inventory consideration. In this chapter, the holistic view model (consideration purchasing planning situation together with production planning situation while feasible capacity constraint) is appropriated and needed to find optimal plan. Such problem can be defined as Multi-level Multi-item Capacitated Lot Sizing Problem with Multi-workstation (MLCLSP-M).

With multi-period and multi-item system, this problem has become a primary importance (Armagan and Kingsman, 2004). As a result various research topics and fields have greatly interests for many decades (Berretta et al., 2005; Brahimi et al., 2006). More about resource problem in the production planning, there are various significant attributes concerned such as capacities, product-resource-assignment, number of resources, product/operation structure, minimal utilization rate, production

coefficient, and setup operation (Federgruen et al., 2007). For planning under limited capacity or a capacitated lot sizing problem (CLSP) is NP-hard or non-deterministic polynomial-time hard (Berretta et al., 2005) indicating that it is unlikely that a polynomial time algorithm can discover for its solution. The extension version of CLSP can be found in many researches within or without combination of such aforementioned resource problem attributes. As time goes by, the extensions of lot sizing problems have been investigated. Even several studies on lot sizing problems could applicable for lot size purchasing planning; no one has really concerned the dynamic situation and limitation of the purchasing. In addition, the dynamic situation of purchasing problems could only be found in the specific models(Ustun and Demi'rtas, 2008). Due to the lacking of particular models and the complication of the system, few studies have determined the integration lot sizing model for purchasing planning, production planning and on-hand inventory (Aksoy and Erenguc, 1988; Brahimi et al., 2006).

Furthermore, this multi-item multi-period decision has affected various costs in the purchasing and production planning system (Armagan and Kingsman, 2004; Robinson and Lawrence, 2004). With high impact of the margin in these business situations, the reducing cost is really necessary. Because the system includes purchasing lot size, production lot size, and on-hand inventory, the total cost can be defined as summary cost of purchasing cost, production cost, and carrying cost. To find the minimum cost, this research proposes the new minimization model that can be presented the all integration of three aspects as aforementioned. The solution of the developed model is the plan for capacitated dynamic lot sizing under multi-period situation that has a minimize total cost (procurement or purchasing cost, production cost and item holding cost). In another word, this research model emphasizes on the reduction of inventories and related unproductive activities and costs. The existing model only considers the assumption that only one item can be operated on one workstation (Chinprateep and Boondiskulchok, 2007). While it works well for a particular one, it is rather difficult to apply to a multi-workstation production process. With the original model, it is easy to overlook a small detailed like that how the impact on the multi-workstation has been made. More importantly it has created a

number of gaps in the management of the production in terms of capacity and utilization. To a very great extent, the model for the multi-workstation capability situation is developed. These environments extend the basic model by assigning the lot size over a workstation in the capable workstation set. Even though this way doesn't increase the setup in one period, it increases a trade-off between utilization capacity and increasing cost with stock consideration. Besides, the trade-offs in this problem are also for between saving in set-up cost of the lot size of purchasing or production and saving in holding cost, for the dynamic parameters in each period and for utilized capacity of the capable workstations. For clarifying the model, we also present a numerical example of this problem.

This chapter is organized as follows: Problem description is presented in Section 3.1. Assumption and notation are provided in Section 3.2 and 3.3, respectively. The model formulation will be presented in Section 3.4. The conclusions of this work and some recommendations for the next are presented in Section 3.5.

### **3.1 Problem Description**

In the system considered here, various items are assembled or manufactured to produce final products. As general production structure, one or more (preceding) items may be used to produce an item, but an item can be used as a component of its succeeding items as given in Bill of Material (BOM). With multi-period multi-workstation consideration, the manufacturing system consists of a number of workstations to process different parts. It is assumed that purchasing or production of each item can require more than one workstation. An example of this can be seen in Figure 3.1. There are ten items (seven production items and three raw materials), and five workstations (three production workstations and two purchasing workstations). The capacity of each workstation is limited by availability of its resources. For example, the capacity of production workstations are limited by capacity of available operating time of machines or workers and the capacity of a purchasing workstation are limited by available raw materials in the supply workstation. While the capacity of

a purchasing workstation is charged by a purchasing lot size, the capacity of production workstation will be charged both from lot-size operation time and setup time. Besides, the capability of a workstation is provided in the Table 3.1.

