ระบบพัสดุคงคลังและการขนส่งสำหรับการเติมเต็มสินค้าหลายชนิดร่วมกันโดยใช้พาหนะที่จำกัดการบรรทุก

นายสมบัติ สินธุเชาวน์

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิศวกรรมศาสตรดุษฎีบัณฑิต
สาขาวิชาวิศวกรรมอุตสาหการ ภาควิชาวิศวกรรมอุตสาหการ
คณะวิศวกรรมศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย
ปีการศึกษา 2546

ISBN: 974-17-4140-5

AN INVENTORY-TRANSPORTATION SYSTEM FOR MULTI-ITEM JOINT REPLENISHMENT WITH LIMITED VEHICLE CAPACITY

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A Dissertation Submitted in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy in Industrial Engineering
Department of Industrial Engineering
Faculty of Engineering
Chulalongkorn University
Academic Year 2003
ISBN: 974-17-4140-5

Thesis Title	An Inventory-Transportation System for Multi-item Joint Replenishment with Limited Vehicle Capacity
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สมบัติ สินธุเชาวน์ :ระบบพัสดุคงคลังและการขนส่งสำหรับการเติมเต็มสินค้าหลายชนิดร่วมกันโดยใช้พาหนะที่จำกัดการบรรทุก (AN INVENTORY-TRANSPORTATION SYSTEM FOR MULTI-ITEM JOINT REPLENISHMENT WITH LIMITED VEHICLE CAPACITY) อาจารย์ที่ปรึกษา : ผ.ศ. ดร. เหรียญ บุญดีสกุลโชค อาจารย์ที่ปรึกษาร่วม : Associate Professor Dr. H. Edwin Romeijn, 123 หน้า. ISBN 974-17-4140-5

ในงานวิจัยนี้ได้ศึกษาระบบการเก็บรวบรวมสินค้าเข้าสู่คลังสินค้า โดยระบบที่ทำการศึกษาประกอบด้วยกลุ่มผู้ผลิตและคลังสินค้าส่วนกลาง กลุ่มผู้ผลิตนี้ตั้งอยู่กระจัดกระจายห่างกันและผลิตสินค้าที่ไม่เหมือนกัน คลังสินสินค้าส่วนกลางเป็นสถานที่เก็บสินค้าสินค้าเหล่านี้ซึ่งถูกรวบรวมมาจากกลุ่มผู้ผลิตโดยรถบรรทุก นอกจากนี้คลังสินสินค้าส่วนกลางต้องตอบสนองความต้องการสินค้าที่เกิดจากผู้ค้าปลีกภายนอกระบบ การศึกษาจะกระทำทั้งในกรณีที่ความต้องการสินค้าของผู้ค้าปลีกเป็นแบบdeterministicและแบบstochastic โดยในการเติมเต็มสินค้าร่วมกันจะใช้นโยบายสินค้าคงคลังแบบปริมาณสั่งซื้อที่ประหยัด (EOQ) คลังสินค้าส่วนกลางจะทำการเติมเต็มสินค้าโดยส่งกลุ่มรถบรรทุกออกไปเก็บรวบรวมสินค้าตามกลุ่มผู้ผลิต รถบรรทุกแต่ละคันจะมีขีดจำกัดในการบรรทุกที่เท่ากันและไม่สามารถเดินทางไปเก็บรวบรวมกลุ่มของสินค้าต่างๆเกินจำนวนครั้งสูงสุดที่กำหนดไว้ ภายใต้นโยบายที่ใช้นี้ รถบรรทุกแต่ละคันจะเก็บรวบรวมกลุ่มสินค้ากลุ่มเดิมในทุกๆครั้งของการเติมเต็มสินค้า ปัญหาที่ศึกษานี้จะถูกแปลงรูปไปเป็นปัญหาการแบ่งกลุ่ม (set partitioning problem) และจะมีการพัฒนาวิธีการแก้ปัญหาทางคณิตศาสตร์เพื่อผสมผสานการตัดสินในด้านพัสดุคงคลังและด้านการขนส่งเข้าด้วยกันโดยมีวัตถุประสงค์ที่จะลดต้นทุนรวมเฉลี่ยให้ต่ำที่สุด มีการพัฒนาวิธี branch-and-price เพื่อหาคำตอบที่ดีที่สุด และใช้วิธี column generation ในการหา lower bound ของต้นทุนรวมเฉลี่ย นอกจากนั้นยังมีการนำเสนอวิธีหาคำตอบที่ดีพอควร วิธีการหาคำตอบที่ดีกว่า และ วิธี very large-scale neighborhood search (VLSN) ซึ่งวิธีเหล่านี้จะให้คำตอบที่ใกล้เคียงกับคำตอบที่ดีที่สุด ส่วนการทดลองจะกระทำโดยการสร้างตัวอย่างแบบสุ่ม ซึ่งผลการทดลองสรุปได้ว่า วิธีการหาคำตอบต่างๆที่พัฒนาขึ้นนั้นสามารถหาคำตอบที่น่าพอใจได้ทั้งในกรณีที่ความต้องการสินค้าของผู้ค้าปลีกเป็นแบบdeterministicและแบบstochastic

ภาควิชา วิศวกรรมอุตสาหการ
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ลายมือชื่อนิสิต.....................
ลายมือชื่ออาจารย์ที่ปรึกษา.....................
ลายมือชื่ออาจารย์ที่ปรึกษาร่วม................

4271835721: MAJOR INDUSTRIAL ENGINEERING
KEYWORD: INVENTORY-ROUTING/TRASPORTATION/MULTI-ITEM INVENTORY REPLENISHMENT/ECONOMIC ORDER QUANTITY/COLUMN GENERATION/VERY LARGE-SCALE NEIGHBORHOOD SEARCH

SOMBAT SINDHUCHAO : AN INVENTORY-TRANSPORTATION SYSTEM FOR MULTI-ITEM JOINT REPLENISHMENT WITH LIMITED VEHICLE CAPACITY. THESIS ADVISOR : ASST. PROF.RIEN BOONDISKULCHOK. THESIS COADVISOR : ASSC. PROF.H. EDWIN ROMEIJN, 123 pp. ISBN 974-17-4140-5

In this research, an inbound commodity collection system is studied. The system consists of a set of geographically dispersed suppliers that manufacture one or more non-identical items, and a central warehouse that stocks these items. The warehouse faces demands for the items from outside retailers. Both deterministic and stochastic demands are considered in a separate case. An economic order quantity (EOQ) inventory policy is applied to jointly replenish the items. The items are collected by a fleet of vehicles that are dispatched from the central warehouse. Each vehicle has an identical limited capacity and must also satisfy a frequency constraint. A policy in which each vehicle always collects the same set of items is adopted. The integrated inventory-transportation problem is formulated as a set partitioning problem and a mathematical programming approach is developed for coordinating inventory and transportation decisions with the objective of minimizing the long-run average inventory and transportation costs which are composed of an inventory holding cost, a fixed ordering cost, a minor ordering cost, a fixed dispatching cost, a stopover cost and a vehicle routing cost. A branch-and-price algorithm is developed to find the optimal assignment of items to vehicles and a lower bound on the total costs is determined by employing a column generation approach. In addition, several greedy heuristics and local search methods are proposed along with a very large-scale neighborhood (VLSN) search algorithm in order to obtain near-optimal solutions for the problem.

Computational tests are also conducted on a set of randomly generated problem instances. The results indicate that the proposed heuristics perform satisfactorily in both deterministic and stochastic cases.

Department Industrial Engineering	Student's signature…………….
Field of study Industrial Engineering	Advisor's signature…………….
Academic year 2003	Co-advisor's signature…………..

ACKNOWLEDGEMENTS

I wish to thank Asst. Prof. Rein Boondiskulchok and Assc. Prof. H. Edwin Romeijn, my advisors, for their advice, guidance and encouragement during my Ph.D. study. I appreciate their commitment to assisting me achieve my research goals. Special thanks are also extended to Asst. Prof. Elif Akçalı for helpful suggestions while I was working on my research at the University of Florida.

I am also grateful for the suggestions, comments and criticisms that Assc. Prof. Chuvej Chansa-ngavej, Asst. Prof. Manop Reodecha, Prof. Fujiwara Okitsugu and Dr. Paveena Chaovalitwongse as members of the examination committee provided me during my study.

I wish to acknowledge the assistance of Huang Wei, Dr. Pricha Pantumsinchai and Asst. Prof. Krung Sinapiromsaran.

In addition, I would like to thank Ubonratchathani University and Chulalongkorn University for providing me scholarships, and the Department of Industrial and Systems Engineering of the University of Florida for giving me an opportunity to perform my research there.

Finally, I wish to express my deepest gratitude to my parents, sisters, brothers and friends for their encouragement, warmness and love that have driven me to become successful.

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CHAPTER 1

INTRODUCTION

1.1 General Background

A supply chain is a network of facilities that performs a series of activities: procuring raw material, transforming raw material into products or services, stocking finished goods and distributing products to customers [Lee and Billington, 1993]. From these mentioned activities, it is clearly seen that inventory control and transportation planning are important aspects of the supply chain management. In some industries such as the food industry, the logistics cost is a huge portion of the cost of products. Therefore, there are more opportunities to reduce the logistics cost than to decrease the production cost [Henkoff , 1994 and Nahmias, 1997].

Due to different sources of uncertainties existing along the supply chain, inventories are always kept to satisfy unexpected demands. When the inventory level drops, a replenishment may be needed in order to raise the inventory back to the desired level. Overstock inventories sometimes would be unprofitable rather than advantageous to an organization. Therefore, efficiency in inventory control is essential. There are many inventory models that can be adopted for efficient inventory management depending on the characteristics of the system. For instance, if the demand rate is constant, the economic order quantity (EOQ) model is appropriate in some degree. However, the fundamental purpose of all replenishment control systems is to resolve the following three issues [Silver, Pyke and Reterson, 1998]:
How often should the inventory status be determined?
When should a replenishment order be placed?
How large should the replenishment order be?

In a distribution process, commodity movement from one place to another involves transportation. For example, a warehouse sends a truck to collect raw materials at a supplier and a plant dispatches a fleet of vehicles to distribute goods to geographically dispersed retailers, etc. Costs of distributing products or collecting raw materials are also considered as a major component of the logistics costs. For that reason, one of the main issues in transportation planning is to design optimal delivery or collection routes for the vehicle subject to restrictions. This problem is known as a vehicle routing problem (VRP) which is an extension of the traveling salesman problem (TSP). For a review of TSP and VRP exact and heuristic algorithms, see Laporte (1992a and 1992b).

Inventory control and transportation planning of an organization are traditionally managed by different departments each of which has its own goal. Consequently, inventory and transportation costs are minimized separately by each department. In general, there is a trade off between the inventory cost and the transportation cost in the logistics system. When attempting to decrease one cost, the other will normally increase. For instance, in an inbound material collection scenario using a simple EOQ inventory policy where a fixed ordering cost could be viewed as a fixed transportation cost, smaller order quantity leads to a lower inventory holding cost but a vehicle needs to be dispatched more frequently to collect materials which generates a higher transportation cost. However, the total inventory and transportation cost in the system can be greatly reduced if inventory control and transportation planning are closely coordinated and decision making in both aspects is cooperated in order to determine the best trade-off between both costs.

In a multi-item inventory system, it is practical to combine groups of items in a single replenishment order to accomplish substantial cost savings [Peterson and Silver ,1979] due to the sharing of fixed replenishment costs. When these replenishment costs contain a transportation cost component, this cost sharing is often a consequence of the ability to share truck and loading equipment between the items. In addition, the design of a vehicle route for visiting a group of retailers (in the case of a distribution system) or a group of suppliers (in the case of a collection system) may have a significant effect on the magnitude of the replenishment costs. Hence, it is desirable to design an efficient joint replenishment strategy that coordinates both inventory control and transportation planning.

1.2 Statement of the Problem

This research involves an inbound commodity collection system that comprises a central warehouse with unlimited space for stocking inventories and geographically dispersed suppliers. Each supplier produces one or more non-identical items. The items face demands from outside retailers. When a manager of the central warehouse decides to replenish the inventories, a fleet of identical vehicles with limited capacity are sent to visit a set of dispersed suppliers for item collection. The frequency of dispatching each vehicle is limited due to the time required for maintaining vehicles and for other responsibilities, and the limited material handling capacity. When the item collection is completed, the vehicle returns to the central warehouse where the items are unloaded and stored.

The problem is to partition items into a number of subsets each of which consists of all different items so as to minimize the total cost per unit time of the integrated inventory-transportation system. For each item, the replenishment quantity and the replenishment interval, must be determined along with the efficient route for the vehicle. See Figure 1.1 for an illustration of the problem. Suppose that there are two groups of products according to a grouping strategy. Group one consists of item I1, I2, I3 and I5 while Group two composes of item I4, I6 and I7. When the replenishment of items in Group one is needed, a vehicle is dispatched to visit in order, according to the shortest distance, supplier S3 for collecting item I5, supplier S2 for item I2 and I3 and supplier S1 for item I1. After the vehicle completes its collection duty, it returns to the central warehouse. The replenishment of items in Group two may not occur at the same time as Group one’s.

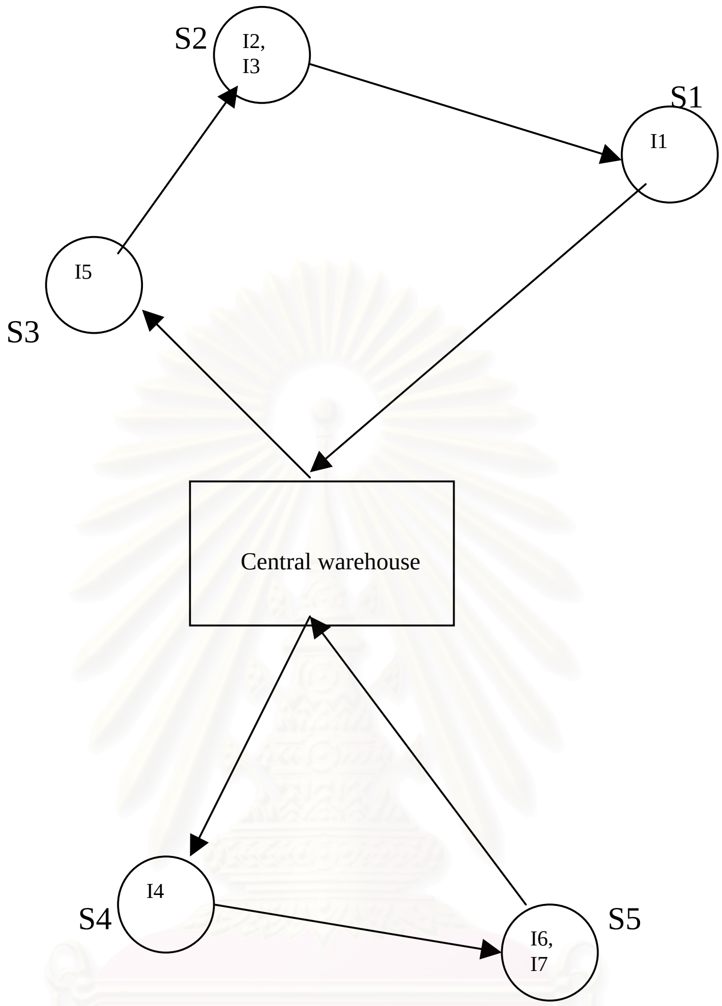

Figure 1.1 Inventory-routing system with multiple items, multiple suppliers and a central warehouse

1.3 Research Objective

The objective of this research is to develop mathematical programming models of the integrated inventory-vehicle routing problem, and propose an exact solution approach several grouping heuristics and improvement algorithms to determine an integrated multi-item replenishment strategy that coordinates inventory control and transportation planning in order to satisfy demands from outside retailers at the minimum average total inventory and transportation costs.

1.4 Research Scope

An integrated inventory-transportation system is studied in this research. This is an inbound commodity collection system that consists of a central warehouse and geographically dispersed multiple suppliers. Each supplier manufactures one or more non-identical items. Transshipment of items between suppliers is not allowed. That is each supplier cannot buy or stock any items produced by other suppliers. The central warehouse has its own fleet of identical capacitated vehicles and an unlimited area for stocking items that face demands from outside retailers. There is a frequency constraint on the number of vehicle dispatching in a given period because of time required for vehicle maintenance and for its other responsibilities and limited material handling capacity at the central warehouse. It is assumed further that each vehicle can be dispatched for item collection with an equal limited frequency. Moreover, the vehicle capacity is assumed to be comparatively larger than accumulated demands of any item in the replenishment interval. This assumption is made to avoid multiple visits for collecting the same product at a particular supplier in one period.

When the inventory replenishment of a particular group of items is needed, the warehouse dispatches a vehicle to collect that group of items from the suppliers. No time window for item collection is considered in the system because suppliers are assumed to operate 24 hours a day and there is no traffic problem. That is the vehicle can pick up the items at any time when it arrives at the location of each supplier. After visiting the suppliers for item collection as planned, the vehicle is driven back to the central warehouse. For each item, lead time is assumed to be fixed for any replenishment and the replenishment interval of any item is assumed to be longer than its lead time. When stock-out situation occurs, the manager of the central warehouse will not expedite any ordering or transporting processes. The model is treated from the viewpoint of the central warehouse and it is assumed that the central warehouse and the suppliers belong to different organizations so any charges to the suppliers from holding stock will not be considered.

The costs in the system are composed of two major costs: inventory costs and transportation costs. On the aspect of inventory costs, there are a joint fixed ordering cost, an item dependent minor ordering cost and an inventory holding cost at the central warehouse. The transportation costs include a fixed dispatching cost, a vehicle routing cost and a fixed stopover cost which is specific for each supplier. It is assumed that every supplier has responsibility of getting the items ready to be picked up at any time so that no shortage or delay occurs for each item collection at any supplier. In addition, the holding cost incurred when the products are in the vehicle is assumed to be very small and can be neglected.

Both deterministic and stochastic demands are studied in separate cases. At first, algorithms are developed for the deterministic problem and then they are applied to solve the stochastic problem. In the deterministic case, it is assumed that each item faces a constant and deterministic demand from outside retailers. As a result, an economic order quantity (EOQ) inventory policy can be adopted at the central warehouse which leads to a joint replenishment of items. In the stochastic case, demands from outside retailers are assumed to be independent, and identically and normally distributed due to a standard periodic review order-up-to level inventory policy adopted by these retailers. However, the stochastic problem is restricted to the case where the average demand of each item is approximately constant with time. The manager of the central warehouse chooses a periodic review fixed order quantity policy for inventory control and the safety stock is considered to prevent the stock-out situation under a given service level determined by management.

All data used in this research (locations of suppliers, demand rates, holding cost rates, minor ordering costs and stopover costs) are randomly generated from uniform distribution.

1.5 Research Contribution

There are various models of integrated inventory-transportation systems which have been studied for the past 20 years. To develop a model, characteristics of the system must be considered. The system may involve a single item or multiple items. A distribution of demand for an item may be deterministic or stochastic. The number of destinations could be one or more. Unsatisfied demand may be backlogged or completely lost. Lead time of each replenishment may be zero, constant or variable. Vehicle capacity may be limited or unlimited. Transportation cost can be fixed or variable or both. A planning horizon may be single, multiple or infinite.

Most researches on integrated inventory-transportation systems deal with a single item multi-retailer deterministic model or simplified ordering cost and transportation cost structures. For example, work of Daganzo, Barns, Hall and Blumenfeld (1985), Dror, Ball and Golden (1985), Chien, Balakrishan and Wong (1989), Gallago and Simchi-Levi (1990), Anily and Federgruen (1990&1993), Anily (1994) and Chan and Simchi-Levi (1998).

For the case of stochastic demands for a single item, Federgruen and Zipkin (1984) study a single period problem but no ordering cost is included in the cost structure. Chaovalitwongse (2000) considers a single period distribution system with multiple capacitated suppliers. For the transportation problem, the author has linear and fixed charge costs but does not capture a vehicle routing cost. Cetinkanu and Lee (2000) deal with Vender-Managed Inventory (VMI) systems. They assume Poisson demands and zero lead time. Again no vehicle routing problem is considered.