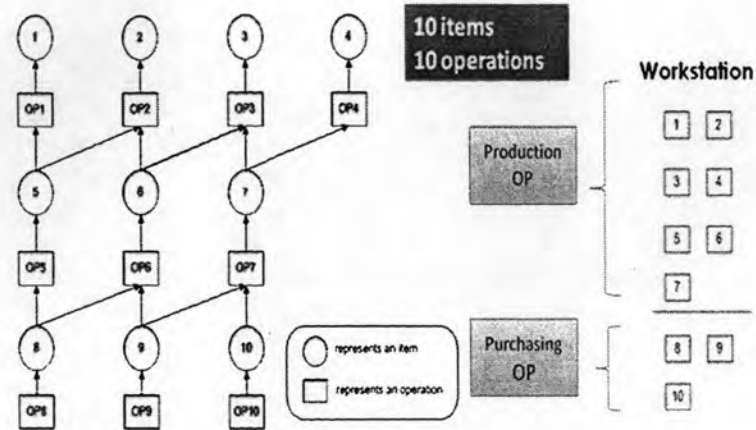


Figure 3.1 An example of BOM

Table 3.1 Capability table.

| Operations | Capable Workstation | Required Item   | Immediate Successor |
|------------|---------------------|-----------------|---------------------|
| OP1        | WS1/WS3             | Item 5          | Item 1              |
| OP2        | WS1/WS3             | Item 5, Item 6  | Item 2              |
| OP3        | WS1/WS2             | Item 6, Item 7  | Item 3              |
| OP4        | WS2                 | Item 7          | Item 4              |
| OP5        | WS3                 | Item 8          | Item 5              |
| OP6        | WS1                 | Item 8, Item 9  | Item 6              |
| OP7        | WS2/WS3             | Item 9, Item 10 | Item 7              |
| OP8        | WS4/WS5             | -               | Item 8              |
| OP9        | WS4                 | -               | Item 9              |
| OP10       | WS5                 | -               | Item 10             |

By way of illustration let us look at the case of item 1; the item 1 can be produced by operation 1 which can be operated on the capacitated workstation 1 and the capacitated workstation 3. Therefore, whenever item 1 is produced, the capacity of workstation 1 or workstation 3 will be charged setup time and operation time (both of which varying by period) for the lot size of item 1. Understanding by this, each workstation may be used may or may not be used. In the circumstance, it

has to be a trade-off not only between fixed cost and variable cost if we want to keep the total cost low but also between charged capacities of the workstation selected.

As multi-workstation; we aim to find a minimum lot size of particular item in a period over multi-period planning horizon consideration. Here, once a product is ordered, the setup time of the production will be occurred and will be considered as sequence independent between orders of different productions. The setup activities for each production workstation incur setup costs and consume setup time. Within a limited capacity expressed in hours during each time period of the workstation, the operation time and the setup time will reduce capacity in period. Assume that the demands for the parts processing vary with time in a deterministic manner and there is no order for a component item from the customers. Therefore, production quantity of a component item can be computed from ordering quantities of end items that require the component item. If an item will not be planned to be used in the next period, it must be kept in warehouse and charged holding costs. Here, the quality of production is neglected. Moreover, all unit costs, prices, and setup times/costs are assumed to be known with dynamic by period. To address this multiple time period formation problem, a mixed integer programming (MIP) model is formulated. The objective of the model is to minimized total cost (setup cost, purchasing cost, production cost and inventory cost) for the entire planning time horizon with given time varying parameters (external demand, availability of raw materials, capacity of workstations, time parameters and cost parameters).

### **3.2 Assumption**

The assumption is the same as the research scope. In the system considered here, various items are assembled or manufactured to produce final products of various types. As the case with assembly systems, items for products are of a tree structure. That is, one or more (preceding) items may be used to produce an item, but an item can be used as a component of at most one (succeeding) item. It is assumed that demands for end items or products, which are determined by orders from

customers, are known and deterministic but may vary over time periods. A customer order is characterized by the type, required quantity and due date of an end item. It is assumed that there is no order for a component item from the customers. Therefore, production quantity of a component item can be computed from order quantities of end items that require the component item. The setup activities incur setup costs and consume setup time. Once a product is ordered, the setup of the production will be occurred. The setup time will reduce capacity in period that it occurs and will be considered as sequence independent between orders of different productions. Processing capacity of each workstation may depend on the number of machines or workers in the workstation. If the parts or products doesn't use in the next period, it must be kept in warehouse and charged holding costs. BOM of production and capacity of workstation in production and capacity of warehouse are assumed to be known and will be concerned in the model. Here, the quality of production is neglected. The availability, varying price and ordering cost will be considered in the purchasing part of the model. Moreover, all unit costs, prices, and setup times or costs are assumed to be known.

This study focuses on MLCLSP problem in finite planning horizon study to minimize total expected cost which includes purchasing cost, production cost and items holding cost.

- (3.1) Production is make to order basis with know in advance order in the finite planning horizon.
- (3.2) Time-window rolls over one period at a time, remaining units and executed orders of raw materials, components, and products are considered as initial value for decision making.
- (3.3) The production capacity, the warehouse capacity and all unit costs are assumed to be known.

- (3.4) The warehouse holding cost is per unit per time and is assumed to be known (One unit of each item uses one space in warehouse).
- (3.5) The varying price and the availability of raw materials are known in advance.
- (3.6) The material and product characteristics including BOM are assumed to be known.
- (3.7) No alternated BOM's are considered in this research.
- (3.8) All transportation both internal and external firm, and the quality control or defect is not considered.
- (3.9) Reworks, defects or repairs are not considered in this model.
- (3.10) There are multiple product can be produced in a period.
- (3.11) The time bucket is considered as big bucket.
- (3.12) Total cost that this research considered includes production cost, items holding cost and purchasing cost.
- (3.13) Although a workstation can operate more than one item, the workstation can operate only one item in each period.
- (3.14) Similarly, an item can be operated only a selected workstation in each period.

### 3.3 Notation

These are the notations which we used in our model.

Sets

|           |   |
|-----------|---|
| $T$       | represents the set of discrete period time in planning horizon,<br>whereby $T = \{1, \dots, NT\}$ |
| $R$       | represents the set of raw material items  |
| $COMP$    | represents the set of component items   |
| $E$       | represents the set of end-items   |
| $I$       | represents the set of items, whereby $I = \{1, \dots, NI\} = R \cup COMP \cup E$                  |
| $WR$      | represents the set of purchasing workstations   |
| $WP$      | represents the set of production workstations   |
| $WS$      | represents the set of workstations, whereby $WS = \{1, \dots, NK\} = WR \cup WP$                  |
| $S(i)$    | represents the set of successor items of item $i$ (By given BOM)                                  |
| $W(i, t)$ | represents the set of successor workstations of item $i$ (By given route sheet)<br>in period $t$  |
| $I(k, t)$ | represents the set of item that workstation $k$ can operate in period $t$                         |

#### Indices

|        |   |
|--------|---|
| $t, l$ | represents the time period index in planning horizon,<br>whereby $t, l \in T$ |
| $i, j$ | represents the item index, whereby $i, j \in I$                               |
| $k$    | represents the workstation index, whereby $k \in WS$                          |

#### Decision Variables

|             |   |
|-------------|---|
| $x_t^{i,k}$ | represents the amount of production item $i$ on workstation $k$ in period $t$   |
| $s_t^i$     | represents the amount of stock item $i$ at the ending of period $t$   |
| $y_t^{i,k}$ | represents the setup decision of the production item $i$ on workstation $k$ in<br>period $t$ , whereby $y_t^{i,k} \in \{0, 1\}$ |

#### Costs

|             |   |
|-------------|---|
| $p_i^{i,k}$ | represents the production unit cost of item $i$ on workstation $k$ in period $t$              |
| $f_i^{i,k}$ | represents the setup cost per unit of production item $i$ on workstation $k$ in<br>period $t$ |



$h_i^t$  represents the holding cost per unit of stock item  $i$  at the ending of period  $t$

### Parameters

$d_i^t$  represents the independent demand item  $i$  in period  $t$

$V$  represents the warehouse capacity

$M_i^{i,k}$  represents  $\min \left\{ \left( \frac{c_i^k - g_i^{i,k} \cdot y_i^{i,k}}{o_i^{i,k}} \right), \sum_{l=1}^t d_l^i, \left( V - \sum_{i \in I} s_i^t \right) \right\}$

$Avail_t^k$  represents the availability of workstation  $k$  during period  $t$

$cap_t^k$  represents the capacity of workstation  $k$  during period  $t$

$o_i^{i,k}$  represents production usage resource of item  $i$  on resource  $k$  during period  $t$

$g_i^{i,k}$  represents setup usage resource of item  $i$  on resource  $k$  during period  $t$

$u^{i,j}$  represents the usage item  $i$  for producing item  $j$ , whereby  $j \in S(i)$