Viswanathan and Mathur (1997) study a multi-item multi-retailer distribution system with the case of deterministic demands for several products at multiple retailers. However, they assume that demands for items are constant and deterministic. Buffa and Munn (1990) analyze a multi-item single retailer stochastic model. They propose a grouping heuristic which is partly based on holding and shipping costs and no vehicle routing cost is considered due to a single destination.

This research is the most similar to that of W.Qu, Bookbinder and Iyogun (1999). They study an integrated inventory-transportation system for multiple items with stochastic demands. They integrate a modified periodic review inventory policy and transportation vehicle routing into one mathematical model. However, they simplify the integrated problem by assuming that a vehicle has unlimited capacity.

An integrated inventory-transportation system considered involves multiple items, a single warehouse and multiple suppliers. Both deterministic and stochastic demands are studied. Furthermore, a vehicle routing problem is also presented in the model which is made more realistic by adding vehicle capacity and frequency constraints. A periodic review inventory policy with a fixed order quantity, which is modified from a simple EOQ model, is adopted for both deterministic and stochastic cases. The EOQ model is studied by Daganzo et al. (1985), Gallago and Simchi-Levi (1990), Anily and Federgruen (1990), Anily and Federgruen (1993) and Anily (1994) in the integrated inventory-transportation system in which there is only a single item. Cost structures which include a holding cost, a major ordering cost, a minor ordering cost, a fixed dispatching cost, a stopover cost and a variable routing cost are realistically captured.

In this research, a branch-and-price algorithm is developed to determine the exact solution to the problem. Several greedy constructive heuristics and local search methods are proposed, and a very large-scale neighborhood (VLSN) search algorithm [see Ahuja et al. (2000) and Ahuja, Ergun, Orlin and Punnen (2002)] is developed to obtain near-optimal solutions for the problem. A column generation approach is applied to construct on the total costs a lower bound which is used to measure the effectiveness of proposed algorithms which can be utilized to solve both deterministic and stochastic problems. The computational results indicate that the proposed algorithms perform satisfactorily for both deterministic and stochastic cases. A developed model of the integrated inventory-transportation system can be applied to any organization that has similar environment to this problem.

1.6 Research Methodology

At first, the integrated inventory-transportation system in the deterministic settings is studied. An exact solution approach and heuristics to find a near-optimal solution are developed. These methods are further applied to solve the stochastic problem. Figure 1.2 shows the research methodology. This research is conducted as the following steps.

1.6.1. Model formulation

Once getting a new idea from literature survey, an integrated inventory-transportation problem is set up and then a single mathematical model in deterministic settings is developed. The stochastic model is developed after the solution approaches to the deterministic problem are obtained. Prior to formulating the model, decision variables which are the order quantity Q and the replenishment interval T as well as the route traveled, parameters, constraints and assumptions of the model are defined.

1.6.2. Solution approach

Firstly, a branch-and-price algorithm is developed and used to solve the inventory-routing problem to optimality. To determine near-optimal solutions, several greedy constructive heuristics are proposed to separate all the items into groups. Each group is replenished independently. Then one or two of neighborhood search methods (One Supplier Move, Supplier Exchange and VLSN) are employed to solve the integrated inventory-transportation problem for the order quantity and the replenishment interval that minimize the total cost of the system. The TSP tour is solved heuristically by using the Arbitrary Insertion heuristic [Rosenkrantz et al. (1977)] and then improved by applying the 2-opt exchange heuristic[Croes (1958), Lin (1965) and Lin and Kernighan (1973)].

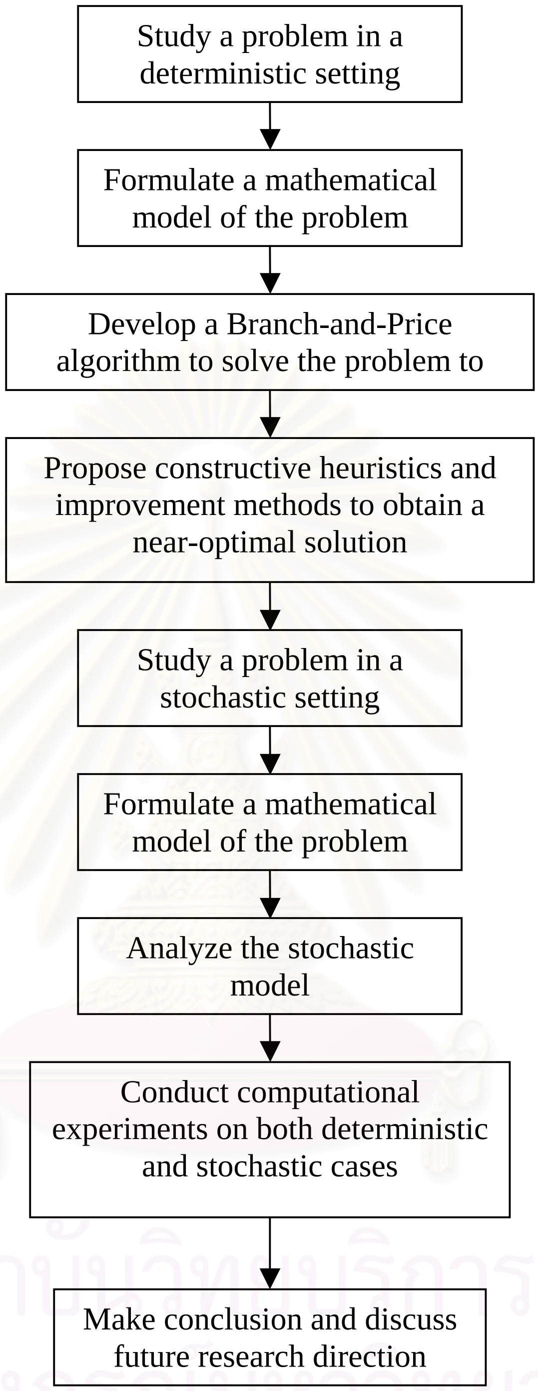

Figure 1.2 Research Methodology

1.6.3. Performance measurement of heuristics

The solutions obtained from the proposed heuristics are merely near optimal. Therefore, how well the proposed heuristics perform is not known. To measure the performance of the heuristics, a lower bound is constructed by employing a column generation approach and it is compared with the solutions obtained from the heuristics.

1.6.4. Sensitivity analysis

To analyze how model parameters effect the solution, the vehicle capacity, the maximum number of trips allowed and the fixed dispatching cost are varied. All the problem instances are randomly generated from a uniform distribution. In the stochastic case, it is assumed demands are normally distributed. Computer codes are written in C++ along with utilization of the CPLEX 8.1 solver.

1.7 Thesis Structure

The outline of this thesis is as follows. The relevant literature is reviewed in Chapter 2. In Chapter 3, the integrated inventory and transportation model in the deterministic settings is formulated under the policy studied as a set partitioning problem. Based on the model formulated, a column generation and a branch-and-price algorithm are developed in Chapter 4, paying particular attention to the pricing subproblem, which is a very challenging optimization problem in its own right. Greedy construction heuristics, local search methods and the VLSN algorithms are proposed and developed in Chapter 5.

In Chapter 6, the integrated inventory and transportation model in the stochastic settings is studied and analyzed. Computational experiments on both deterministic and stochastic problems are conducted in Chapter 7. The sensitivity analysis on some parameters is also carried out in this chapter. Finally, this research is summarized and future research directions are provided in Chapter 8.

CHAPTER 2

LITERATURE REVIEW

Firstly the integrated inventory-transportation systems are discussed focusing on characteristics of the systems, cost structures and model formation. The literature review of the integrated inventory-transportation systems is classified by the number of items: single and multiple. Secondly, the joint replenishment strategies are reviewed with classification of a characteristic of demand: deterministic and stochastic. In addition, grouping strategies for the multi-item joint replenishment and the integrated inventory-transportation system are also examined. For the review of integrated inventory-transportation deterministic models with both single item and multi-item cases, see Bertazzi and Speranza (1999).

2.1 Integrated inventory-transportation systems

2.1.1 Single item cases

Inventory and transportation are two of the important elements in supply chain management which has recently emerged as a major and interesting topic in operations research and operations management. See Thomas and Griffin (1996) for review of coordinated supply chain management. In distribution systems, if inventory control and transportation planning are closely coordinated, the total system cost can be greatly reduced. The main interest in the past was to study inventory problems and transportation problems separately, without paying attention to the entire system. Until early 1980's, Federgruen and Zipkin (1984), to the best of our knowledge, are the first to integrate the allocation and routing problems in a single model. They study the allocation of a scarce resource from a central depot to many retailers using a fleet of capacitated vehicles and consider random demands in a single period model. The problem is formulated as a nonlinear integer program. Interchange heuristics for the deterministic vehicle routing problem are modified to solve the problem. They also derive an exact algorithm for the problem using Benders' decomposition method which decomposes the main problem into one nonlinear inventory allocation subproblem and a number of traveling salesman subproblems. With this approach, substantial cost saving can be achieved. The decomposition method presented by Federgruen and Zipkin provides the basis for the solution algorithm to the model developed by Qu et al (1999).

An analytic approach for minimizing the distribution freight by capacitated trucks from a supplier to many customers is developed by Burns, Hall, Blumenfeld and Daganzo (1985) whose work is the first one to integrate transportation and inventory costs explicitly in decision making over an infinite time horizon. Nevertheless, their cost structure, opposed to our model, does not include ordering. In their paper, two distribution strategies, direct shipping and peddling, are analyzed and compared. The analysis for direct shipping is consistent with EOQ model. For peddling, formulas derived require the spatial density for customers, rather than the precise customer locations. The results show that the optimal shipment size for direct shipping is the economic order quantity while the one for peddling is a full truck.

A direct shipping strategy is studied by Gallego and Simchi-Levi (1990) as well. Their system includes a single warehouse, a single item and multiple retailers. In contrast to the proposed model here, demand is assumed to be constant with retailer specific rate. They show the benefits of direct shipping.

With the assumption of a single commodity with deterministic demands in a multi-period setting, Dror, Ball and Golden (1985) describe and computationally compare two algorithms, the assignment routing approach and the modified routing approach for a distribution system that consists of a central depot and multiple customers who process a known capacity and have a constant consumption rate. The authors present two formulations of the problem, the vehicle assignment formulation and the day assignment formulation. Cost structures include the routing cost and the future costs associated with the inventory. The future costs are used for consideration of assigning customers to a vehicle on a particular day in the vehicle assignment problem and assigning customers to days in the day assignment problem. No inventory costs are expressed in the models.

Dror and Ball (1987) study the inventory routing problem similar to the one of Dror, Ball and Golden (1985). Unlike the previous work, they formulate the problem in short duration, not in an annual period. Single customer deterministic and stochastic models are derived and then extended to the multi-customer problem. They present a procedure for reducing the annual distribution problem to a single period problem by including penalty costs within the single-period model that reflects the long term effect of decisions made during that period.

One of interesting work under a single item deterministic demand model is presented by Chien, Balakrishnan and Wong (1989). They develop an integrated inventory allocation and vehicle routing model with the objective of maximizing profit, unlike other models that have the objective of minimizing total costs. However, they do not capture inventory ordering costs. The integrated problem is formulated as a mixed integer program and then decomposed into two subproblems: the inventory allocation subproblem and the customer assignment/vehicle utilization subproblem. The authors use a Lagrangian relaxation approach and a heuristic method to generate upper bounds and lower bounds respectively.

Anily and Federgruen (1990) consider single item distribution systems with one depot and many geographically dispersed retailers who keep inventories. The planning horizon is infinite. Unlike our model, no ordering cost is included in the cost structure. In addition, the demand rate is constant and assumed to be integer multiples of some base rate. The problem is studied within a specific class of replenishment strategies in which there is a collection of regions covering all retailers, each of which may belong to several regions and each region satisfies a faction of total demand. When a vehicle visits a retailer in a particular region, it must visit every retailer in that region as well. This scheme is also adopted in our work. The authors apply the Modified Circular Regional Partitioning Scheme to partition the set of demand points and propose the Combined Routing and Replenishment Strategies Algorithm to compute lower and upper bounds for total costs.

Anily and Federgruen (1993) develop a two-echelon distribution system from their previous work. The extension is that inventory can be kept at retailers as well as at a central warehouse while other main assumptions are not changed. Earlier methods are extended to manage this more complicated problem.

Anily (1994) also extends the work of Anily and Federgruen (1990) by employing general holding cost rates. One different aspect from the previous work is that the assignment of retailers to routes in the previous work is based merely on their geographical location while this one is based on both the geographical location of retailers and holding cost rates. An experiment study for both capacitated and uncapacitated systems is presented to demonstrate the algorithm's efficiency.

Chan, Federgruen and Simchi-Levi (1998) investigate the asymptotic effectiveness of the Zero Inventory Ordering and Fixed Partition Policies for a one origin multi-destination single item network in an infinite planning horizon and a deterministic setting. Vehicle capacity and frequency constraints are imposed. Only holding cost with an identical rate for all retailers is captured in the inventory problem. Computational results are given to show the effectiveness of the proposed strategy. The proposed policy in this research bears some resemblance to the class of Fixed Partition policies introduced by Bramel and Simchi-Levi (1995&1997) for an inventory-routing problem in which a single item is distributed among retailers. Under this policy, a group of retailers are partitioned into a number of regions each of which is served separately and independently and when a retailer in a region is visited by a vehicle, every retailer in the same region is visited as well. Although such policies are generally not optimal, they are important from a practical standpoint, as they are easy to implement. In particular, they allow for efficient integration of several business functions.

Chan and Simchi-Levi (1998) study a three-level distribution system which consists of a single outside vendor, a number of warehouses and multiple retailers. They simplify the problem by considering a single item with a constant, retailer specific demand rate. Plus, only a holding cost is involved in the inventory cost structure. An efficient algorithm for the integrated inventory control and vehicle routing problem is proposed. They show that, in an effective strategy which minimizes the asymptotic long run average cost, each warehouse receives fully loaded trucks from the outside vendor but never holds of deliveries to the retailers and that each retailer is served by exactly on warehouse.

One of the most recent works in the integrated inventory-transportation system is presented by Chaovalitwongse (2000). The author analyzes a single-period distribution system where multiple capacitated warehouses supply multiple retailers with a single commodity. Like this research to some degree, demands are assumed to be stochastic. Nevertheless, a vehicle routing is not incorporated in the model. The transportation costs include linear and fixed charge costs. The linear transportation cost model is solved by the Lagrange multiplier approach while the fixed charge cost model is solved by the developed dynamic slope scaling procedure (DSSP) scenario-based heuristic. The Lagrangian relaxation based DSSP heuristic that generates better solutions is also proposed for the fixed charge cost model.

An interesting concept in supply chain management is that a supplier has responsibility for managing inventories at retailers by reviewing the retailer's inventory levels and making decisions regarding the quantity and timing of resupply. This is called a Vendor-Managed Inventory (VMI) system which is studied by Cetinkaya and Lee (2000). They develop for the case of a single item with Poisson demands an analytical model coordinating inventory and transportation decisions for a VMI supplier who employs a special kind of (s, S) policy with s= 0 for inventory replenishment. Zero lead time is assumed and no vehicle routing cost is considered in the model.

2.1.2 Multi-item cases

Now the case of multiple items is discussed. The number of researches on multi-item inventory-transportation systems is much smaller than the one in a single item case. Among those are Buffa and Munn (1990), Ben-Khedher and Yano (1994), Chanda and Fishr (1994), Viswanathan and Mathur (1997), Beertazzi and Speranza (1999), Fumero and Vercellis (1999) and Qu, Bookbinder and Iyogun (1999).

One of the work which is most similar to this research belongs to Qu, Bookbinder and Iyogun (1999). In fact, the idea of this research is motivated by their paper. Qu, Bookbinder and Iyogun (1999) develop an integrated inventory and transportation system for joint replenishment with a modified periodic policy in which each replenishment period is an integer multiple of a base period. Like our problem in some aspects, this is an inbound material-collection problem with a central warehouse sending an uncapacitated vehicle to collect multiple items at geographically dispersed suppliers in multiple periods and a stochastic setting. A heuristic decomposition method is proposed to solve the problem by separating the model into two subproblems namely conventional inventory and vehicle routing models. The inventory subproblem is solved item by item while the transportation one is solved period by period. A lower bound is also constructed to test the effectiveness of the heuristic which performs satisfactorily.

The problem studied here differs from the work of Qu, et al. (1999) in that a periodic review inventory policy with a fixed order quantity is exploited instead of a modified periodic policy with an order-up-to level. This allows a vehicle capacity constraint to be included in the mathematical model. Another major difference is that it is assumed that each vehicle has identical limited capacity. Moreover, a frequency constraint is also encompassed in the model.

Under a modified periodic policy with an order-up-to level and unlimited vehicle capacity, it is known that at the time of review, order must be placed to raise the inventory up to the maximum level and a vehicle is dispatched to collect items no matter what total order quantity is. In the case when the combined order quantity is very small compared to the actual vehicle capacity, this policy does not seem to work efficiently. In other words, when the inventory level of every item at the time of review is not much below the order-up-to level, it is likely that this replenishment strategy will not lead to cost saving due to high transportation costs per unit replenished. This policy can’t be used in our problem because of the capacity constraint incorporated in the model. That is it is not guaranteed that the total order quantity of items in the group will not exceed the vehicle capacity.

The first multi-stage grouping algorithm for a stochastic model with both inventory and transportation costs realistically modeled is proposed by Buffa and Munn (1990). Firstly, they rank items based on holding and shipping costs, then test if additional grouping is economic and finally apply a grouping heuristic to form the groups. They model transportation cost sensibly as a function of cycle time, shipping distance and weight. In addition, lead time is assumed to be dependent on transit time. However, they simplify the problem by considering a single destination. Moreover, no vehicle routing problem is included in the model. The total logistic cost of the groups obtained by the proposed algorithm is compared with the minimum one determined by a complete enumeration.

Ben-Khedher and Yano (1994) combine a bin-packing problem with a multi-item joint replenishment problem. The system considered is composed of a single supplier, an assembly facility and multiple items which face deterministic demands and are packed into containers shipped by identical capacitated trucks. Opposed to this research, they assume zero lead time and only a fixed cost proportional to the number of trucks shipped is included in the transportation cost structure. They develop a heuristic solution procedure starting by relaxing container integrality constraints. The solution to the relaxed problem is then modified by sequentially considering each item and optimally scheduling the fractional containers.

Viswanathan and Mathur (1997) integrate a vehicle routing problem and inventory decisions in a single warehouse multi-retailer multi-item distribution system with deterministic demands. The cost structure of their model is similar to the one of our proposed model. In addition to the holding cost, major and minor ordering costs are associated with three components of the transportation cost: fixed dispatching, stopover and variable routing costs. They propose a stationary nested joint replenishment policy (SNJRP) heuristic to solve the problem where replenishment intervals are power of two multiples of a base planning period.

Bertazzi and Speranza (1999) extend the single supplier single customer multi-item environment to the multistage supply chain networks where multiple items are shipped from a common origin to a common destination through one or several intermediate nodes. A periodic shipping strategy is determined to minimize the total inventory and transportation costs based on shipping frequencies on each link that may be the same or different. In this more global setting, the supply and demand rates are assumed to be constant and equal for each item. Plus, no stock-out is allowed. They concentrate on the formulation and evaluation of the total inventory cost. On the other hand, unlike our proposed model, they don't capture a vehicle routing cost. Six heuristic algorithms are presented and evaluated.

Interesting work on integrated production and distribution systems belongs to Chanda and Fisher (1994) and Fumero and Vercellis (1999). Chanda and Fisher (1994) investigate the value of coordinating production and distribution planning in a multi-period setting whereas Fumero and Vercellis (1999) consider a multi-period system that consists of a single plant producing multiple items with limited resource. Items face constant demand rates and are distributed to several customers by a fleet of capacitated vehicles. The problem is formulated as a mixed integer program and solved by Lagrangean relaxation to obtain both lower bounds and heuristic feasible solutions. To demonstrate the effectiveness of the proposed solution scheme that separates the production and distribution decisions, computational results on randomly generated problems are provided.