## 3.4 Mathematical Model

The model of multi-level multi-item multi-workstation lot-sizing with capacity (MLCLSP-M) constraints for purchasing and production planning under warehouse limited consideration can be formulated as follows:

$$\text{Min} \sum_{t \in T} \sum_{i \in I} \sum_{k \in WS} (p_i^{i,k} \cdot x_i^{i,k} + f_i^{i,k} \cdot y_i^{i,k}) + \sum_{t \in T} \sum_{i \in I} (h_i^t \cdot s_i^t) \quad (3.1)$$

Subject to

$$s_{i-1}^t + \sum_{k \in W(i,t)} x_i^{i,k} = d_i^t + s_i^t, \forall i \in E, \forall t \in T \quad (3.2)$$

$$s_{i-1}^t + \sum_{k \in W(i,t)} x_i^{i,k} = \sum_{j \in S(i)} \left( u^{i,j} \cdot \sum_{k \in W(j,t)} x_j^{j,k} \right) + s_i^t, \forall i \in I - E, \forall t \in T \quad (3.3)$$

$$\sum_{i \in I(k,t)} x_i^{i,k} \leq Avail_t^k, \forall k \in WR, \forall t \in T \quad (3.4)$$

$$\sum_{i \in I(k,t)} (o_i^{i,k} \cdot x_i^{i,k} + g_i^{i,k} \cdot y_i^{i,k}) \leq cap_i^k, \forall k \in WP, \forall t \in T \quad (3.5)$$

$$\sum_{i \in I} s_i^i \leq V, \forall t \in T \quad (3.6)$$

$$s_0^i = s_{NT}^i = 0, \forall i \in I \quad (3.7)$$

$$\sum_{k \in WS} y_i^{i,k} \leq 1, \forall i \in I, \forall t \in T \quad (3.8)$$

$$\sum_{i \in I} y_i^{i,k} \leq 1, \forall k \in WS, \forall t \in T \quad (3.9)$$

$$x_i^{i,k} \leq M_i^{i,k} \cdot y_i^{i,k}, \forall i \in I, \forall t \in T, \forall k \in W(i,t) \quad (3.10)$$

$$x_i^{i,k} \geq 0, \forall i \in I, \forall k \in W(i,t), \forall t \in T \quad (3.11)$$

$$s_i^i \geq 0, \forall i \in I, \forall t \in T \quad (3.12)$$

$$y_i^{i,k} \in \{0,1\}, \forall i \in I, \forall k \in W(i,t), \forall t \in T \quad (3.13)$$

Here  $M_i^{i,k}$  is a big value or an upper bound of  $x_i^{i,k}$ , which can be determined as the maximum amount of the lot size. Let the index  $k$  represents each workstation in the workstation set,  $WS (k \in WS)$ , which is the set composed of all elements of the set of purchasing workstations ( $WR$ ) and all elements of the set of production workstations ( $WP$ ), understanding by this the set  $WS = WR \cup WP$ . The set  $I$  includes all items which composed of all elements of the set of raw material items ( $R$ ), all elements of the set of components ( $COMP$ ) and all elements of the set of end-items ( $E$ ). The index  $i$  represents each item in the set  $I$ , and the index  $t$  represents each period in the planning period set  $T$ , understanding by this  $i \in I$  and  $t \in T$ . With multi-level product structure, all end-items have only external demand and others have only internal demand or dependent demand. Besides, we introduce two new set which are  $W(i,t)$  and  $I(k,t)$ . The first set represents the set of capable workstations of item  $i$  in period  $t$ . The second set represents the set of capable items of workstation  $k$  in period  $t$ . The  $x_i^{i,k}$  and  $y_i^{i,k}$  are lot size and ordering (setup) decision variables for item  $i$  on workstation  $k \in W(i,t)$  in period  $t$ , respectively. The  $s_i^i$  are ending inventory decision

variables for item  $i$  in period  $t$ . The cost parameters are the purchasing unit price or production unit cost,  $p_i^{t,k}$ , ordering cost or setup cost,  $f_i^{t,k}$ , and the holding cost,  $h_i^t$ . The time parameters are the operation time,  $o_i^{t,k}$  and setup time,  $g_i^{t,k}$ . The capacity parameters for all workstation  $k \in WP$  in period  $t$  are represented as  $cap_t^k$  and for all  $k \in WR$  in period  $t$  are represented as  $Avail_t^k$ .

The objective function (3.1) aims at minimizing purchasing, production, setup, and holding cost. The constraints (3.2) and (3.3) are the balance constraints for end-items and immediate items, respectively. It is important to have a BOM relation representation for the balance material constraints. Let  $S(i)$  represents the set of immediate successors of item  $i$ . The usage parameters,  $u^{i,j}$ , represent the number of units of item  $i$  required for producing one unit of the immediate successors item  $j$  whereby  $j \in S(i)$ . In constraints (3.4), purchasing lot size must not exceed the availability of raw materials on a workstation  $k$  in period  $t$ ,  $Avail_t^k$ . Capacity production constraints (3.5) make sure that required capacity for setup and operation must not exceed available capacity. For each period, all holding-items are kept in one capacitated warehouse. The constraints (3.6) are the limited warehouse capacity ( $V$ ). We assume that there is no item in warehouse at the first period and the end of planning horizon as represented in constraints (3.7). To ensure that only one item will be assigned to one workstation in each period and one workstation will operate one item in each period, the constraint (3.8) and (3.9) are respectively added. The constraints (3.10) ensure that the setup variables are set to be 1 if there is positive production or purchasing lot-size in period  $t$ . Constraints (3.11) and (3.12) restricted all variables  $x_i^{t,k}$  and  $s_i^t$  to non-negative, respectively. Finally, constraints (3.13) enforce  $y_i^{t,k}$  to binary.

### 3.5 Numerical Example

In this section, we solved a numerical example of the proposed model using the mathematical model as presented. We consider a scenario with ten-item ten-workstation over a planning horizon of four periods. The product structure is shown in

Figure 3.1 whereas the capability matrix is given in Table 3.1. In order to analyze the mathematical model's characteristics, test instances were generated. The pattern of the generation was introduced in (Chandra and Grabis, 2001). The demand series, the products' structure (general or assembly) are exactly the same as they described. However, this research problem is an extension version of their problem with multi-workstation and one warehouse consideration. Therefore, the production cost, setup cost/ordering cost, operation time, setup time and capacity/Availability were initially generated for this research as shown in Table 3.2.

Table 3.2 All parameters initially generated for this research

| Parameters  | Data Generation  |
|-------------|--|
| $p_i^{i,k}$ | For production : NORM(12,5) , For purchasing : NORM (45,9)   |
| $f_i^{i,k}$ | For production : NORM(570,190) , For purchasing : NORM (820,250)   |
| $o_i^{i,k}$ | For production : NORM(15,3) , For purchasing : 0   |
| $g_i^{i,k}$ | For production : NORM(70,14) , For purchasing : 0  |
| $V$         | 10% of the summation of all workstations' capacity for all period divided by product of number of workstations and number of items                   |
| $Avail_i^k$ | Maximum value of summation of all demand divided by %utilization for all items in period   |
| $cap_i^k$   | Maximum value of summation of all demand (subtracted with setup time then divided by operation time) divided by %utilization for all items in period |

We select the problem G 0 1 1 3 0 which defines a test instance with

- General product/operation structure (G)
- Random setup time (0)
- Slight demand variations (1)
- 90% resource utilization on all resources (1)
- Test set specific TBO profile (3)
- No seasonality in the demand series (0)

However, we use problem size 10-item 10-workstation with four periods (the product structure is same as the standard library) to show this problem. To solve this Mixed Integer Programming (MIP) problem we used AMPL/CPLEX 8.0.0 solver. Results of the example are shown in Table 3.3.