2.2 Multi-item joint replenishment inventory systems

In addition to the integrated inventory-transportation literature, inventory literature that is related to this work is also surveyed. Multi-item joint replenishment inventory systems is concentrated. Most of the work in this area involves deterministic models. For example, Shu (1971), Nocturne (1973), Silver (1976), Goyal and Belton (1979), etc. In a stochastic setting, more work on a continuous review system than on a periodic review system is explored, especially on a continuous can-order policy considered closely by Goyal and Satir (1989) who review joint replenishment inventory models for both deterministic and stochastic cases. We start this section with review in deterministic models.

2.2.1 Deterministic models

Shu (1971) and Nocturne (1973) analyze a deterministic model which is applicable to the batch processing industry where a batch of item is blended and subsequently packaged into various types of containers. This can be viewed as a joint replenishment. Under an infinite planning horizon and a continuous time model, Shu (1971) finds the conditions under which the total set-up and holding cost is minimized by packaging the smallest demand item with less frequency of packaging than the rest of the group. However, Nocturne (1973) demonstrates that Shu's solution does not always lead to an optimal solution. In addition, Nocturne formulates a multi-item joint replenishment problem and also provides a graphical solution of optimal ordering frequencies for the two-item case.

Silver (1976) studies a multi-item lot sizing problem when item demands are constant over a finite horizon. The cost structure includes a major setup cost for each replenishment, a minor setup cost for each item included in the replenishment and a carrying cost. The author uses the EOQ concept to propose a simple procedure of determining order quantities, a group replenishment interval and replenishment periods of each item. Goyal and Belton (1979) improve performance of Silver's method by modifying the item selection rule.

For a multi- item deterministic demand single supplier system, Goyal (1974) develops an algorithm for determining the optimal ordering quantity and the relative ordering frequency for each item. Like our proposed policy, this optimal ordering policy may be viewed as a periodic review policy with a fixed order quantity.

Another interesting inventory policy is a power-of-two policy. Work related to the power-of-two policy is studied by Jackson, Maxwell and Muckstadt (1985), Mucktadt and Roundy (1987) and Iyogun and Atkins (1993).

Jackson, Maxwell and Muckstadt (1985) develop a model for the joint replenishment problem over a finite planning horizon in a manufacturing system under the restriction that constant reorder intervals must be power-of-two multiples of a base planning period.

Mucktadt and Roundy (1987) apply a power-of-two policy and a stationary nested policy for a multi-echelon distribution system while Iyogun and Atkins (1993) propose a power-of-two heuristic for a multi-stage multi-item distribution network which is decomposed into facilities-in-series problems to obtain a lower bound.

2.2.2 Stochastic models

For a joint replenishment inventory system in a stochastic setting, several inventory policies, both in continuous and periodic reviews, have been investigated.

For a continuous review model, Pantumsinchai (1992) and Cheung (1998) study QS policies where all items are replenished to their base stock levels Si whenever the combined usage of all items reaches Q while Balintfy (1964), Silver (1974) and Federgruen, Groenevelt and Tijms (1984) study the can-order (S, c, s) policy where inventory levels are continuously reviewed. Whenever the inventory position of item i drops to its must order point si or lower, a replenishment is triggered to raise its inventory position to an order-up-to-level Si. At the same time, any other items with inventory positions at or below their individual can-order point c are included in this replenishment in order to raise their inventory levels to the order-up-to-levels.

Federgruen, Groenevelt and Tijms (1984) present an algorithm to search for an optimal can-order (S, c, s) coordinated control rule. The proposed algorithm differs from Silver (1974) and Thompstone and Silver (1975) in that it can handle nonzero lead times for the case of compound Poisson demands.

The can-order (S, c, s) policy is compared with a simple periodic (Ri, Ti) policy proposed by Atkins and Iyogun (1988). For this simple periodic (Ri, Ti) policy, item i is raised to Ri every Ti period. The computational study shows that the simple periodic policy seems to show considerable promise over the can-order one.

Pantumsinchai (1992) compares the QS policy with the can-order policy and with the simple periodic policy. The computational results indicate that no one policy is superior to the others and in the situation where the stock-out cost is low and the major setup cost is high relative to the minor setup cost, the QS policy performs significantly well.

Like Atkins and Iyogun (1988), Chakravarty and Martin (1988) and Eynan and Kropp (1998) also work on periodic review inventory policies. Chakravarty and Martin (1988) develop the item grouping strategy for a coordinate inventory replenishment under a normally distributed demands environment whereas Eynan and Kropp (1998) propose simple heuristics for solving the multi-item joint replenishment problem under stochastic demands with normal distribution.

A new class of policies called Periodic Review (s, S) Policies for joint replenishment inventory systems in a stochastic setting is proposed by Viswanathan (1997). Under this policy, Inventories of all items are reviewed once every t units of time. Item i is ordered up to the level Si, if its inventory position is less than or equal to si at time of review.

The author compares the (s, S) policy with other four policies suggested in the literature: MP, QS, Can-order and Independent Control policies. From computational results, although the P(s, S) policy is only marginally better than the MP policy, unlike the MP policy, the P(s, S) policy generally dominates all the other policies over wide range of problem parameters.

2.3 Grouping Strategy

In this section, grouping strategies of previous work in the multi-item joint replenishment and the integrated inventory-transportation system are discussed.

2.3.1 Multi-item joint replenishment

When a distribution system consists of multiple items that can be jointly replenished, a question of which items should be grouped together in order to minimize total related costs is arisen. In fact, the optimal grouping set of items can be determined by completely enumerating every possible group but when the number of multiple items is high, it is impractical to perform a complete enumeration due to the combinatorial nature of this grouping problem [Buffa and Munn, 1990].

In a multi-item system, how items are partitioned into groups and jointly replenished partly relies on an inventory policy adopted. Under a can-order policy (S,c,s) studied by Federgruen, Groenevett and Tijms (1984), Balintfy (1964) and Silver (1974), an item whose inventory position continuously reviewed drops to a must order point s triggers a replenishment to build up its inventory position to an order-up-to level S and other items whose inventory positions are at or below their can order points c will be replenished as well. Therefore, under this policy, the can order point of each item determines whether it will be added in the joint replenishment or not. It is clearly seen that a group of items in each replenishment may vary.

Another joint ordering inventory policy with a continuous review presented by Pantumsinchai (1992) is a QS policy where a group reorder point is a device to initiate a replenishment to raise the inventory position of each item to its base stock level Si. As a result, any item that has an inventory position below its base stock level at the time an order is demanded will be incorporated in the group refilled.

Some previous literature in the joint replenishment concentrates on a periodic review inventory policy. Among them are Jackson, Maxwell and Muckstadt (1985), Muckstadt and Roundy (1987), Chakravarty and Martin (1988), Viswanathan (1997) and Eynan and Kropp (1998).

Jackson, Maxwell and Muckstadt (1985), Muckstadt and Roundy (1987) and Eynan and Kropp (1998) adopt a power of two policy where a reorder interval of each item must be a power of two multiple of the base planning period. Consequently, in each replenishment, items will be grouped for joint ordering according to their reorder intervals. Items that have lower reorder intervals are included in a group of items that are replenished and have larger reorder intervals.

Eynan and Kropp (1998) also study the case where an item's replenishment interval is an integer multiple of the base cycle which is acquired by taking a derivative on the cost function. The iteration of solving the basic cycle continues until a marginal difference of the total cost between consecutive iterations is succeeded. Similar to the case of the power of two policy, items are grouped for joint ordering according to their replenishment intervals.

Chakravarty and Martin (1988) propose a grouping strategy for the stochastic demand environment by assuming consecutiveness in all individual optimal replenishment intervals that are then ranked in non-decreasing order. The shortest-path approach and the ranking process of replenishment intervals are repeatedly employed to determine the minimum cost grouping system.

Viswanathan (1997) introduce a Periodic Review (s,S) Policy for a multi-item inventory system where inventory of every item is reviewed every constant period t. At the time of the review, any items whose inventory levels are at or below their own s are grouped together for a single order and replenished up to their highest levels S. By originating a value of a review interval and assuming that the item yields only the minor setup cost, the order-up-to level S and the reorder point s of each item are calculated independently from others. Then the best review interval is searched in the small step. Under this policy, the items are possibly different in each replenishment relying on their inventory levels at the review time.

Goyal (1974) and Silver (1976) apply the EOQ concept for the multi-item single supplier joint replenishment in a deterministic setting. Goyal (1974) determines the economic order quantity of each item from the economic number of purchase whereas Silver (1976) obtains the economic order quantity from the time interval between replenishments of the group. However, they use the same idea that the replenishment interval of each item is an integer multiple ki of the base interval. Therefore, the item’s integer multiple ki which is solved by differentiating the total cost function decides which items will be combined in the single order.

2.3.2 Integrated Inventory-Transportation System

In the integrated inventory-transportation model, retailers, suppliers or products can be categorized in the grouping strategy depending on the characteristics of the integrated system. For example, for a single product multi-retailer system studied by Federgruen and Zipkin (1984) and Anily and Federgruen (1990), retailers are grouped. On the other hand, for a multi-item integrated system presented by Buffa and Munn (1990), Viswanathan and Mathur (1997) and Qu, Bookbinder and Iyogun (1999), the items are partitioned into groups.

Federgruen and Zipkin (1984) formulate the integrated inventory-routing problem as a non-linear integer program and then solve the inventory allocation problem first to determine the amount of the scarce item shipped to each customer in the group assigned to each vehicle. A modified interchange heuristic is utilized to alter the partition of customers among vehicles.

Anily and Federgruen (1990) use the EOQ policy for a single-item multi-retailer distribution system where each retailer receives a fixed quantity of the item every constant time interval. A Modified Circular Regional Partitioning Scheme is applied to separate demand points of retailers into groups. In this scheme, the circle that encloses all demand points and has the radius as the distance between the depot and the furthest demand point are divided into K successive sectors each of which contains the equal number of demand points mq and possibly one extra sector having a fewer number of demand points. Then using circular cuts, each sector is partitioned into subregions each of which encompasses m equal demand points. As we see, only geographical locations of retailers are considered for this grouping strategy but no inventory or transportation cost involved.

An inbound consolidation for a multi-item replenishment with stochastic demands is examined by Buffa and Munn (1990). They practically model the integrated inventory and transportation costs and present the grouping algorithm which is composed of a ranking procedure, a continue rule and a grouping heuristic. In the ranking procedure, firstly each item is reordered separately. Then they are arranged in a non-decreasing order of the amount reflecting the sum of holding cost and shipping cost. In the continue rule, the combination of groups is performed if the shipping weight of each group falls in the elastic range of the unit freight rate function and if the primary trade-off in acquiring the optimal cycle time of each group is between shipping and holding costs. In the grouping heuristic, the selection of groups for merging is based on the physical characteristics of value and weight as well as the value of marginal holding-shipping cost ratio (MHSR). At each stage, the receptor group which is the group with the largest MHSR is merged with the donor group, the adjacent group in the ranking scheme with the lowest MHSR. It is observed that the holding and shipping costs are taken into account for grouping items. Nevertheless, no vehicle routing cost is involved as a result of a common carrier considered in this system.

A stationary nested joint replenishment policy (SNJRP) is proposed by Viswanathan and Mathur (1997) for distribution systems with one warehouse and multiple retailers who keep multiple items that face deterministic demands. Under this policy, the replenishment intervals of items are power of two multiple of the base planning period. Accordingly, the items are grouped based on their replenishment intervals which are computed using a modification of the standard EOQ formula where the marginal cost incurred if an item is collectively replenished with the items already embraced in the nested set of items is employed as the setup cost. The items that have equal replenishment intervals are kept in the same group and replenished simultaneously. Both uncapacitated and capacitated cases are examined. For the uncapacitated case, all items are included in a single cluster consisting off several groups each of which has the nested replenishment interval. On the other hand, for the capacitated case, several clusters are created and there are several groups in each cluster.

An integrated inventory-transportation model dealing with an inbound material collection for a multi-item multi-supplier system with unlimited vehicle capacity in a stochastic setting is developed by Qu, Bookbinder and Iyogun (1999). With the modified periodic inventory policy, the authors separate items into two types, base items and non-base items, and group them based on an item’s replenishment cycle which is adjusted to be an integer multiple of a base planning period. The replenishment interval of the non-base items is calculated using the shared transportation cost along with minor ordering cost, holding cost and backlogging cost while the one of the base items which are restocked every replenishment period is determined using the joint fixed ordering cost in addition to those costs considered for the non-base items. The non-base items that possess an identical replenishment cycle are grouped and replenished together with the base items and sometimes with other non-base items at equally space epochs according to their replenishment period.

CHAPTER 3

AN INTEGRATED INVENTORY-TRANSPORTATION SYSTEM UNDER DETERMINISTIC DEMANDS

3.1 A description of the problem

In this chapter, an inbound commodity collection system in a deterministic setting is considered. The system consists of a central warehouse and a set of geographically dispersed suppliers. Each supplier produces one or more non-identical items, each of which faces constant and deterministic demand from outside retailers. The central warehouse has an unlimited area for stocking items and uses a fleet of vehicles to collect the items from its suppliers. These vehicles have limited capacity, and they are also subject to frequency constraints that limit the number of trips that each vehicle can make per time unit. The frequency constraint may, for example, be caused by the time required for vehicle maintenance and other responsibilities, or by the fact that material handling capacity is limited. Lead time of each replenishment is constant and the replenishment interval of any item is assumed to be longer than its lead time.

It seems unlikely that it is possible to identify an optimal strategy for this problem. But more importantly, even if such an optimal strategy could be found efficiently, it would likely be too complex to be implementable in practice. Nevertheless, some progress has been made in this direction recently with the work of Adelman (2003), who develops an approximate dynamic programming approach that finds high quality policies without imposing any a priori policy structure for inventory routing problems where only the routing costs are taken into account. Due to the difficulty, as well as perhaps the undesirability from the point of view of implementability, of finding truly optimal policies, it is common practice in inventory-routing problems to consider a given policy structure up front, and focus on finding optimal parameters for that policy.

A policy where the set of items is partitioned into disjoint groups is adopted. Each group of items is assigned to a vehicle. The vehicle leaves the central warehouse, visits the set of suppliers corresponding to the items in its group, and returns to the warehouse, where the items are unloaded and stored. It is assumed that no item can be assigned to more than one group, i.e. the orders cannot be split across multiple vehicles. However, it is not necessary for items produced by the same supplier to be in the same group, i.e., a supplier can be visited by multiple vehicles. Finally, the fact that the central warehouse faces a constant demand for each item leads to a joint replenishment of all items in a group using an economic order quantity (EOQ) policy.

3.2 Notation

All notation used in chapter 3 for the deterministic model is defined as follows.

S set of items in the system (stored by the central warehouse).

S subset of items $S \subseteq \mathrm{S}$.

m number of available vehicles.

n total number of items.

i index for vehicles $(i = 1,2,...,m)$.

k index for vehicles $(k = 1,2,...,m)$.

j subscript denoting item $(j = 1,2,...,n)$.

D_j demand rate for item j.

h_j inventory holding cost rate for item j.

Q_j replenishment quantity of item j.

K fixed joint ordering cost plus fixed dispatching cost.

C vehicle capacity.

F maximum number of trips allowed for each vehicle.

$TSP(S)$ optimal vehicle rout for visiting suppliers of items in subset S.

$L(S)$ fixed transportation costs plus fixed joint ordering cost.

$D(S)$ aggregate demand rate for all items in subset S.

$h(S)$ weighted average unit holding cost for items in subset S.

$T(S)$ replenishment interval for items in subset S.

$Q(S)$ aggregate replenishment quantity for all items in subset S.

Q_j^* optimal replenishment quantity of item j.

$Q^*(S)$ optimal aggregate replenishment quantity for all items in subset S.

$T^*(S)$ optimal replenishment interval for all items in subset S.

$S^{(i)}$ subset of items assigned to vehicle i.

$c(S)$ total inventory-transportation cost for all items in subset S.

3.3 Costs

In the deterministic problem, it is assumed that the costs of the integrated inventory-routing system include the inventory holding cost at the central warehouse, the joint ordering cost, the vehicle dispatching cost, and the vehicle routing cost.

3.3.1 Inventory Holding Cost

The inventory holding cost is proportional to the average inventory kept at the central warehouse and incurred at a constant rate per unit item per year. The inventory holding cost rate of each item may be different. The model is treated from the viewpoint of the central warehouse and it is assumed that the central warehouse and the suppliers belong to different organizations. Therefore, any charges to the suppliers from holding stock will not be considered.

3.3.2 Joint Ordering Cost

This joint ordering cost is fixed for every replenishment, regardless of which items and what quantity are replenished. It's assumed to be associated only with the ordering process and charged when a manager of the central warehouse decides to replenish the stock.

3.3.3 Vehicle Dispatching Cost

The vehicle dispatching cost is constant and equal for all the vehicles. It is incurred whenever a vehicle is dispatched to collect a set of items assigned to it. Wage of a driver per trip could be viewed as this cost.

3.3.4 Vehicle Routing Cost

This component of transportation costs is a variable cost depending on the distance traveled by a vehicle. However, it is constant for a specific set of items assigned to the vehicle. That is it is fixed for replenishment of the same set of items. An example of this cost is gas expenses.

3.4 Model Formulation

In the integrated inventory-transportation system, it is assumed that there are the set of items stored by the central warehouse which is denoted by S. Item j ($j \in$ S) faces a deterministic demand rate D_j. The items are collected from the suppliers using a fleet of m vehicles. The total system costs consist of the holding costs associated with each item, which are incurred at a constant rate of h_j per unit per year for item j ($j \in$ S), as well as fixed costs. These fixed costs include fixed ordering costs and fixed vehicle dispatching costs, as well as the total vehicle routing costs associated with a trip, which is the cost of a Traveling Salesman Problem (TSP) where the cities are the warehouse and the suppliers of the items collected in the trip. For convenience, the fixed joint ordering and dispatching costs are combined in a single term K per vehicle per trip. Using a policy where each item is assigned to a single group that is replenished repeatedly using a given vehicle, the inventory-routing problem is then to determine the subsets of items that are replenished with a single vehicle, as well as the corresponding replenishment quantities, the replenishment interval and the optimal vehicle routes, that minimize the average total inventory and transportation cost per unit time.

3.4.1 No vehicle capacity and frequency constraints

Firstly, the average total inventory and transportation costs per unit time for a given set of items $S \subseteq$ S assigned to a vehicle, and under the simplifying assumption that the vehicle is uncapacitated and does not face a frequency constraint, is determined. It is assumed that the fixed cost associated with this set is of the form

$$L(S) = K + TSP(S)$$

where $TSP(S)$ denotes the cost of the optimal TSP route for visiting the suppliers corresponding to the items in S, and K represents any other fixed costs associated with using the vehicle plus the fixed joint ordering cost. If the time between replenishments of the items in S is denoted by $T(S)$, then the corresponding replenishment quantities are given by

$$Q_j = D_j T(S)$$

for all items j in S. The total inventory-transportation costs per unit time incurred for replenishing the items in S as a function of the replenishment interval $T(S)$ is equal to

$$\frac{L(S)}{T(S)} + \frac{1}{2}\sum_{j\in S} h_j D_j T(S)$$

It is convenient to define the aggregate demand and weighted average unit holding costs for subset S as follows:

$$D(S) = \sum_{j\in S} D_j$$
$$h(S) = \frac{\sum_{j\in S} h_j D_j}{D(S)}$$

The cost function can then be rewritten as

$$\frac{L(S)}{T(S)} + \frac{1}{2} h(S) D(S) T(S)$$

which is a standard EOQ-type cost function and thus immediately yields that the optimal replenishment time for set S is equal to

$$T^*(S) = \sqrt{\frac{2L(S)}{h(S)D(S)}}$$

Alternatively, the total cost function can be formulated in terms of the aggregate replenishment quantity

$$Q(S) = \sum_{j \in S} Q_j$$

Note that the individual item replenishment quantities have to satisfy

$$\frac{Q_j}{Q(S)} = \frac{D_j}{D(S)} \quad \text{for} \quad j \in S$$

or

$$Q_j = \frac{D_j}{D(S)} Q(S) \quad \text{for} \quad j \in S$$

Since $Q_j = D_j T(S)$ for all $j \in S$, it is clear that $Q(S) = D(S)T(S)$. The total inventory-transportation costs per unit time incurred for replenishing the items in S can now equivalently be written as

$$L(S)\frac{D(S)}{Q(S)} + \frac{1}{2}h(S)Q(S)$$

This is again a standard EOQ-cost function, and leads to the optimal aggregate replenishment quantity for subset S

$$Q^*(S) = \sqrt{\frac{2D(S)L(S)}{h(S)}}$$

Using either approach, we obtain that the optimal replenishment quantities for the individual items are equal to

$$Q_j^* = D_j T^*(S) = \frac{D_j}{D(S)} Q^*(S) = D_j \sqrt{\frac{2L(S)}{D(S)h(S)}} \qquad \text{for} \quad j \in S$$

The corresponding optimal inventory-transportation costs are equal to

$$c(S) = \sqrt{2D(S)L(S)h(S)}$$

The integrated inventory-routing problem can now be formulated as a set partitioning problem:

$$\min \sum_{i=1}^{m} c(S^{(i)})$$

subject to

$$\bigcup_{i=1}^{m} S^{(i)} = \mathrm{S}$$

$$S^{(i)} \cap S^{(k)} = \phi \qquad \text{for all } i,k = 1,2,\ldots,m; \quad i \neq k.$$

3.4.2 Vehicle capacity constraint

In the capacitated case where each vehicle has identical limited capacity C, if the optimal aggregate replenishment quantity obtained from the EOQ formula is larger than the vehicle capacity, it has to be reduced to be equal to the vehicle capacity in order to satisfy the capacity constraint. As a result, a similar partitioning problem can be obtained but with

$$Q^{*}(S) = \min\left\{\sqrt{\frac{2D(S)L(S)}{h(S)}}, C\right\}$$

and corresponding optimal inventory-transportation costs equal to

$$c(S) = \begin{cases} \sqrt{2D(S)L(S)h(S)} & \text{if } \sqrt{\dfrac{2D(S)L(S)}{h(S)}} \leq C \\ L(S)\dfrac{D(S)}{C} + \dfrac{1}{2}h(S)C & \text{if } \sqrt{\dfrac{2D(S)L(S)}{h(S)}} \geq C \end{cases}$$

3.4.3 Frequency constraint

The vehicles may face only a frequency constraint. In other words, the vehicles cannot travel more than a maximum number of trips allowed per year F due to the time required for vehicle maintenance and other responsibilities, or the limited material handling capacity. This means that the number of trips per year is bounded from above or, equivalently, the replenishment interval is bounded from below. The minimum length of the replenishment interval would be $1/F$. If the optimal aggregate replenishment quantity obtained from the EOQ formula is smaller than the smallest aggregate replenishment quantity $D(S)/F$ that still satisfies the frequency constraint, then the optimal aggregate replenishment quantity collected by the vehicles must be increased to become $D(S)/F$. That is

$$Q^*(S) = \max\left\{\sqrt{\frac{2D(S)L(S)}{h(S)}}, \frac{D(S)}{F}\right\}$$

and corresponding optimal inventory-transportation costs equal to

$$c(S) = \begin{cases} \sqrt{2D(S)L(S)h(S)} & \text{if } \sqrt{\dfrac{2D(S)L(S)}{h(S)}} \geq \dfrac{D(S)}{F} \\ L(S)F + \dfrac{1}{2}h(S)\dfrac{D(S)}{F} & \text{if } \sqrt{\dfrac{2D(S)L(S)}{h(S)}} \leq \dfrac{D(S)}{F} \end{cases}$$

3.4.4 Vehicle capacity and frequency constraints

In the case where the vehicles face both vehicle capacity and frequency constraints, the aggregate replenishment quantity must be larger than or equal to $D(S)/F$, and smaller than or equal to C. It is clearly seen that the set $S \subseteq \mathrm{S}$ is a feasible subset of items only if

$$D(S) \leq CF$$

With both the vehicle capacity and frequency constraints included in the model, the aggregate replenishment quantity for the items in a feasible subset S can be determined from

$$Q^*(S) = \begin{cases} \dfrac{D(S)}{F} & \text{if } \sqrt{\dfrac{2D(S)L(S)}{h(S)}} \le \dfrac{D(S)}{F} \\ \sqrt{\dfrac{2D(S)L(S)}{h(S)}} & \text{if } \dfrac{D(S)}{F} \le \sqrt{\dfrac{2D(S)L(S)}{h(S)}} \le C \\ C & \text{if } C \le \sqrt{\dfrac{2D(S)L(S)}{h(S)}} \end{cases} \tag{3.1}$$

or

$$Q^*(S) = \max\left\{\frac{D(S)}{F}, \min\left\{\sqrt{\frac{2D(S)L(S)}{h(S)}}, C\right\}\right\}$$

and the corresponding optimal costs can be obtained from

$$c(S) = \begin{cases} L(S)F + \dfrac{1}{2}h(S)\dfrac{D(S)}{F} & \text{if } \sqrt{\dfrac{2D(S)L(S)}{h(S)}} \le \dfrac{D(S)}{F} \\ \sqrt{2D(S)L(S)h(S)} & \text{if } \dfrac{D(S)}{F} \le \sqrt{\dfrac{2D(S)L(S)}{h(S)}} \le C \\ L(S)\dfrac{D(S)}{C} + \dfrac{1}{2}h(S)C & \text{if } C \le \sqrt{\dfrac{2D(S)L(S)}{h(S)}} \end{cases} \tag{3.2}$$

In this case, the set partitioning problem becomes

$$\min \sum_{i=1}^{m} c(S^{(i)}) \tag{3.3}$$

subject to

$$S^{(i)} \subseteq \mathrm{S}$$

$$\bigcup_{i=1}^{m} S^{(i)} = \mathrm{S}$$

$$D(S^{(i)}) \le CF \qquad \text{for all } i = 1,2,\ldots,m$$

$$S^{(i)} \cap S^{(k)} = \phi \qquad \text{for all } i,k = 1,2,\ldots,m; \quad i \ne k.$$

CHAPTER 4

AN EXACT SOLUTION APPROACH USING BRANCH-AND-PRICE

An exact solution approach, the Branch-and-Price algorithm, is developed in this chapter. The branch-and-price algorithm can be used to solve our inventory-routing problem to optimality. This algorithm is based on a column generation approach to the set partitioning formulation of the problem. After formulating this problem as an integer programming problem, its LP-relaxation is then solved via column generation. In this approach, the problem is solved iteratively with only a limited number of candidate subsets for the vehicles. In each iteration, a subproblem, called the pricing problem, is solved. Each subproblem will either verify that the current solution is optimal for the entire problem, or identify one or more subsets that should be added to the limited model. This solution method for solving the LP-relaxation of the set partitioning problem is incorporated in a branch-and-bound algorithm if the optimal solution of the LP-relaxation is fractional. The procedure of the branch-and-price algorithm is summarized in Figure 4.1. Applications of this methodology have been applied to other set partitioning problems, such as the generalized assignment problem [see Savelsbergh (1997)], the multi-period single-sourcing problem [see Freling et al. (1999)], a continuous-time version of that model [see Huang et al. (2003)], a joint location-inventory model [see Shen et al. (2003)], and the crew scheduling problem [see Barnhart et al. (1998)].

4.1 Notation

Here is additional notation mentioned in this chapter.

N_i number of feasible subsets of items that can be assigned to vehicle i.

l superscript denoting subsets of items.

α_{ij}^l $\alpha_{ij}^l = 1$ if item j is in subset l of items for vehicle i and $\alpha_{ij}^l = 0$ otherwise.

α_i^l binary vector representing subset l of items for vehicle i.

y_i^l $y_i^l = 1$ if subset l of items is assigned to vehicle i and $y_i^l = 0$ otherwise.

$c_i(\alpha_i^l)$ total inventory-transportation cost of subset l of items assigned to vehicle i.

μ_j dual variable associated with item constraints $(j = 1,2,...,n)$.

δ_i dual variable associated with vehicle constraints $(i = 1,2,...,m)$.

$\hat{\mu}_j$ optimal dual solution associated with item constraints $(j = 1,2,...,n)$.

$\hat{\delta}_i$ optimal dual solution associated with vehicle constraints $(i = 1,2,...,m)$.

z feasible subset of items.

z_j $z_j = 1$ if item j is in subset (column) z and $z_j = 0$ otherwise.

C_i vehicle capacity for vehicle i.

F_i maximum number of trips allowed for each vehicle i.

J^0 set of items whose z_j is fixed to 0.

J^1 set of items whose z_j is fixed to 1.

J set of items whose z_j has not been fixed.

$\underline{c}(z)$ lower bound of total inventory-transportation cost of subset z of items.

4.2 A column generation approach to the set partitioning formulation

At the beginning, the set partitioning problem will be formulated as an integer programming problem. Without loss of generality, it is assumed that there are n items, and $S = \{1,2,...,n\}$. Then, let N_i denote the number of feasible candidate subsets of items that can be assigned to vehicle i. Each of these subsets is represented by a binary vector

$$\alpha_i^l = (\alpha_{i1}^l,...,\alpha_{in}^l)^T$$

where $\alpha_{ij}^l = 1$ if item j is in candidate subset l for vehicle i, and $\alpha_{ij}^l = 0$ otherwise. Letting $c_i(.)$ denote the cost function for vehicle i (as derived in Chapter 3 for a generic vehicle), the cost of subset l of vehicle i can be obtained by $c_i(\alpha_i^l)$. Note that a binary incidence vector of a subset of S is used rather than the subset itself as the argument of c_i. When it is convenient, this also will be done for all set functions introduced in Chapter 3. Finally, with the introduction of a binary variable y_i^l that takes on the value 1 if subset l is chosen for vehicle i, and 0 otherwise. The set partitioning problem can then be reformulated as

$$\min \sum_{i=1}^{m} \sum_{l=1}^{N_i} c_i(\alpha_i^l) y_i^l$$

subject to (P)

$$\sum_{i=1}^{m} \sum_{l=1}^{N_i} \alpha_{ij}^l y_i^l = 1 \qquad j = 1,2,...,n \qquad (4.1)$$

$$\sum_{l=1}^{N_i} y_i^l = 1 \qquad i = 1,2,...,m \qquad (4.2)$$

$$y_i^l \in \{0,1\} \qquad l = 1,2,...,N_i, i = 1,2,...,m$$

The first n constraints (item constraints) ensure that each item is collected by exactly one vehicle, while the next m constraints (vehicle constraints) state that only one feasible subset of items can be assigned to each vehicle. It is clear that the number of variables in this problem grows extremely rapidly in the number of items considered, which would make even solving the LP-relaxation of (P) a daunting task. However, since it is expected that most variables will have a value of zero in the optimal solution, a column generation approach is applied to solving LP(P), the LP-relaxation of (P).

In this approach, at first only a small number of subsets (columns) for each vehicle are considered. These can, for example, be obtained using a heuristic. After obtaining the solution to the master problem, a subproblem called the pricing problem is solved in order to either identify columns that would provide a better objective value if they would be added to the problem, or conclude that the current solution is optimal. This process is then repeated iteratively until the optimal solution is indeed obtained. To check for optimality of an intermediate solution we consider the dual problem (D) of LP(P). Letting μ_j denote the dual variables associated with constraints (4.1) ($j = 1,2,...,n$) and δ_i the dual variables associated with constraints (4.2) ($i = 1,2,...,m$), and noting that the binary constraints are replaced by nonnegativity constraints in the LP-relaxation of (P), we obtain

$$\max \sum_{j=1}^{n} \mu_j + \sum_{i=1}^{m} \delta_i$$

subject to (D)

$$\sum_{j=1}^{n} \alpha_{ij}^{l} \mu_j + \delta_i \le c_i(\alpha_i^l) \qquad l = 1,2,\ldots,N_i, i = 1,2,\ldots,m \qquad (4.3)$$

$$\mu_j \quad \text{free} \qquad j = 1,2,\ldots,n$$

$$\delta_i \quad \text{free} \qquad i = 1,2,\ldots,m$$

Now suppose that we have the optimal primal and, in particular, dual solution to a restricted version of LP(P) and (D) in which only a subset of the columns has been taken into account. Extending the primal optimal solution with the implicit zero values of all omitted variables, we find that if the corresponding dual solution is feasible for the entire dual problem (D), then the current solution is optimal.

4.3 The pricing problem

The pricing problem aims to find, for each vehicle i, the feasible subset (represented by a binary vector α_i), for which the corresponding constraint in (D) is most violated. Denoting the decision variable representing a feasible subset for vehicle i by z, and the optimal dual solution to the restricted version of LP(P) by $(\hat{\mu}, \hat{\delta})$, the pricing problem for vehicle i can be formulated as follows:

$$\max \sum_{j=1}^{n} \hat{\mu}_j z_j - c_i(z)$$

subject to (PP i)

$$\sum_{j=1}^{n} D_j z_j \le C_i F_i$$

$$z_j \in \{0,1\} \qquad \text{for } j = 1,2,\ldots,n$$

where C_i denotes the capacity of vehicle i, and F_i denotes its maximum frequency. If the optimal solution value of (PP i) is no more than $-\hat{\delta}_i$, then all dual constraints for vehicle i are satisfied. If the optimal solution value of (PP i) exceeds $-\hat{\delta}_i$, the corresponding optimal solution yields a subset for vehicle i that may improve the solution if added to the limited set partitioning problem.

In the remainder of this section, a branch-and-bound algorithm that can be used to solve the pricing problem (PP i) to optimality will be developed. For notational convenience, we will omit the index i indicating the vehicle, and consider a general pricing problem (PP). Similar to often used branch-and-bound strategies for the knapsack problem [see, e.g., Martello and Toth (1990)], the binary variables z_j will be branched. Therefore, each node of the branch-and-bound tree is characterized by a partition of the items in S into the following three sets: J^0, J^1, and J:

$$\begin{aligned} J^0 &= \{ j \in \mathrm{S} : z_j \text{ has been fixed to } 0 \} \\ J^1 &= \{ j \in \mathrm{S} : z_j \text{ has been fixed to } 1 \} \\ J &= \{ j \in \mathrm{S} : z_j \text{ has not been fixed} \} \end{aligned}$$

Note that z_j can be set to 0 for all items j such that $\hat{\mu}_j < 0$ without loss of optimality, which may significantly reduce the size of the problem. So, these items will be assumed to always be included in the set J^0. Now an upper bound on the objective function value of (PP) in a node of the tree can be found as follows.

Note that the function c given in equation (3.2) in Chapter 3 can be bounded by noting that the fixed costs are given by

$$L(z) = K + TSP(z)$$

so that we can bound these from below by

$$L(z) \geq K + TSP(J^1)$$

to obtain

$$c(z) \geq \underline{c}(z)$$

where

$$\underline{c}(z) = \begin{cases} (K + TSP(J^1))F + \frac{1}{2}h(z)\frac{D(z)}{F} & \text{if } \sqrt{\frac{2D(z)(K + TSP(J^1))}{h(z)}} \leq \frac{D(z)}{F} \\ \sqrt{2D(z)(K + TSP(J^1))h(z)} & \text{if } \frac{D(z)}{F} \leq \sqrt{\frac{2D(z)(K + TSP(J^1))}{h(z)}} \leq C \\ (K + TSP(J^1))\frac{D(z)}{C} + \frac{1}{2}h(z)C & \text{if } C \leq \sqrt{\frac{2D(z)(K + TSP(J^1))}{h(z)}} \end{cases}$$

If the number of items in J are small, the following problem may be efficiently solved

$$\max \sum_{j=1}^{n} \hat{\mu}_j z_j - \underline{c}(z)$$

subject to $(\overline{\text{PP}}^1)$

$$\sum_{j=1}^{n} D_j z_j \leq CF$$

$$z_j = 0 \qquad \text{for } j \in J^0$$

$$z_j = 1 \qquad \text{for } j \in J^1$$

$$z_j \in \{0,1\} \qquad \text{for } j \in J$$

by complete enumeration, which will clearly provide a valid upper bound in the current node of the branch-and-bound tree. However, in general we will need to find an upper bound that can be computed more efficiently.

Clearly, an alternative lower bound on the costs can be found by, in addition to using the lower bound on $L(z)$, ignoring the capacity and frequency constraints, which yields

$$c(z) \geq \sqrt{2D(z)(K + TSP(J^1))h(z)}$$

Recalling the definition of the aggregate demand and inventory holding cost functions, the lower bound may be rewritten as

$$c(z) \geq \sqrt{2(K + TSP(J^1))\sum_{j=1}^{n} h_j D_j z_j}$$

An upper bound on the solution value of (PP) given the sets J^0 and J^1 can now be determined by solving the following optimization problem

$$\max \sum_{j=1}^{n} \hat{\mu}_j z_j - \sqrt{2(K + TSP(J^1))\sum_{j=1}^{n} h_j D_j z_j}$$

subject to $(\overline{\text{PP}}^2)$

$$z_j = 0 \quad \text{for} \quad j \in J^0$$
$$z_j = 1 \quad \text{for} \quad j \in J^1$$
$$z_j \in \{0,1\} \quad \text{for} \quad j \in J$$

where the capacity and cardinality constraints have also been ignored. This problem can now be solved efficiently using a result from Huang et al. (2003). This result says that, if the $|J|$ relevant items are renumbered and sorted in non-increasing order of the ratio

$$\frac{\hat{\mu}_j}{h_j D_j} \qquad (4.4)$$

the optimal solution will be of the form

$$z_j^* = \begin{cases} 1 & \text{for} \quad j = 1,2,...,k \\ 0 & \text{for} \quad j = k+1,...,|J| \end{cases}$$

for some $k = 0,...,|J|$.

Since our relaxation yields an integral solution, this solution cannot be used to guide the branching. We will instead use a strategy that has been successfully applied to many knapsack and related problems. This is a depth-first-search strategy that first explores the subtree in which the variable corresponding to the most promising item is set to one. For this problem, it means that the subproblem in which the unassigned item with the largest positive ratio (4.4) is added to J^1 will be considered first.

Since any feasible solution to the pricing problem for vehicle i with a value that exceeds -$\hat{\delta}_i$ provides a subset of items that is attractive, it is not strictly necessary to solve the pricing problem to optimality, especially in the early stages of the column generation procedure. Therefore, the branch-and-bound procedure for (PP) usually is implemented heuristically by finding an approximate bound in each node. That is, we find a value that will often, but not necessarily, be an upper bound to the objective function value in the current node of the tree. This approximation is based on considering solutions by sequentially adding items according to the ranking scheme given by the ratio (4.4). Observing the capacity and cardinality constraints, we then choose the solution that maximizes the objective function of $(\overline{\text{PP}}^1)$. Since this procedure does not necessarily find the optimal solution to $(\overline{\text{PP}}^1)$, the corresponding bound is not exact, and therefore the solution to the pricing problem obtained is not necessarily optimal.

4.4 Branching

We now return to our main problem (P). If the optimal solution of LP(P) obtained by using the column generation approach is not integral, we need to use branch-and-price. It remains to discuss the corresponding branching strategy. As has been mentioned by several authors [e.g., Freling et al. (1999)], branching on the subset selection variables y_i^l in the set partitioning formulation is problematic, since excluding a subset from consideration would require finding the second best solution to the pricing problem. However, the subset selection variables can be transformed to assignment variables that indicate the fraction of an item that is included in a subset:

$$x_{ij} = \sum_{l=1}^{N_i} \alpha_{ij}^l y_i^l$$

Clearly, x is integral if y is integral. In a node of the branch-and-price tree, we can now branch on fractional assignment variables. This corresponds to requiring or disallowing an item to be replenished by vehicle i. Item j is assigned to vehicle i if the fractional assignment variable x_{ij} is fixed to 1. On the other hand, item j cannot be assigned to vehicle i if the fractional assignment variable x_{ij} is fixed to 0. To solve the problem at the branching node, the column generation and pricing are applied as before.