Table 3.3 Result of an example

Total Cost = 154276 and Solve time = 5.4375 seconds

$x_i^{t,k}$

| t=1 |    |    |    |     |    |    |     |     |     |     | t=2 |    |    |    |     |    |     |     |     |   |     | t=3 |    |    |    |     |     |   |     |    |     |     | t=4 |     |    |    |     |    |     |    |    |   |     |
|-----|----|----|----|-----|----|----|-----|-----|-----|-----|-----|----|----|----|-----|----|-----|-----|-----|---|-----|-----|----|----|----|-----|-----|---|-----|----|-----|-----|-----|-----|----|----|-----|----|-----|----|----|---|-----|
| k   |    |    |    |     |    |    |     |     |     |     | k   |    |    |    |     |    |     |     |     |   |     | k   |    |    |    |     |     |   |     |    |     |     | k   |     |    |    |     |    |     |    |    |   |     |
| i   | 1  | 2  | 3  | 4   | 5  | 6  | 7   | 8   | 9   | 10  | i   | 1  | 2  | 3  | 4   | 5  | 6   | 7   | 8   | 9 | 10  | i   | 1  | 2  | 3  | 4   | 5   | 6 | 7   | 8  | 9   | 10  | i   | 1   | 2  | 3  | 4   | 5  | 6   | 7  | 8  | 9 | 10  |
| 1   | 64 | 0  | 0  | 0   | 0  | 0  | 0   | 0   | 0   | 0   | 1   | 80 | 0  | 0  | 0   | 0  | 0   | 0   | 0   | 0 | 0   | 1   | 0  | 0  | 0  | 0   | 0   | 0 | 0   | 73 | 0   | 0   | 1   | 0   | 0  | 0  | 0   | 0  | 0   | 75 | 0  | 0 | 0   |
| 2   | 0  | 28 | 0  | 0   | 0  | 0  | 0   | 0   | 0   | 0   | 2   | 0  | 29 | 0  | 0   | 0  | 0   | 0   | 0   | 0 | 0   | 2   | 31 | 0  | 0  | 0   | 0   | 0 | 0   | 0  | 0   | 0   | 2   | 0   | 32 | 0  | 0   | 0  | 0   | 0  | 0  | 0 | 0   |
| 3   | 0  | 0  | 48 | 0   | 0  | 0  | 0   | 0   | 0   | 0   | 3   | 0  | 0  | 0  | 0   | 0  | 54  | 0   | 0   | 0 | 0   | 3   | 0  | 51 | 0  | 0   | 0   | 0 | 0   | 0  | 0   | 0   | 3   | 0   | 0  | 0  | 0   | 52 | 0   | 0  | 0  | 0 | 0   |
| 4   | 0  | 0  | 0  | 0   | 0  | 0  | 116 | 0   | 0   | 0   | 4   | 0  | 0  | 0  | 0   | 97 | 0   | 0   | 0   | 0 | 0   | 4   | 0  | 0  | 0  | 0   | 104 | 0 | 0   | 0  | 0   | 0   | 4   | 0   | 0  | 0  | 104 | 0  | 0   | 0  | 0  | 0 | 0   |
| 5   | 0  | 0  | 0  | 0   | 0  | 92 | 0   | 0   | 0   | 0   | 5   | 0  | 0  | 0  | 0   | 0  | 127 | 0   | 0   | 0 | 0   | 5   | 0  | 0  | 86 | 0   | 0   | 0 | 0   | 0  | 0   | 0   | 5   | 107 | 0  | 0  | 0   | 0  | 0   | 0  | 0  | 0 | 0   |
| 6   | 0  | 0  | 0  | 0   | 78 | 0  | 0   | 0   | 0   | 0   | 6   | 0  | 0  | 80 | 0   | 0  | 0   | 0   | 0   | 0 | 0   | 6   | 0  | 0  | 0  | 111 | 0   | 0 | 0   | 0  | 0   | 0   | 6   | 0   | 0  | 56 | 0   | 0  | 0   | 0  | 0  | 0 | 0   |
| 7   | 0  | 0  | 0  | 164 | 0  | 0  | 0   | 0   | 0   | 0   | 7   | 0  | 0  | 0  | 151 | 0  | 0   | 0   | 0   | 0 | 0   | 7   | 0  | 0  | 0  | 0   | 155 | 0 | 0   | 0  | 0   | 0   | 7   | 0   | 0  | 0  | 0   | 0  | 156 | 0  | 0  | 0 | 0   |
| 8   | 0  | 0  | 0  | 0   | 0  | 0  | 0   | 102 | 0   | 0   | 8   | 0  | 0  | 0  | 0   | 0  | 0   | 121 | 0   | 0 | 0   | 8   | 0  | 0  | 0  | 0   | 0   | 0 | 116 | 0  | 0   | 0   | 8   | 0   | 0  | 0  | 0   | 0  | 0   | 0  | 73 | 0 | 0   |
| 9   | 0  | 0  | 0  | 0   | 0  | 0  | 0   | 0   | 170 | 0   | 9   | 0  | 0  | 0  | 0   | 0  | 0   | 0   | 207 | 0 | 0   | 9   | 0  | 0  | 0  | 0   | 0   | 0 | 0   | 0  | 208 | 0   | 9   | 0   | 0  | 0  | 0   | 0  | 0   | 0  | 0  | 0 | 152 |
| 10  | 0  | 0  | 0  | 0   | 0  | 0  | 0   | 0   | 0   | 242 | 10  | 0  | 0  | 0  | 0   | 0  | 0   | 0   | 0   | 0 | 231 | 10  | 0  | 0  | 0  | 0   | 0   | 0 | 0   | 0  | 0   | 266 | 10  | 0   | 0  | 0  | 0   | 0  | 0   | 0  | 0  | 0 | 212 |