If the solution to the problem contains y_i^l with a value that is negative or greater than one, this means that the set partitioning problem is infeasible in that node. This may be because there are not enough subsets of items included in the problem to allow for a feasible solution. In this case, at least one subset of items needs to be added to the existing ones to obtain a feasible solution and then the column generation and pricing are repeated in that node.

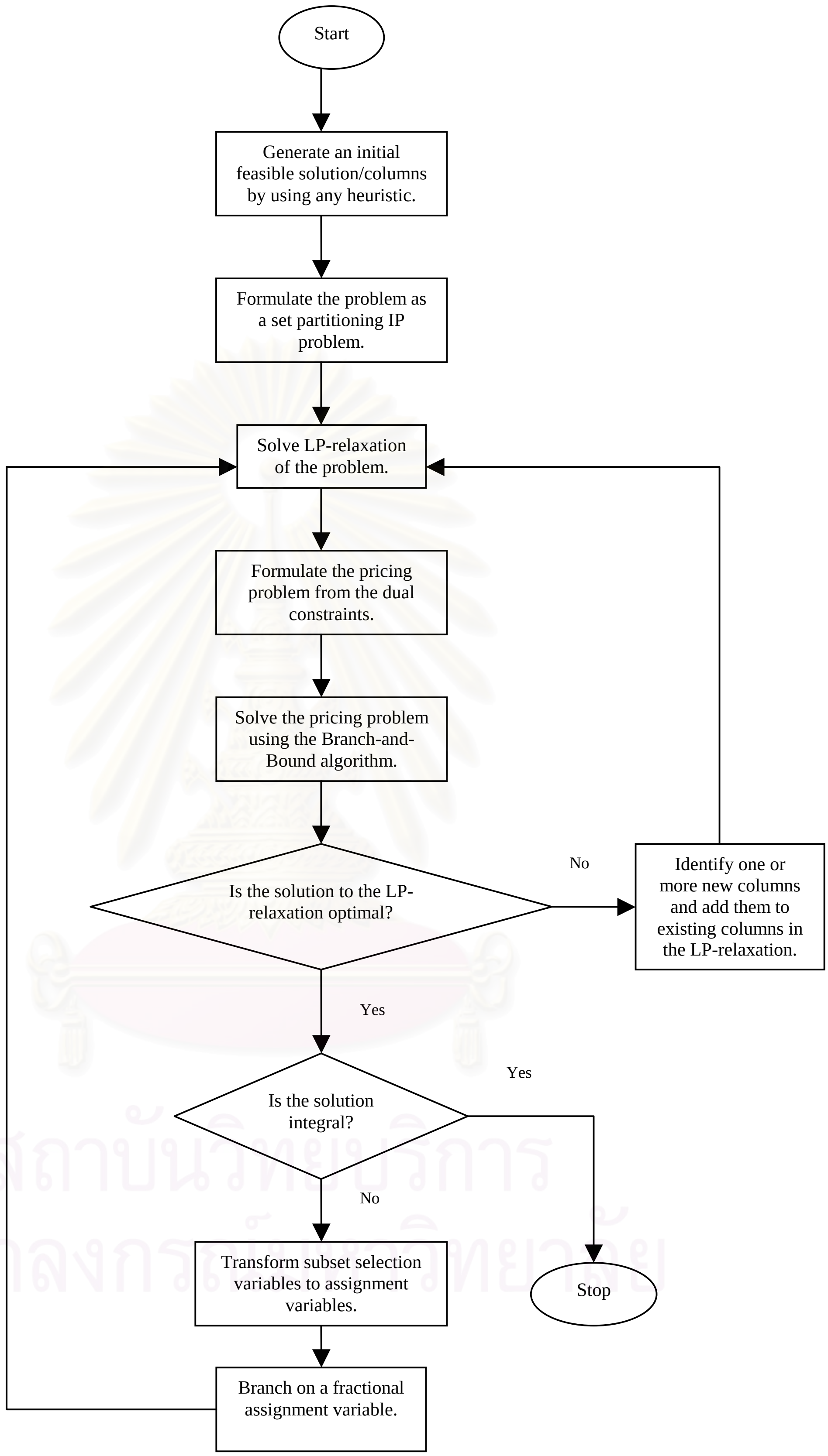

Figure 4.1 Branch-and-Price Algorithm for the integrated inventory-transportation problem

CHAPTER 5

CONSTRUCTIVE HEURISTICS AND IMPROVEMENT ALGORITHMS

In Chapter 4, a branch-and-price algorithm has been developed to solve the inventory-routing problem described in Chapter 3. Clearly, only relatively small problem instances can be solved to find an exact solution in reasonable time. The computational effort is likely to increase rapidly when the number of items, suppliers, and/or the vehicle capacities increase. Therefore, heuristic approaches to the problem are focused in this chapter. In particular, two constructive heuristics will be described. These heuristics can be used to find an initial feasible solution by constructing routes for the vehicles either sequentially or simultaneously. In addition, several neighborhood search algorithms that can be used to improve a solution found by the constructive heuristics will also developed..

5.1 Constructive Heuristics

5.1.1 Distance Ratio (DR) heuristic

The first heuristic constructs routes sequentially for one vehicle at a time. The idea of the heuristic is to add items to a vehicle whose supplier is (a) located far away from the warehouse, but (b) close to at least one supplier that is already visited in the route. In that case, it is attractive to add the item to the vehicle under consideration rather than supplying this item with another vehicle. Items are added until no more items can be added without violating the capacity and/or frequency constraints. Initially, when the route for a vehicle is empty, this criterion says that the item that is located furthest away from the warehouse should be chosen.

When a group of items is assigned to a vehicle and no more items can be added, the cost associated with the vehicle is estimated by solving its associated TSP heuristically. Firstly, a TSP tour is constructed by using the Arbitrary Insertion (AI) heuristic [see Rosenkrantz et al. (1977)]. To improve the vehicle tour, the 2-opt exchange heuristic studied by Croes (1958), Lin (1965), and Lin and Kernighan (1973) is utilized.

In the remainder, let $d_{j_1 j_2}$ denote the distance (cost) from the supplier of item j_1 to the supplier of item j_2, for all j_1, j_2 $\in$ S. Similarly, let d_{0j} and d_{j0} denote the distance from the warehouse to the supplier of item j and from the supplier of item j to the warehouse. The procedure of the DR heuristic is described as follows.

DR heuristic

Step0. Initialize an empty route for the next vehicle.

Step1. For each of ungrouped items that can be added to the vehicle without violating its capacity constraint, say j, determine its distance-ratio as the minimum value of $d_{jj'} / d_{0j}$ over all items j' served by the current vehicle. If the current vehicle does not contain any items, let the distance-ratio be $1/d_{0j}$. If no such items exist, go to Step 3.

Step2. Find the item with the smallest distance-ratio, assign it to the vehicle, and return to Step 1.

Step3. If all items have been assigned to a vehicle, go to Step 4. Otherwise, if there are available vehicles left, return to Step 0.

Step4. Find a TSP tour for all vehicles using the AI heuristic to construct a route and the 2-opt exchange heuristic to improve the tour.

As an alternative, we have explored the possibility of choosing the first item in the vehicle to be the unassigned item that has the smallest replenishment interval when replenished individually. In this case, Steps 0 and 1 in the algorithm are replaced by

Step0. Initialize an empty route for the next vehicle, and find the ungrouped item with the smallest individual replenishment interval that can be added to this vehicle without violating its capacity constraint. Add that item to the vehicle.

Step1. For each ungrouped item that can be added to the vehicle without violating its capacity constraint, say j, determine its distance-ratio as the minimum value of $d_{jj'} / d_{0j}$ over all items j' served by the current vehicle. If no such items exist, go to Step 3.

5.1.2 Arbitrary Item Insertion (AII) heuristic

This heuristic is based on the Arbitrary Insertion (AI) heuristic for the TSP [see Rosenkrantz et al. (1977)]. This heuristic starts with an empty route for each vehicle. On each iteration, given a partial route for each vehicle, the total insertion costs are calculated for each unassigned item and each possible slot in each partial route. The insertion costs estimate the additional total inventory-transportation costs to be incurred if an item is inserted in a given slot in a route. This estimation is obtained by first finding the traditional TSP insertion costs associated with inserting the item in a route, and next computing the total costs incurred by the vehicle with this additional item. More formally, consider a vehicle, say i, and a given pair of items, say j_1 and j_2, that are visited consecutively in the current route for that vehicle. Moreover, let $L(S^{(i)})$ denote the current costs associated with the route for assigned items in $S^{(i)}$. Then, ignoring for simplicity the capacity and frequency constraints, the corresponding insertion costs are:

$$\sqrt{2D(S^{(i)} \cup \{j\})(L(S^{(i)}) + d_{j_1 j} + d_{jj_2} - d_{j_1 j_2})h(S^{(i)} \cup \{j\})} - \sqrt{2D(S^{(i)})L(S^{(i)})h(S^{(i)})}$$

Analogously, the insertion costs can be derived in the presence of capacity and frequency constraints.

AII heuristic

Step0. Initialize an empty route for each vehicle.

Step1. Randomly select an unassigned item for insertion, and determine its insertion costs corresponding to each slot in each vehicle's partial route, for each vehicle to which the item can feasibly be assigned.

Step2. Find the minimum insertion cost for this item, and insert the item in the corresponding slot.

Step3. If all items have not been assigned, return to Step 1. Otherwise, improve, if possible, the TSP tour for each vehicle by applying a 2-opt exchange heuristic and stop.

5.2 Improvement Algorithms

Neighborhood search algorithms are often the most effective approaches available for solving partitioning problems which is a difficult class of combinatorial optimization problems. They usually begin with an initial feasible solution which is then repeatedly replaced by an improved solution until no further improvements can be found or some termination criterion is satisfied. For the problem studied, five different neighborhood search algorithms are proposed to improve the solutions obtained from constructive heuristics. The first two, which are called One Supplier Move (OSM) and Supplier Exchange (SE), are based on 1- and 2-exchange heuristics for the Vehicle Routing Problem (VRP) [see, e.g., Toth and Vigo (2002)]. The third and fourth are combination of OSM and SE. The last neighborhood search algorithm that will be considered is the very large scale neighborhood (VLSN) search algorithm.

5.2.1 One Supplier Move (OSM)

In the OSM method, a supplier is moved from one vehicle to another one. That is, all items from a supplier that are replenished using a given vehicle are moved to another vehicle. A move is only performed when it is feasible and results in a cost savings, and the search continues until no more solution improvement can be obtained. To reduce the computational time, the involved TSP problem is not solved to optimality, but the insertion heuristic will be utilized to estimate the cost change resulting from the move.

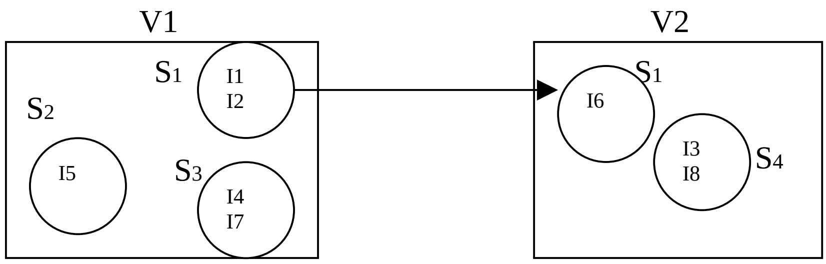

Figure 5.1 Illustrating a One Supplier Move

In Figure 5.1, an example of OSM is illustrated. Items I1 and I2 from supplier S1 that are replenished by vehicle V1 are moved to vehicle V2 which also visits supplier S1 for collecting item I6. As a result, vehicle V1 collects item I5 from supplier S2 and items I4 and I7 from supplier S3 while vehicle V2 collects items I1, I2 and I6 from supplier S1 and items I3 and I8 from supplier S4.

OSM improvement heuristic

Step0. Sequentially select the next vehicle to consider. If all vehicles have been considered without making any improving move, stop. Otherwise, return to the first vehicle.

Step1. Sequentially select the next supplier in the current vehicle to consider for moving. If all suppliers have been considered, go to step0.

Step2. Consider only a feasible move. Determine the (approximate) cost changes of moving this supplier along with its items, which are replenished with the current vehicle, to each of the other vehicles.

Step3. Perform the move that results in the largest cost savings, if any, and go to Step1.

5.2.2 Supplier Exchange (SE)

In the SE approach, in each step, a group of items with a common supplier currently replenished by one vehicle are exchanged with a group of items with a common supplier that is currently replenished by another vehicle. The exchange occurs only when it is feasible and incurs cost reduction.

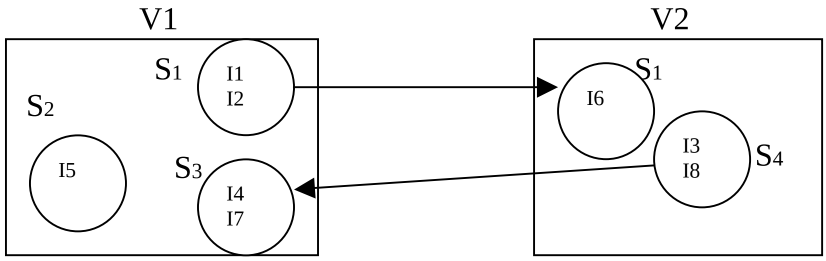

Figure 5.2 Illustrating a Supplier Exchange

As shown in Figure 5.2, Items I1 and I2 from supplier S1 that are replenished by vehicle V1 are moved to vehicle V2. At the same time, items I3 and I8 from supplier S4 that are replenished by vehicle V2 are moved to vehicle V1. Note that vehicle V2 will finally visit only supplier S1 for items I1, I2 and I6 while vehicle V1 will visits all other suppliers for collecting their items.

SE improvement heuristic

Step0. Sequentially select the next vehicle to consider. If all vehicles have been considered without making any improving move, stop. Otherwise, return to the first vehicle.

Step1. Sequentially select the next supplier in the current vehicle to consider for moving. If all suppliers have been considered, go to step 0.

Step2. Consider only a feasible exchange. Determine the (approximate) cost changes of exchanging this supplier along with its items that are replenished with the current vehicle with any group of items with a common supplier currently replenished by another vehicle.

Step3. Perform the exchange that results in the largest cost savings, if any, and go to Step1.

5.2.3 One Supplier Move-Supplier Exchange (OSM-SE)

The OSM-SE improvement method combines the OSM approach with the SE approach. In this algorithm, the OSM method is applied first to improve an initial feasible solution obtained from constructive heuristics. Then, the improved solution is further improved by applying the SE method.

5.2.4 Supplier Exchange- One Supplier Move (SE-OSM)

This improvement algorithm also combines the OSM approach with the SE approach. However, the SE-OSM approach starts with the SE method which is then followed by the OSM approach , in contrast to the OSM-SE method.

5.2.5 Very Large-Scale Neighborhood Search (VLSN)

As mentioned before, One Supplier Move (OSM) and Supplier Exchange (SE), are based on 1- and 2-exchange heuristics. The 1- and 2-exchange heuristics have been developed and applied to the traditional vehicle routing problem with some success. However, these methods search for an improved solution in a relatively small neighborhood of the current solution. Much better results may be expected if larger neighborhoods can be searched. Rather than extending the 1- and 2-exchange heuristics to our inventory-routing problem, a Very Large-Scale Neighborhood (VLSN) Search algorithm will be developed. Using this technique, very large neighborhoods can be explored implicitly through solving a subproblem, rather than explicitly by enumeration, as is common practice with small neighborhood search methods. This technique has relatively recently been developed and applied with much success to several hard combinatorial optimization problems. For example, the technique has been applied to vehicle routing problems [see Thompson and Psaraftis (1993), Gendreau et al. (1998), and Fahrion and Wrede (1990)], minimum makespan machine scheduling [see Frangioni et al. (2000)] and other scheduling problems [see Thompson and Psaraftis (1993)], the capacitated minimum spanning tree problem [see Ahuja et al. (2001-2 and 2001-3)], and several single-sourcing problems [see Ahuja et al. (2002) and Huang et al. (2003)]. Surveys of VLSN can be found in Ahuja et al. (2000) and Ahuja, Ergun, Orlin and Punnen (2002).

The VLSN algorithms that we propose can be viewed as extensions of 1- and 2-exchange heuristics for VRPs. In the first algorithm, which is called Supplier-VLSN (S-VLSN), we consider a neighborhood of solutions that can be reached by moving groups of items with a common supplier that are currently replenished by one vehicle to another vehicle. In particular, we consider simultaneous moves of this form, where each of a subset of the vehicles exchanges one group of items by another. A solution is called a neighbor of a given solution if it can be reached through a set of moves of the following form: for some sequence of distinct vehicles $i_1,...,i_k$, a group of items is moved from vehicle i_1 to vehicle i_2, while simultaneously a group of items is moved from vehicle i_2 to vehicle $i_3,...$, a group of items is moved from vehicle i_{k-1} to vehicle i_k, and a group of items is moved from vehicle i_k to vehicle i_1. This type of exchange is called a cyclic exchange. An even larger neighborhood is obtained when sets of moves that do not include the last one are considered, that is, one vehicle ``loses'' a group of items without ``gaining'' one, while another vehicle ``gains'' a group of items without ``losing'' one. Those type of exchanges are called path exchanges. The second algorithm, which is called item-VLSN (I-VLSN), is similar to S-VLSN, with the distinction that groups of items with a common supplier are now not moved, but single items only.

Efficient methods for identifying an improving neighbor without explicit enumeration and evaluation of all neighbors in the neighborhood are based on a characterization of the neighborhood through a so-called improvement graph [see Ahuja et al. (2000 and 2002)], which captures all information needed to evaluate any exchange. The improvement graph for cyclic exchange can be constructed by creating a node corresponding to each item (or group of items) that is a candidate for exchange. Then, an arc is created from one node to another if it is possible to move the item(s) corresponding to the first node to the vehicle that currently replenishes the item(s) corresponding to the second node, while removing these latter items from their vehicle. The arc costs in the improving graph are defined to be the change in costs due to the move incurred by the ``receiving'' vehicle. To allow also path exchanges, the improvement graph is extended by a node for each vehicle as well as a dummy node. Then, an arc with appropriate cost is created from an item-node to a vehicle-node if it is possible to move the item(s) corresponding to the item-node to the vehicle corresponding to the vehicle-node without removing any items from that vehicle. Similarly, an arc with appropriate cost is created from the dummy node to each item-node, modeling the fact that an item may leave a vehicle without being replaced by one. Finally, zero-cost arcs are created from the vehicle-nodes to the dummy node.

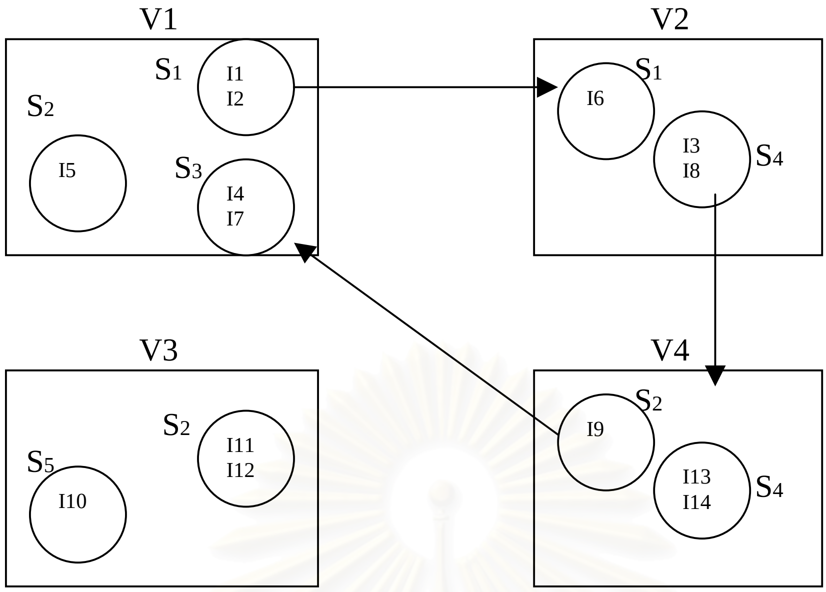

Figure 5.3 Illustrating the Supplier-VLSN for the Cyclic Exchange

Each neighbor is now represented by a so-called subset-disjoint cycle in the improvement graph, that is, a cycle whose nodes correspond to distinct vehicles. (Note that a cycle that does not contain the dummy node corresponds to a cyclic exchange, and a cycle that does contain the dummy node corresponds to a path exchange.) Furthermore, the cost change from the current solution to the neighbor is equal to the total cost of the corresponding cycle in the improvement graph. As shown by Thompson and Orlin (1989) and Thompson and Psaraftis (1993), the problem of finding an improving neighbor therefore reduces to the problem of finding a negative-cost subset-disjoint cycle in the improvement graph. However, the problem of determining whether there exists a subset-disjoint cycle in the improvement graph is NP-complete, and the problem of finding a negative cost subset-disjoint cycle is NP-hard [see Thompson (1988), Thompson and Orlin (1989),and Thompson and Psaraftis (1993)]. We will employ heuristics for this problem that have been developed by Ahuja et al. (2001-1, 2001-2 and 2001-3), and appear to be highly effective in practice.

In Figure 5.3, the S-VLSN for the cyclic exchange is illustrated. In this cycle, items I1 and I2 from supplier S1 are moved from vehicle V1 to vehicle V2, items I3 and I8 from supplier S4 are moved from vehicle V2 to vehicle V4 and item I9 from supplier S2 is moved from vehicle V4 to vehicle V1. The total cost of this cyclic exchange can be calculated from

$$c(S^{(1)} \setminus \{\mathrm{I1},\mathrm{I2}\} \cup \{\mathrm{I9}\}) - c(S^{(1)}) + c(S^{(2)} \setminus \{\mathrm{I3},\mathrm{I8}\} \cup \{\mathrm{I1},\mathrm{I2}\}) - c(S^{(2)}) +$$
$$c(S^{(4)} \setminus \{\mathrm{I9}\} \cup \{\mathrm{I3},\mathrm{I8}\}) - c(S^{(4)}))$$

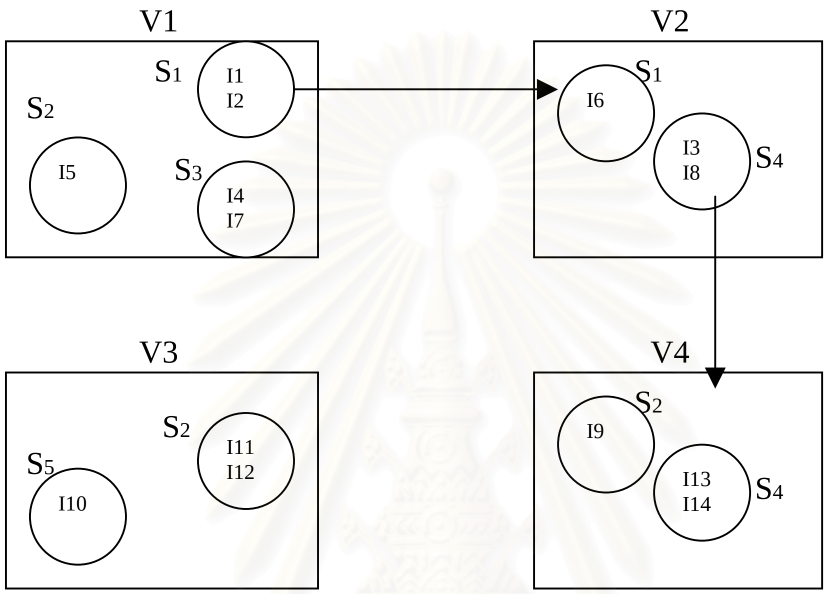

Figure 5.4 Illustrating the Supplier-VLSN for the Path Exchange

Figure 5.4 shows the S-VLSN for the path exchange. In this path exchange, items I1 and I2 from supplier S1 are moved from vehicle V1 to vehicle V2 and items I3 and I8 from supplier S4 are moved from vehicle V2 to vehicle V4 while no item is moved out from vehicle V4 and no item is moved to vehicle V1. The total cost of this path exchange can be calculated from

$$c(S^{(1)} \setminus \{\mathrm{I1},\mathrm{I2}\}) - c(S^{(1)}) + c(S^{(2)} \setminus \{\mathrm{I3},\mathrm{I8}\} \cup \{\mathrm{I1},\mathrm{I2}\}) - c(S^{(2)}) +$$
$$c(S^{(4)} \cup \{\mathrm{I3},\mathrm{I8}\}) - c(S^{(4)}))$$

CHAPTER 6

AN INTEGRATED INVENTORY-TRANSPORTATION SYSTEM UNDER STOCHASTIC DEMANDS

6.1 Description of the problem

An inbound commodity collection system in a stochastic setting is studied in this chapter. Literature that studies the integrated inventory-transportation system with multiple items usually simplifies the problem with one of the following assumptions: Demand rates are constant and determinist [see Viswanathan and Mathur (1997)] and the vehicle capacity is unlimited [see Qu et al. (1999)]. In the problem studied, stochastic demands and capacitated vehicles will be considered simultaneously. The characteristics of the integrated inventory-transportation system studied here are mostly the same as the ones in the deterministic case. There is a central warehouse that dispatches a given number of capacitated vehicles to visit a set of geographically dispersed suppliers for collecting non-identical items. Each supplier manufactures at least one item. The frequency of dispatching each vehicle per unit time is limited. The vehicles return to the warehouse after completing their duty. In contrast to the deterministic case, demands from outside retailers are assumed to be independent and identically distributed because each retailer faces stochastic demands for its items from customers and a standard periodic review order-up-to level inventory policy is adopted at each retailer. The probability distributions of demands of all items are in the same form but the mean and standard deviation of demands for an item may be different from others'. The normal distribution will be examined. This problem is restricted to the case where the average demand of each item is approximately constant with time. With limited vehicle capacity and demand uncertainty, the manager of the central warehouse selects a periodic review fixed order quantity policy for inventory control to ensure that the replenishment quantity of items of any subset can be picked up by a single vehicle. In addition, the central warehouse will hold more inventories than in the deterministic case in order to prevent the stock-out situation under a given service level determined by management. These additional inventories are called Safety Stock. Other assumptions, such as the central warehouse has unlimited space for stocking inventories, lead time is fixed for any replenishment and no shortage or delay occurs at any suppliers, remain unchanged.

6.2 Notation

Notation mentioned in Chapter 3 will be used in this chapter and additional notation for the stochastic model is given below.

σ_j standard deviation of demands for item j in unit item per unit time.

SS_j safety stock of item j.

T_i replenishment interval of items collected by vehicle i.

p service level or probability of no stock out per replenishment cycle.

Z_p random variable that has standard normal distribution at the service level p.

n_i number of items collected by vehicle i.

$M(S)$ aggregate minor ordering cost for items in subset S .

$O(S)$ aggregate stopover cost for items in subset S.

$f(Q)$ average total inventory-transportation cost in term of aggregate order quantity Q.

$f^{/}(Q)$ first derivative of $f(Q)$

6.3 Additional Costs

In addition to the costs considered in the deterministic case, minor ordering and stopover costs will be incorporated in the stochastic model. However, in the model formulation, these two costs will be assumed to be zero for convenience. Then a special case where they are included in the cost structure will be studied.

6.3.1 Minor Ordering Cost

An item dependent minor order cost is constant and incurred for any item included in the replenishment. It does not depend on the order quantity.

6.3.2 Stopover Cost

The stopover cost is considered as a part of transportation costs. It is charged when the vehicle stops at the supplier's location for item collection. This cost is fixed but specific for each supplier.

6.4 Model Formulation

In the integrated inventory-transportation system studied in this chapter, demands for each item from outside retailers are assumed to follow the normal distribution. The EOQ inventory policy which is the same as the one used in the deterministic problem can still be applied to the probabilistic problem. However, the modification of the total cost function is needed due to the uncertainty of demands. Under this policy, the review period of a particular subset of items is fixed and the fixed replenishment quantity for each item is determined with the objective of minimizing the total cost without violating vehicle capacity and frequency constraints and satisfying demands at a specific service level as well.

In general, when facing unknown and varied demands, the inventory manager always keeps the inventory level higher than the one when demands are deterministic in order to prevent the stockout situation. As a result, the inventory holding cost in the stochastic model is higher than the one in the deterministic model. The increasing inventory holding cost is incurred by safety stock. The safety stock is an average inventory just before the replenishment order arrives at the warehouse. There are many different ways to establish the safety stock. See Silver and Peterson (1985) for details. In this problem, the safety stock of each item is determined, based on the service level which is usually specified by management. The service level is the probability of no stockout per replenishment cycle. It is assume that the manager of the central warehouse sets the same service level p for all items.

6.4.1 No Minor Ordering and Stopover Costs

Now suppose the mean and the standard deviation of demands for item j are D_j units per year and σ_j units per year respectively. Therefore, demands for item j

picked up by vehicle i over its replenishment interval T_i are also normally distributed with mean $D_j T_i$ and standard deviation $\sigma_j T_i$ or

$$N(D_j T_i, \sigma_j^2 T_i)$$

The safety stock SS of item j can be calculated from the following :

$$SS_j = Z_p \sigma_j \sqrt{T_i}$$

Where Z_p is a random variable that follows the standard normal distribution with mean 0 and standard deviation 1 at the service level p.

Apparently, adding the safety stock means increasing the inventory holding cost. Consequently, a mathematical model for this problem in the stochastic setting can be formulated by including the inventory holding cost incurred by the safety stock in the average total inventory-transportation cost function (3.2) derived in Chapter 3. For item j in subset S assigned to vehicle i, the average inventory cost due to its safety stock is the inventory holding cost rate multiplied by the safety stock. That is

$$h_j Z_p \sigma_j \sqrt{T_i}$$

The average total inventory holding cost of this subset of items with the service level p can be formulated as

$$\frac{1}{2} h(S) Q(S) + Z_P \sqrt{\frac{Q(S)}{D(S)}} \sum_{j \in S} h_j \sigma_j$$

Meanwhile, the assumptions about the transportation part remain the same as the ones in the deterministic case. The minor order cost and the stopover cost will not be taken into consideration. In the other words, they are assumed to be zero for now. As a result, the average total transportation cost can be derived in the same way as

$$L(S)\frac{D(S)}{Q(S)}$$

$L(S)$ can be determined after a subset of items assigned to the vehicle is known. Therefore, the average total integrated inventory-transportation cost of items in subset S is

$$c(S) = L(S)\frac{D(S)}{Q(S)} + \frac{1}{2}h(S)Q(S) + Z_P\sqrt{\frac{Q(S)}{D(S)}}\sum_{j\in S}(h_j\sigma_j) \qquad (6.1)$$

Subject to:

$$D(S) \le CF$$

$$Q(S) > 0$$

If there are m vehicles available, the integrated inventory-transportation problem under stochastic demands can still be considered as a partitioning problem.

$$\min \sum_{i=1}^{m} c(S^{(i)}) \qquad \text{(PPP)}$$

Subject to:

$$\bigcup_{i=1}^{m} S^{(i)} = \mathrm{S}$$

$$S^{(i)} \cap S^{(k)} = \phi \quad \text{for all } i \ne k.$$

$S^{(i)}$ denotes a subset of items collected by vehicle i.

6.4.2 Minor Ordering Cost

Now the case when there are item dependent minor ordering costs is considered. The minor ordering cost is constant and incurred for any specific item included in the replenishment. $M(S)$ is denoted as an aggregate minor ordering cost which is the summation of minor ordering costs of all items in the subset S. Therefore, the aggregate minor ordering cost of a particular subset S of items assigned to the vehicle is also constant for every replenishment interval. When a subset of items S is determined for the vehicle i, the aggregate minor ordering cost can be considered as one of components of fixed costs of that vehicle. So, from (6.1), we obtain

$$c(S) = L(S)\frac{D(S)}{Q(S)} + \frac{1}{2}h(S)Q(S) + Z_P\sqrt{\frac{Q(S)}{D(S)}}\sum_{j\in S}(h_j\sigma_j) \qquad (6.2)$$

where $L(S) = K + TSP(S) + M(S)$

Subject to:

$$D(S) \le CF$$

$$Q(S) > 0$$

6.4.3 Stopover Cost

Assume that there is a fixed cost incurred when the vehicle visits a supplier for item collection. This cost is called the stopover cost which is specific for each supplier and does not depend on items or quantities picked up. As a result, the aggregate stopover cost of a subset S of items, denoted by $O(S)$, is also fixed for every replenishment interval. For a particular subset of items assigned to the vehicle, the aggregate stopover cost can be combined to the existing fixed costs in (6.2). Therefore, the average total integrated inventory-transportation cost of items in subset S assigned to vehicle i is

$$c(S) = L(S)\frac{D(S)}{Q(S)} + \frac{1}{2}h(S)Q(S) + Z_P\sqrt{\frac{Q(S)}{D(S)}}\sum_{j\in S}(h_j\sigma_j) \qquad (6.3)$$

where $L(S) = K + TSP(S) + M(S) + O(S)$

Subject to:

$$D(S) \le CF$$
$$Q(S) > 0$$

If the minor ordering and stopover costs are included in the model, the integrated inventory-transportation problem under stochastic demands can still be formulated as a set partitioning problem and the only difference is the term of the average integrated inventory-transportation cost of each subset of items $c(S)$.

6.5 Analysis of the Cost Function

For a purpose of analyzing the total cost function, a single subset of items S will be considered and the minor ordering and stopover costs are assumed to be zero. From the average total cost function (6.1)

$$c(S) = L(S)\frac{D(S)}{Q(S)} + \frac{1}{2}h(S)Q(S) + Z_P\sqrt{\frac{Q(S)}{D(S)}}\sum_{j\in S}(h_j\sigma_j)$$

where $L(S) = K + TSP(S)$

Let

$A = L(S)D(S)$, $B = \frac{1}{2}h(S)$ and $C = \frac{Z_P}{\sqrt{D(S)}}\sum_{j\in S}(h_j\sigma_j)$

and $f(Q)$ be the average total cost function in term of the aggregate order quantity Q. For convenience, the term S will be omitted for now. Then, the total cost function (6.1) can be rewritten as

$$f(Q) = \frac{A}{Q} + BQ + C\sqrt{Q} \qquad (6.4)$$

A, B and C can be viewed as constant terms if which items are in the subset is known. Then, the total cost function can be analyzed. Suppose there are three items, items 1, 2 and 3, in subset S and $L(S)$=\$50, p=0.975, D_1=120, D_2=150, D_3=200, h_1=100, h_2=100, h_3=120, σ_1=24, σ_2=30 and σ_3=40.

Now A, B and C can be determined and A=23500, B=54.26 and C=922.16. Different values of the total cost function can be obtained by changing the value of Q without considering the capacity and frequency constraints. With these values, the graph of the relationship between the total integrated cost and the aggregate order quantity of items of that particular group can be obtained and shown in Figure 6.1.

As seen from Figure 6.1, at the first stage, when the aggregate order quantity Q increases, the total cost decreases rapidly and reaches the lowest point as $Q \approx 40$. This is because A is much greater than both B and C. So when Q increases, the decreasing rate of the cost associated with A/Q is much more than the increasing rate of the costs associated with both BQ and $C\sqrt{Q}$ combined. While Q is increasing, the total cost continues falling down until the amounts of decreasing and increasing costs are equal which incurs the minimum total cost. After that, the total cost keeps raising up as Q increases. This can be explained that when Q is very large, the first term A/Q becomes near zero while the other two terms, BQ and $C\sqrt{Q}$, keep increasing. This behavior is similar to the simple EOQ model.

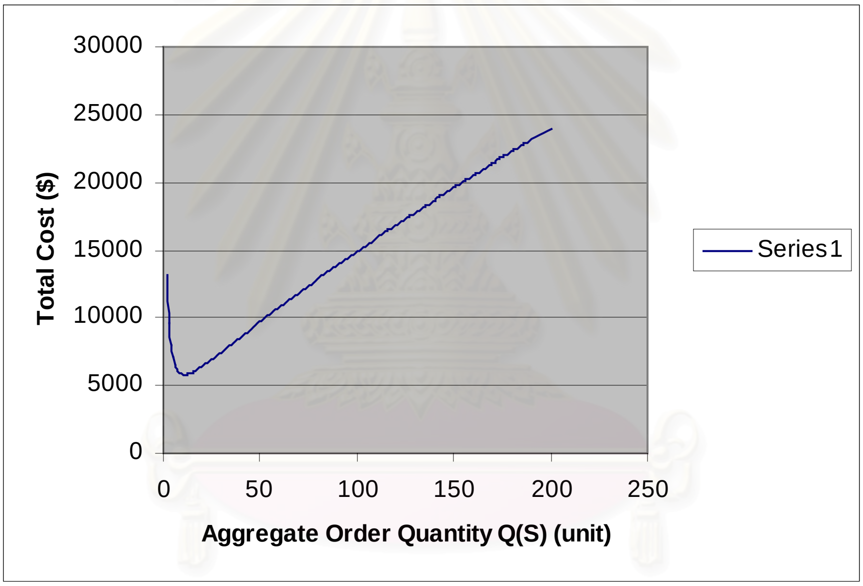

Figure 6.1. Relationship between the total cost and the aggregated order quantity

From Figure 6.1, the total integrated cost function is convex in the range of Q=0 to $Q \approx 40$ which is the inflection point. This can be easily proved which is shown in Appendix A. After the inflection point, the cost function is concave. Now a question of “Is the local optimum also the global optimum?” has arisen. This question is answered by proving in Appendix A that the local optimal solution to the problem is also the global optimal solution. In Appendix B, the order quantity determined from (6.2) is compared with the EOQ. That the optimal order quantity Q of the model studied is always smaller than the EOQ is proved.

6.6 Solution Approaches

In this section, the solution methods to solve the integrated inventory-transportation problem under stochastic demands with the minor ordering and stopover cost components added are focused.

6.6.1 Exact Solution Approach and Heuristics

As mentioned before, this problem can be formulated as the set partitioning problem. In addition, adding the minor ordering cost, the stopover cost and the inventory holding cost due to the safety stock to the deterministic model does not impact on the main concept of algorithms proposed in Chapters 4 and 5. As a result, the column generation, the branch-and-price algorithm, the constructive heuristics and the improvement algorithms including VLSN approaches developed in the deterministic case can still be applied to the problem. However, in determining the aggregate replenishment quantity $Q(S)$ of each vehicle, the standard EOQ formula cannot be used because of the inventory holding cost term due to the safety stock. Therefore, the bisection method [see Kincaid and Cheney (1996)] known as the method of interval halving is adopted to solve for the optimal replenishment quantity $Q^*(S)$. Because the local optimum is also the global optimum, $Q^*(S)$ will be determined from the first derivative of the total cost function $f(Q(S))$. From the graph showing the relationship between the first derivative $f^{/}(Q(S))$ and $Q(S)$, it intersects the X-axis at only one point which provides the optimal replenishment quantity $Q^*(S)$ as shown in Figure 6.2. As a result, the bisection method is easily applied to the stochastic problem. In Appendix B, it has been proved that $Q^*(S)$ is always smaller than the EOQ which is $\sqrt{A/B}$. Consequently, the initial interval for halving in the bisection method can be set to $[0, \sqrt{A/B}\,]$.