$y_i^{t,k}$

| t=1 |    |    |    |     |    |    |     |     |     |     | t=2 |    |    |    |     |    |     |     |     |   |     | t=3 |    |    |    |     |     |   |     |    |     |     | t=4 |     |    |    |     |    |     |   |    |   |     |
|-----|----|----|----|-----|----|----|-----|-----|-----|-----|-----|----|----|----|-----|----|-----|-----|-----|---|-----|-----|----|----|----|-----|-----|---|-----|----|-----|-----|-----|-----|----|----|-----|----|-----|---|----|---|-----|
| k   |    |    |    |     |    |    |     |     |     |     | k   |    |    |    |     |    |     |     |     |   |     | k   |    |    |    |     |     |   |     |    |     |     | k   |     |    |    |     |    |     |   |    |   |     |
| i   | 1  | 2  | 3  | 4   | 5  | 6  | 7   | 8   | 9   | 10  | i   | 1  | 2  | 3  | 4   | 5  | 6   | 7   | 8   | 9 | 10  | i   | 1  | 2  | 3  | 4   | 5   | 6 | 7   | 8  | 9   | 10  | i   | 1   | 2  | 3  | 4   | 5  | 6   | 7 | 8  | 9 | 10  |
| 1   | 64 | 0  | 0  | 0   | 0  | 0  | 0   | 0   | 0   | 0   | 1   | 80 | 0  | 0  | 0   | 0  | 0   | 0   | 0   | 0 | 0   | 1   | 0  | 0  | 0  | 0   | 0   | 0 | 0   | 73 | 0   | 0   | 1   | 0   | 0  | 0  | 0   | 0  | 75  | 0 | 0  | 0 | 0   |
| 2   | 0  | 28 | 0  | 0   | 0  | 0  | 0   | 0   | 0   | 0   | 2   | 0  | 29 | 0  | 0   | 0  | 0   | 0   | 0   | 0 | 0   | 2   | 31 | 0  | 0  | 0   | 0   | 0 | 0   | 0  | 0   | 0   | 2   | 0   | 32 | 0  | 0   | 0  | 0   | 0 | 0  | 0 | 0   |
| 3   | 0  | 0  | 48 | 0   | 0  | 0  | 0   | 0   | 0   | 0   | 3   | 0  | 0  | 0  | 0   | 0  | 54  | 0   | 0   | 0 | 0   | 3   | 0  | 51 | 0  | 0   | 0   | 0 | 0   | 0  | 0   | 0   | 3   | 0   | 0  | 0  | 0   | 52 | 0   | 0 | 0  | 0 | 0   |
| 4   | 0  | 0  | 0  | 0   | 0  | 0  | 116 | 0   | 0   | 0   | 4   | 0  | 0  | 0  | 0   | 97 | 0   | 0   | 0   | 0 | 0   | 4   | 0  | 0  | 0  | 0   | 104 | 0 | 0   | 0  | 0   | 0   | 4   | 0   | 0  | 0  | 104 | 0  | 0   | 0 | 0  | 0 | 0   |
| 5   | 0  | 0  | 0  | 0   | 0  | 92 | 0   | 0   | 0   | 0   | 5   | 0  | 0  | 0  | 0   | 0  | 127 | 0   | 0   | 0 | 0   | 5   | 0  | 0  | 86 | 0   | 0   | 0 | 0   | 0  | 0   | 0   | 5   | 107 | 0  | 0  | 0   | 0  | 0   | 0 | 0  | 0 | 0   |
| 6   | 0  | 0  | 0  | 0   | 78 | 0  | 0   | 0   | 0   | 0   | 6   | 0  | 0  | 80 | 0   | 0  | 0   | 0   | 0   | 0 | 0   | 6   | 0  | 0  | 0  | 111 | 0   | 0 | 0   | 0  | 0   | 0   | 6   | 0   | 0  | 56 | 0   | 0  | 0   | 0 | 0  | 0 | 0   |
| 7   | 0  | 0  | 0  | 164 | 0  | 0  | 0   | 0   | 0   | 0   | 7   | 0  | 0  | 0  | 151 | 0  | 0   | 0   | 0   | 0 | 0   | 7   | 0  | 0  | 0  | 0   | 155 | 0 | 0   | 0  | 0   | 0   | 7   | 0   | 0  | 0  | 0   | 0  | 156 | 0 | 0  | 0 | 0   |
| 8   | 0  | 0  | 0  | 0   | 0  | 0  | 0   | 102 | 0   | 0   | 8   | 0  | 0  | 0  | 0   | 0  | 0   | 121 | 0   | 0 | 0   | 8   | 0  | 0  | 0  | 0   | 0   | 0 | 116 | 0  | 0   | 0   | 8   | 0   | 0  | 0  | 0   | 0  | 0   | 0 | 73 | 0 | 0   |
| 9   | 0  | 0  | 0  | 0   | 0  | 0  | 0   | 0   | 170 | 0   | 9   | 0  | 0  | 0  | 0   | 0  | 0   | 0   | 207 | 0 | 0   | 9   | 0  | 0  | 0  | 0   | 0   | 0 | 0   | 0  | 208 | 0   | 9   | 0   | 0  | 0  | 0   | 0  | 0   | 0 | 0  | 0 | 152 |
| 10  | 0  | 0  | 0  | 0   | 0  | 0  | 0   | 0   | 0   | 242 | 10  | 0  | 0  | 0  | 0   | 0  | 0   | 0   | 0   | 0 | 231 | 10  | 0  | 0  | 0  | 0   | 0   | 0 | 0   | 0  | 0   | 266 | 10  | 0   | 0  | 0  | 0   | 0  | 0   | 0 | 0  | 0 | 212 |