If the vehicle capacity and frequency constraints are taken into consideration, the optimal total costs and the optimal aggregate replenishment quantity of items in subset S assigned to vehicle i can be obtained from

$$c(S) = L(S)\frac{D(S)}{Q(S)} + \frac{1}{2}h(S)Q(S) + Z_P\sqrt{\frac{Q(S)}{D(S)}}\sum_{j\in S}(h_j\sigma_j) \qquad (6.5)$$

where $L(S) = K + TSP(S) + M(S) + O(S)$

Subject to:

$$D(S) \leq CF$$

$$Q(S) > 0$$

where $$Q(S) = \begin{cases} \dfrac{D(S)}{F} & \text{if} \quad Q^*(S) \leq \dfrac{D(S)}{F} \\ Q^*(S) & \text{if} \quad \dfrac{D(S)}{F} \leq Q^*(S) \leq C \\ C & \text{if} \quad C \leq Q^*(S) \end{cases}$$

and $Q^*(S)$ is the optimal replenishment quantity determined by the bisection method [see Kincaid and Cheney (1996)].

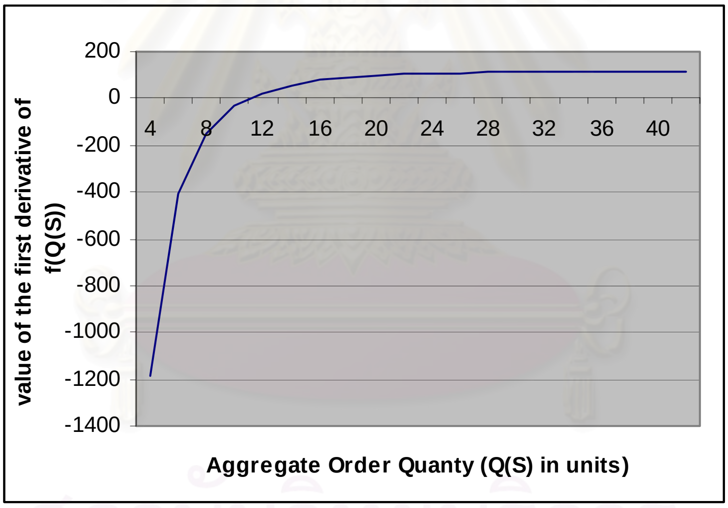

Figure 6.2 Relationship between $f^{/}(Q(S))$ and the aggregated order quantity $Q(S)$

6.6.2 The pricing problem

All the same notations as in the deterministic part will be used. The procedures of the column generation approach and the branch-and-bound strategies used in both deterministic and stochastic models are unchanged so only how to bound on the function $c(z)$ will be discussed. In fact, $L(z)$ is still bounded from below by

$$L(z) \geq K + TSP(J^1) + M(z) + O(z) = L(J^1)$$

to obtain

$$c(z) \geq \underline{c}(z)$$

where

$$\underline{c}(z) = L(J^1)\frac{D(z)}{Q(z)} + \frac{1}{2}h(z)Q(z) + Z_p\sqrt{\frac{Q(z)}{D(z)}}\sum_{j\in z} h_j\sigma_j \tag{6.6}$$

where

$$Q(z) = \begin{cases} \dfrac{D(z)}{F} & \text{if} \quad Q^*(z) \leq \dfrac{D(z)}{F} \\ Q^*(z) & \text{if} \quad \dfrac{D(z)}{F} \leq Q^*(z) \leq C \\ C & \text{if} \quad C \leq Q^*(z) \end{cases}$$

The optimal $Q^*(z)$ is computed by using the bisection method.

Now the following problem can be solved

$$\max \sum_{j=1}^{n} \hat{\mu}_j z_j - \underline{c}(z)$$

subject to

$$\sum_{j=1}^{n} D_j z_j \leq CF$$

$$z_j = 0 \qquad \text{for } j \in J^0$$

$$z_j = 1 \qquad \text{for } j \in J^1$$

$$z_j \in \{0,1\} \quad \text{for } j \in J$$

by applying the same algorithms developed in the deterministic case.

CHAPTER 7
COMPUTATIONAL EXPERIMENTS

In order to measure the performance of the proposed algorithms, computational experiments on both deterministic and stochastic models have been performed using a randomly generated set of test instances. In this chapter, the performance of the column generation and exact branch-and-price algorithm on a number of small instances will be discussed first. As was expected, the branch-and-price method is very time-consuming. However, these tests can be used to assess the tightness of the lower bound obtained using the column generation solution to the LP-relaxation of the set partitioning formulation. To reduce the computational time used in the branch-and-price method, the solution obtained from the I-VLSN algorithm is used as an upper bound at each node. In the remaining experiments, the performance of the proposed constructive and improvement heuristics will be compared, and the quality of the solutions obtained will be assessed by comparing the heuristic costs to the column generation lower bound on the optimal costs.

7.1 Generation of the test instances and implementation

Both the deterministic and stochastic models are tested by using random generated data, like other literature [See Chan and Simchi-Levi (1998)]. For every experiment, random instances have been generated as follows: The demand rate (demand per unit time) for each item is randomly generated from the uniform distribution on [100,300], and the inventory holding cost rate (cost per unit per unit time) for each item is randomly generated from the uniform distribution on [1,15]. The items are randomly assigned to one of 10 suppliers, while ensuring that each of these suppliers manufactures at least one item. The locations of the warehouse and suppliers are generated uniformly in the square $[0,20]^2 \subset R^2$, and Euclidean distances are used to measure transportation costs, with unit cost per unit distance traveled.

A base case has been defined as follows: Each vehicle has a capacity of C =150 units, the fixed transportation cost is set to K =50, and the maximum number of trips allowed per time unit by each vehicle is F =10. The size of an instance is identified by the number of items, n, and the number of vehicles, m.

In the experiments on the deterministic model, four different sizes have been considered: 15 items and 3 vehicles, 30 items and 6 vehicles, 40 items and 8 vehicles, and 50 items and 10 vehicles, where the number of vehicles has been chosen to ensure that a solution with this number of vehicles indeed exists given the capacity constraints. In addition, a sensitivity analysis of the computational results has been performed for changes in the capacity, frequency, and fixed cost parameters, where the number of available vehicles has been adjusted accordingly.

For the stochastic case, the model with minor ordering and stopover costs included and the one without these costs are examined. For each model, four different sizes have been chosen: 15 items and 3 vehicles, 20 items and 4 vehicles, 25 items and 5 vehicles, and 30 items and 6 vehicles. The reason for testing on smaller problems for the stochastic model is that the column generation approach to obtain the lower bound takes much longer time than the one in the deterministic model. For the case with minor ordering and stopover costs, the minor ordering and stopover costs are randomly generated from the uniform distribution on [0,5]. The base case for this model is defined as follows: C =150, F =10, K =50, p =0.975 and the percent of demands for the standard deviation for all items is set to 20 %. In addition to analyzing the effect of varying the capacity, frequency, and fixed cost parameters, the changes in the service level and the standard deviation of demands are also examined. However, a sensitivity analysis has been conducted for only the stochastic model with the minor ordering and stopover costs included.

All the algorithms and heuristics have been implemented in the C++ programming language on a PC with a 1.80 GHz Intel Pentium 4 CPU and 240 MB of RAM. The CPLEX 8.1 solver is used to obtain the optimal solution to the TSP and the solutions to the LP-relaxation of the set partitioning problem in the column generation procedure.

7.2 Experiments on the deterministic model

7.2.1 Performance of branch-and-price and quality of the lower bounding procedure

The goal of the first experiment is to test the computational efficiency of the branch-and-price algorithm and assess the quality of the lower bound obtained from the column generation procedure. For this experiment, the branch-and-price algorithm has been used to obtain optimal solutions for all 15-item instances, as well as the corresponding lower bounds. The results of this test on the 10 instances are given in Table 7.1.

It can be concluded that the branch-and-price algorithm is very time-consuming, even for these small problem instances. However, the column generation procedure is able to find a reasonably tight lower bound to the optimal costs efficiently, with an average gap between the optimal cost and the lower bound of approximately 2.5 %, and an average computational time of 13.5 CPU seconds.

Problem	LB	Optimal	%deviation
1	2778.1	2778.1	0.00
2	2645.8	2645.8	0.00
3	2545.2	2598.6	2.10
4	2669.7	2761.2	3.43
5	2557.6	2726.0	6.58
6	2563.9	2699.3	5.28
7	2511.3	2526.7	0.61
8	2378.3	2426.2	2.02
9	2556.2	2577.2	0.82
10	2710.0	2825.5	4.26
avg	2591.6	2656.4	2.51
max	2778.1	2825.5	6.58
Time(sec)	13.5	13571.3	

Table 7.1 Optimal cost vs. lower bound (n = 15, m = 3).

7.2.2 Performance of the constructive heuristics and improvement algorithms

In the second set of experiments on the deterministic model, the goal is to assess the quality of the solutions obtained by the constructive heuristics and improvement algorithms. Firstly, each instance is solved by using the constructive heuristics. Next, these initial solutions are improved by using the OSM, SE, OSM-SE, SE-OSM, I-VLSN and S-VLSN improvement algorithms. Then the objective function value found by the heuristic is compared to the lower bound for each instance using the column generation approach. In addition, for the instances with $n=15$ items the heuristic costs are also compared to the optimal costs. The results of these experiments are given in Tables 7.2-7.4. From Table 7.2, the DR heuristic combined with the I-VLSN algorithm provides the best solution to the problem with $n=15$ and $m=3$. The average cost is only 0.76% higher than the optimal solution.

The average and maximum cost deviations from the lower bound of the solutions obtained by both constructive heuristics and all the improvement algorithms are reported in Tables 7.2-7.4 and Figures 7.1-7.6. The results show that the DR heuristic outperforms the AII heuristic both with respect to the average and the maximum error over all 10 instances. In addition, the gap between the lower bound and the solution obtained from the DR heuristic improves with increasing problem size, while this effect is absent from the AII heuristic. This can be explained that in the AII heuristic items of the same supplier may not be assigned to the same vehicle because an item is randomly selected for insertion. When the number of items increases, it is more unlikely that all items of the same supplier are assigned to the same vehicle. On the other hand, in the DR heuristic, items of the same supplier are assigned to the same vehicle if the vehicle capacity is still satisfied. From Tables 7.2-7.4, the solutions improved by the SE, OSM-SE and SE-OSM algorithms are not significantly different for the DR and AII heuristics. In other words, these improvement algorithms provide solutions with almost the same level of quality, regardless of constructive heuristics used to obtain the initial solutions. The SE approach seems to perform better than the OSM method. However, the OSM-SE and the SE-OSM approaches perform equally well. With respect to the VLSN improvement algorithms, I-VLSN outperformed S-VLSN for all cases, except for the largest problem with n =50 and the starting solution found by AII. In fact, the I-VLSN works better than any other improvement algorithms. The I-VLSN algorithm based on the DR solution on average provided the solution with smallest error, with a maximum average error of 3.28 %. Moreover, as for the DR heuristic, the gap between the lower bound and the solution obtained from the I-VLSN decreases when the number of items increases.

Heuristics		Error bound (%)						
		Heuristic	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
DR	avg	5.29	3.51	3.66	2.11	2.41	0.76	2.37
	max	14.11	14.11	9.04	9.04	9.04	5.26	9.04
AII	avg	7.31	5.23	3.13	2.29	2.49	1.05	2.89
	max	15.99	15.70	8.37	6.73	6.73	6.04	9.04

Table 7.2 Error with respect to optimal solution of solutions obtained from the constructive heuristics and improvement algorithms (n = 15, m = 3).

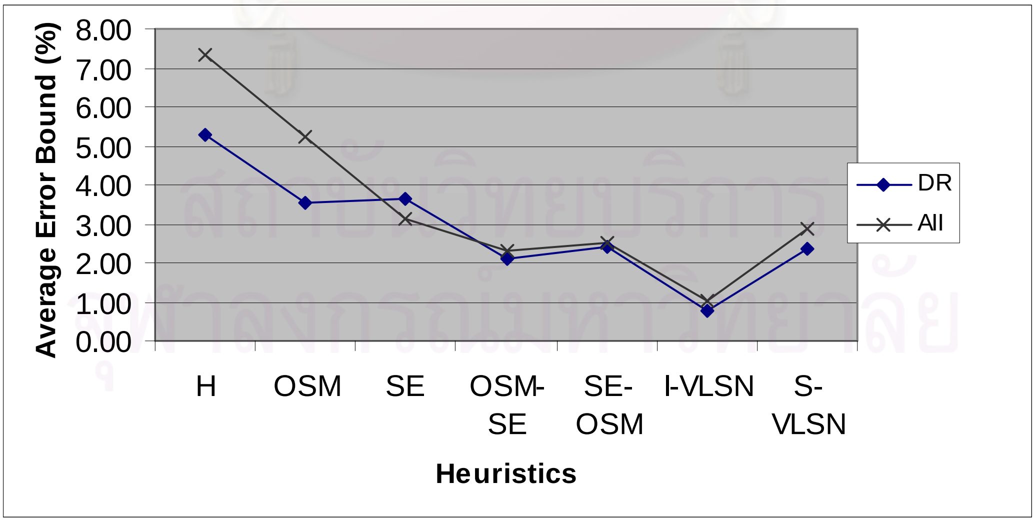

Figure 7.1 Average error with respect to optimal solution of solutions obtained from the constructive heuristics (H=heuristic) and improvement algorithms (n = 15, m = 3).

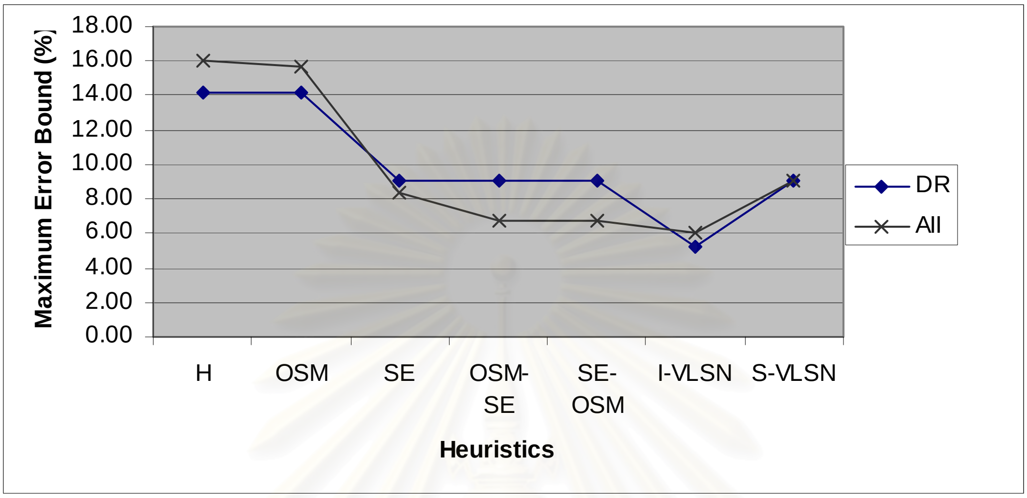

Figure 7.2 Maximum error with respect to optimal solution of solutions obtained from the constructive heuristics (H=heuristic) and improvement algorithms (n = 15, m = 3).

Size			Error bound (%)						
n	m		DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
15	3	avg	8.81	6.98	6.47	4.67	5.18	3.28	4.91
		max	14.11	14.11	9.26	9.04	9.04	6.92	9.68
30	6	avg	8.25	6.84	5.93	6.10	5.62	2.84	5.63
		max	11.32	8.78	8.49	8.08	8.08	6.73	7.46
40	8	avg	5.53	4.82	4.47	4.04	3.88	2.69	3.83
		max	8.21	7.80	7.03	6.15	6.08	3.20	6.08
50	10	avg	5.43	4.68	4.69	4.06	4.24	2.37	3.70
		max	8.48	6.70	7.40	5.63	6.31	3.31	5.64

Table 7.3 Error bounds of solutions obtained from the DR heuristic and improvement algorithms.

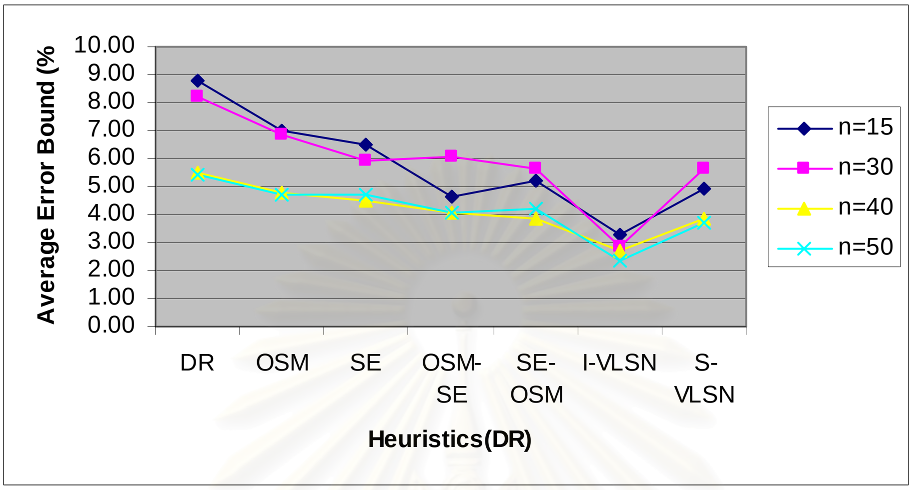

Figure 7.3 Average error of solutions obtained from the DR heuristic and improvement algorithms.

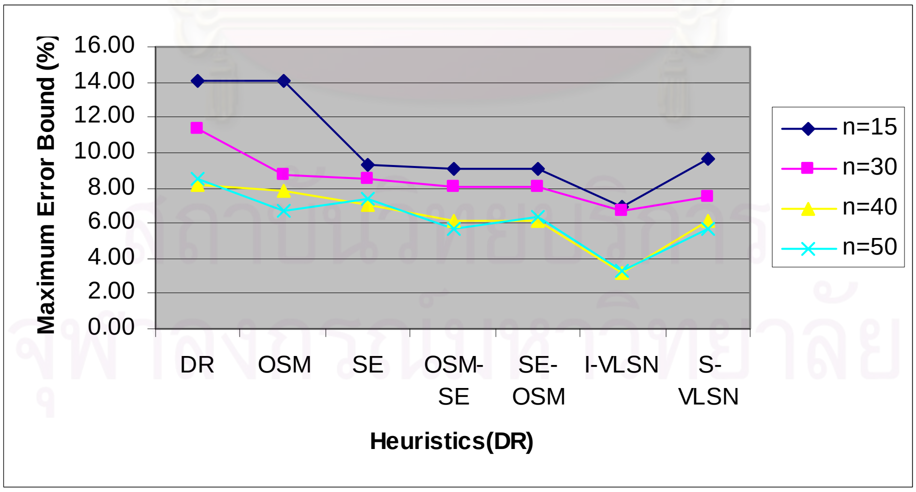

Figure 7.4 Maximum error of solutions obtained from the DR heuristic and improvement algorithms.

Size			Error bound (%)						
n	m		AII	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
15	3	avg	9.98	7.84	5.69	4.84	5.05	3.57	5.45
		max	16.94	15.70	9.39	9.82	9.39	6.58	9.33
30	6	avg	12.18	10.21	6.06	5.21	5.55	3.89	4.94
		max	16.71	13.97	9.11	7.35	9.11	6.51	6.46
40	8	avg	13.00	9.80	4.78	5.14	4.74	3.64	4.30
		max	16.19	14.45	7.46	6.04	7.44	4.58	6.16
50	10	avg	13.31	11.24	4.24	4.24	4.17	4.04	3.76
		max	17.74	14.07	5.71	6.61	5.71	7.44	5.00

Table 7.4 Error bounds of solutions obtained from the AII heuristic and improvement algorithms.