$s_i'$

| i |   |   |     |    |    |     |   |     |    |    |
|---|---|---|-----|----|----|-----|---|-----|----|----|
| i | 1 | 2 | 3   | 4  | 5  | 6   | 7 | 8   | 9  | 10 |
| 1 | 0 | 0 | 0   | 22 | 0  | 2.1 | 0 | 10  | 0  | 0  |
| 2 | 0 | 0 | 0.5 | 0  | 18 | 0   | 0 | 4.2 | 0  | 0  |
| 3 | 0 | 0 | 0   | 0  | 0  | 28  | 0 | 34  | 11 | 0  |
| 4 | 0 | 0 | 0   | 0  | 0  | 0   | 0 | 0   | 0  | 0  |

### 3.6 Conclusion

This paper develops a mathematical model for an integrated purchasing and production lot-sizing problem with multi-workstation, which is very frequently confronted in real life and which is still more interesting to the operations researchers. The determinations of the multi-period multi-item multi-level lot-sizing under the consideration of the availability of raw materials and capacity of workstations to minimize cost are studied in this paper. The dynamic purchasing cost, production cost and holding cost are examined in the problem.

The benefits of the mathematical model can be used for determining the optimal lot sizing under minimized cost criteria. This leads to a reduction in purchasing cost, production cost and on-hand inventory cost. While all constraints are met, there are not only a trade-off between saving in set-up cost of the lot size of purchasing or production and saving in holding cost, but also a trade-off between the dynamic parameters in each period. Besides, under the circumstance of multi-level product structure, it can be applied for finding material requirement planning extended with dynamic parameters of limited availability, capacity and warehouse. We strongly believe that this model corresponds more closely to actual lot-sizing problems which arise in practice, particularly those occurring in natural supply industries. For example, a food processing industry usually has to face the dynamic pricing and limited availability of natural raw materials. Its production process regularly operates with daily skilled labors. Therefore, the capacity will be changed every period.

In addition, how this model reduces cost can be clearly illustrated in various issues. One of the most obvious issues is a lot size policy. In the given example, if the major target is to satisfy orders, the conventional lot size policy is a lot-for-lot policy. However, because of the capacity limitation constraints, the insufficient resources will be using the exceed resources in the previous period. According to a feasible solution, the unnecessary setup cost or ordering cost and the unnecessary holding cost are occurred in the system, because the policy uses only the information of the external

demands and no consideration of the others, such as dynamic cost parameters and dynamic resource limitations.

Generally the lot-sizing MIP models are often very large in practice even advanced solvers such as CPLEX are unable to identify provably-optimal solutions in acceptable computational time (Berretta and Rodrigues, 2004; Silvio et al., 2008). That the developed model might be classified as a capacitated lot sizing with setup time model, which is NP-hard problem it can be solved to optimality only with a huge computational effort.

Clearly it takes an impracticable amount of computer time and memory, motivating the development of the alternative approaches. Therefore, further researches are encouraged to find a heuristic algorithm that can solve large-scale problems to near-optimality with a reasonable computational time.