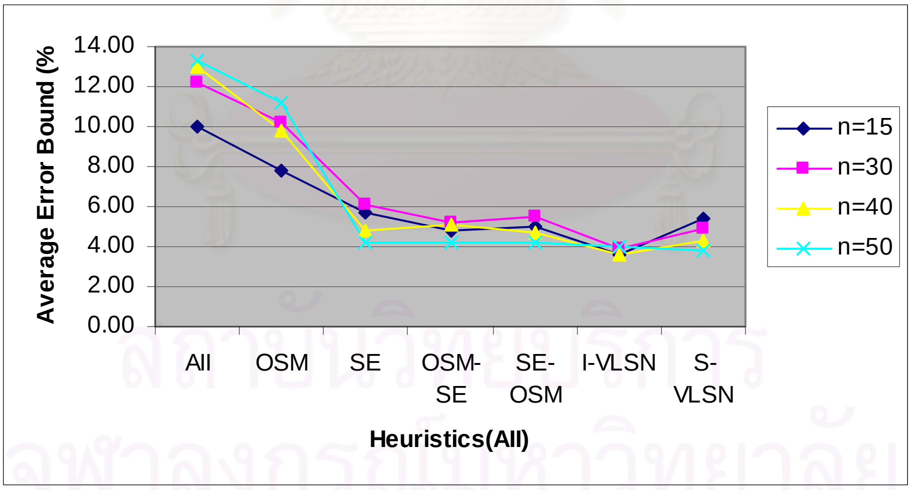

Figure 7.5 Average error of solutions obtained from the AII heuristic and improvement algorithms.

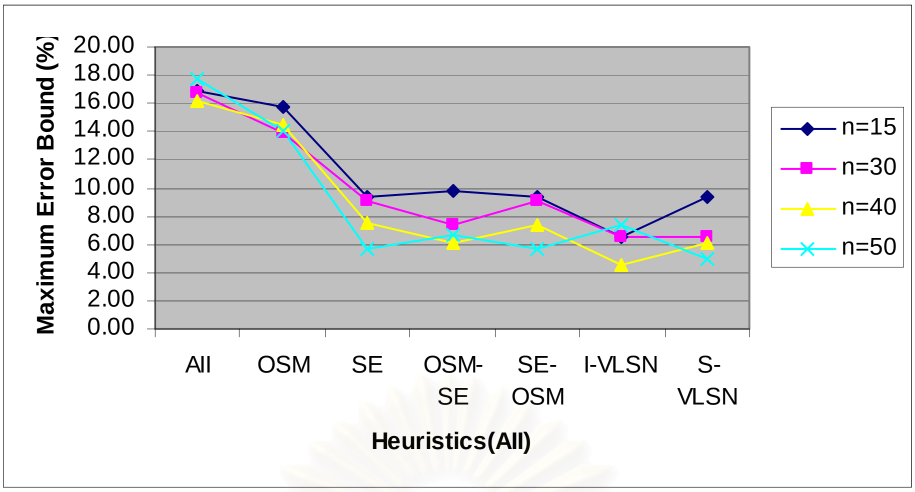

Figure 7.6 Maximum error of solutions obtained from the AII heuristic and improvement algorithms.

Finally, the computation time needed for finding the lower bound as well as for the heuristics is provided in Tables 7.5-7.6. It is immediately seen that the heuristics are extremely efficient, finding a solution in usually less than a second. On the other hand, the time needed for computing the lower bound is very time-consuming, increasing dramatically as the problem size increases. However, as seen in Tables 7.3-7.4, the solution quality of the heuristics is very good, and improves as the problem size increases. Therefore, the need for computing the lower bound in practice decreases as the problem size increases. In the view of the VLSN algorithms, the I-VLSN requires more computational time than the S-VLSN does. This is because there are more nodes in the I-VLSN. As a result, constructing and updating the improvement graph in the I–VLSN takes more computational effort.

Size		Average computational time (sec)							
n	m	LB	DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
15	3	13.5	0.000	0.000	0.000	0.000	0.005	0.017	0.006
30	6	849.5	0.000	0.036	0.002	0.037	0.003	0.136	0.008
40	8	14207.6	0.002	0.009	0.000	0.011	0.000	0.217	0.011
50	10	48672.0	0.003	0.002	0.003	0.005	0.005	0.433	0.016

Table 7.5 Average computational time for the column generation method (LB), the DR heuristic and the improvement algorithms.

Size		Average computational time (sec)							
n	m	LB	AII	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
15	3	13.5	0.000	0.034	0.002	0.036	0.003	0.027	0.011
30	6	849.5	0.000	0.008	0.002	0.011	0.002	0.235	0.038
40	8	14207.6	0.002	0.014	0.002	0.022	0.003	0.525	0.081
50	10	48672.0	0.006	0.013	0.006	0.019	0.006	1.378	0.133

Table 7.6 Average computational time for the column generation method (LB), the AII heuristic and the improvement algorithms.

7.2.3 Sensitivity analysis

In order to investigate the impact of changing some of the problem parameters on the computational performance of the proposed heuristics and improvement algorithms, a third set of experiments have been conducted. Using the 40-item instances as a base case, the vehicle capacity C, the maximum number of trips per vehicle F and the fixed (dispatching and joint ordering) costs K have been varied from 100 to 200, from 8 to 12, and from 0 to 100 respectively. In each case, the number of vehicles is adjusted to suit the capacity constraints. Based on the results in Tables 7.3-7.4, the best constructive heuristic, DR, and all the improvement algorithms have been chosen for performing the sensitivity analysis.

The results in Tables 7.7-7.9 and Figures 7.7-7.12 show that the gap between the lower bound and the solution obtained from almost all methods decreases on average for problems with smaller vehicle capacity, smaller maximum number of trips per time unit, or smaller fixed transportation cost. However, the error of the solution obtained from the I-VLSN method decreases as the fixed transportation cost increases. The error bound also seems to be more stable across instances when the vehicle capacity or the maximum number of trips is smaller. In addition, the changes of the maximum number of trips allowed seem to have less impact on the average error. As for the changes in the fixed costs K, it is surprising that the I-VLSN approach is outperformed by the OSM-SE, SE-OSM and S-VLSN algorithms when $K = 0$.

C	m		Error bound (%)						
			DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
100	12	avg	5.34	4.52	4.47	4.23	4.28	1.91	4.39
		max	6.66	5.97	5.83	5.83	5.83	3.05	5.97
150	8	avg	5.53	4.82	4.47	4.04	3.88	2.69	3.83
		max	8.21	7.80	7.03	6.15	6.08	3.20	6.08
200	6	avg	7.45	7.07	6.46	6.24	6.21	2.89	5.63
		max	11.25	11.25	8.99	8.38	8.38	4.59	8.00

Table 7.7 Error bounds when the vehicle capacity C is varied.

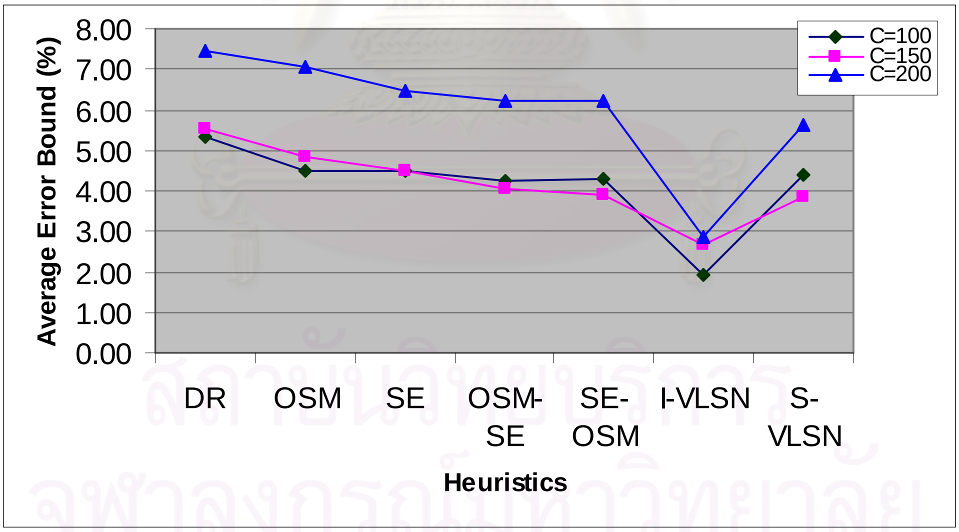

Figure 7.7 Average error bounds when the vehicle capacity C is varied.

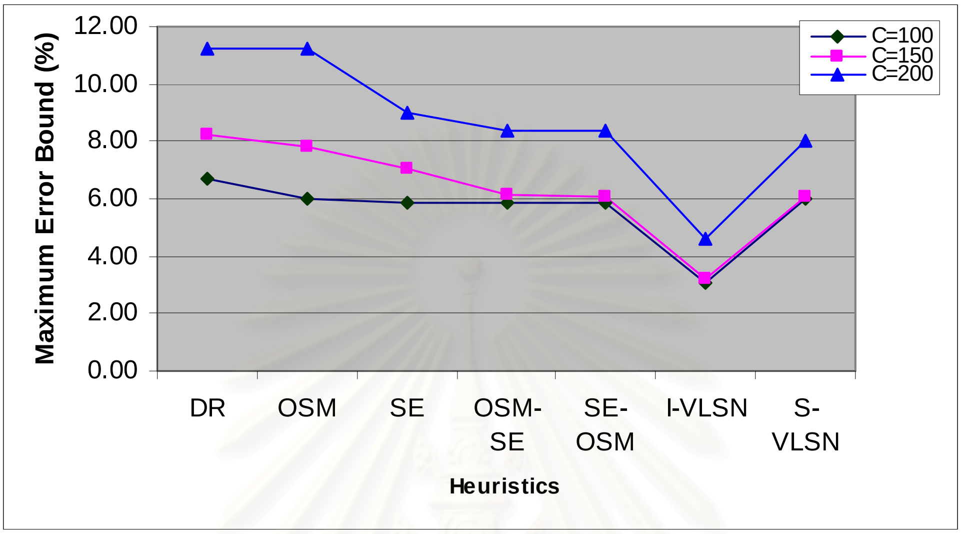

Figure 7.8 Maximum error bounds when the vehicle capacity C is varied.

F	m		Error bound (%)						
			DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
8	10	avg	5.45	4.12	4.72	3.83	4.11	2.24	4.02
		max	6.63	5.79	5.81	5.35	5.46	2.67	5.35
10	8	avg	5.53	4.82	4.47	4.04	3.88	2.69	3.83
		max	8.21	7.80	7.03	6.15	6.08	3.20	6.08
12	7	avg	5.12	4.35	4.14	4.09	3.82	2.94	3.76
		max	10.00	6.41	4.97	6.08	4.94	4.50	4.89

Table 7.8 Error bounds when the maximum number of trips allowed F is varied.

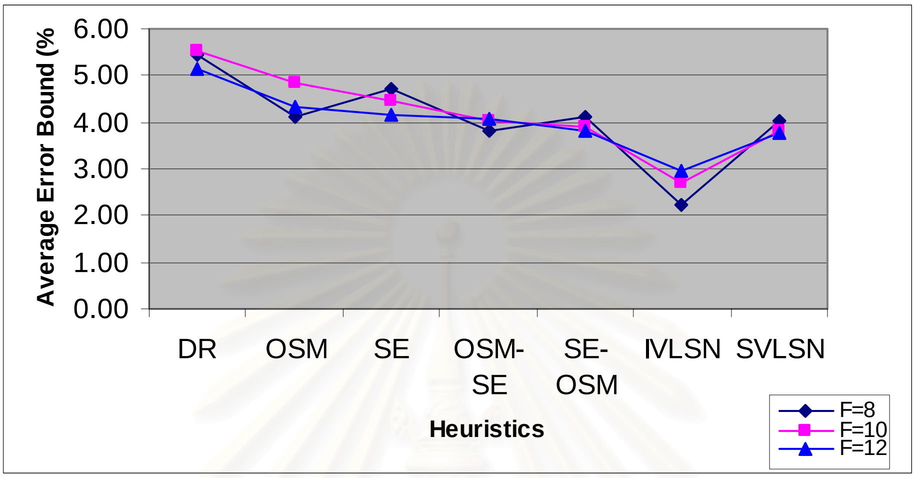

Figure 7.9 Average error bounds when the maximum number of trips allowed F is varied.

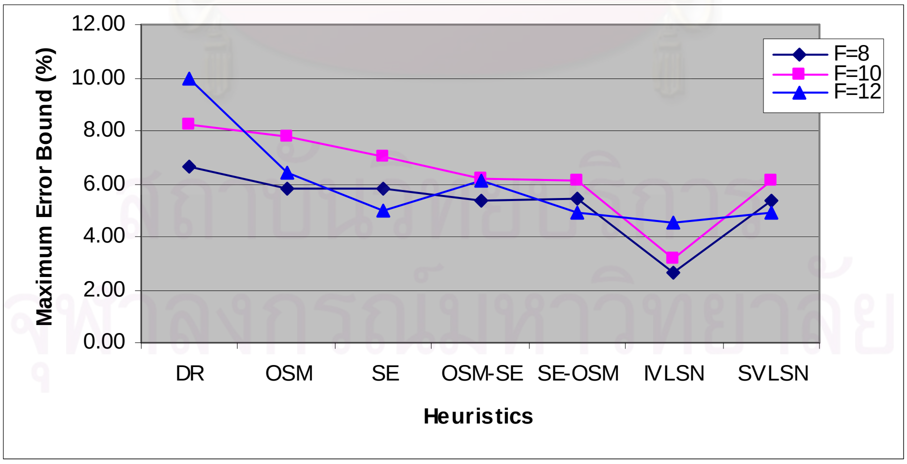

Figure 7.10 Maximum error bounds when the maximum number of trips allowed F is varied.

K		Error bound (%)						
		DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
0	avg	4.34	3.07	2.53	1.99	2.12	3.49	2.42
	max	7.07	4.93	5.88	3.76	4.08	5.96	4.36
20	avg	5.26	4.77	4.04	3.83	3.65	3.29	3.36
	max	7.93	7.93	5.77	5.68	5.68	5.95	5.12
50	avg	5.53	4.82	4.47	4.04	3.88	2.69	3.83
	max	8.21	7.80	7.03	6.15	6.08	3.20	6.08
100	avg	4.06	3.55	3.47	3.00	3.03	2.04	2.98
	max	6.79	5.64	5.94	5.06	5.06	2.95	5.04

Table 7.9 Error bounds when the fixed transportation cost K is varied.

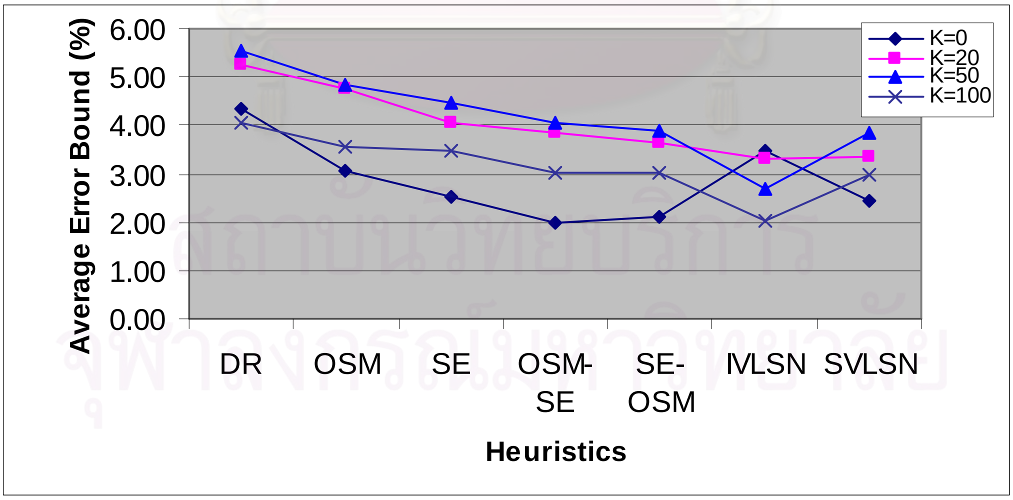

Figure 7.11 Average error bounds when the fixed transportation cost K is varied.

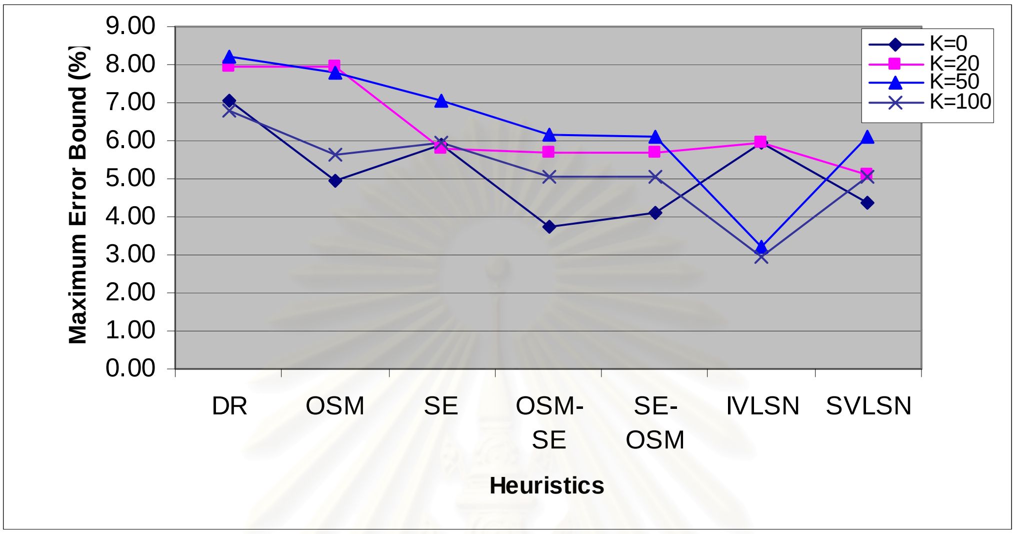

Figure 7.12 Maximum error bounds when the fixed transportation cost K is varied.

7.3 Experiments on the stochastic model

7.3.1 Performance of branch-and-price and quality of the lower bounding procedure

In this experiment, the case where the minor ordering and stopover costs are not considered and the case where the minor ordering and stopover costs are included in the model are investigated and for each case, ten of 15-item instances are tested. The results of both cases are shown in Tables 7.10-7.11. The branch-and-price algorithm is still very time-consuming for determining the optimal solution to the problem of the stochastic model. However, the column generation approach can find a tighter lower bound, with an average gap between the optimal cost and the lower bound of approximately 1.7 % for the case without the minor ordering and stopover costs and only 1.31 % for the case with the minor ordering and stopover costs.

Problem	LB	Optimal	%deviation
1	6193.9	6193.9	0.00
2	5501.6	5501.6	0.00
3	5332.2	5412.0	1.50
4	5612.8	5766.4	2.74
5	5676.9	5866.1	3.33
6	5270.1	5477.1	3.93
7	5480.8	5498.7	0.33
8	4927.4	5006.9	1.61
9	5658.3	5681.6	0.41
10	5743.0	5921.8	3.11
avg	6193.9	5632.6	1.70
max	5539.7	6193.9	3.93
Time(sec)	14.9	14357.9	

Table 7.10 Optimal cost vs. lower bound for the stochastic case without minor ordering and stopover costs (n = 15, m = 3).

Problem	LB	Optimal	%deviation
1	6852.4	6852.4	0.00
2	6279.4	6279.4	0.00
3	5990.5	6056.1	1.10
4	6202.8	6316.4	1.83
5	6340.9	6519.1	2.81
6	5970.3	6172.2	3.38
7	6106.9	6116.2	0.15
8	5481.0	5549.9	1.26
9	6278.6	6292.7	0.23
10	6453.8	6602.3	2.30
avg	6195.7	6852.4	1.31
max	6852.4	6275.7	3.38
Time(sec)	36.9	5499.7	

Table 7.11 Optimal cost vs. lower bound for the stochastic case with minor ordering and stopover costs (n = 15, m = 3).

7.3.2 Performance of the constructive heuristics and improvement algorithms

Firstly, the model without the minor ordering and stopover costs is tested. The purpose of these experiments is the same as the one for the deterministic model. In Table 7.12 and Figures 7.13-7.14 showing the error with respect to optimal solution for small problem instances with n =15, the AII heuristic seems to outperform the DR heuristic. The solution obtained from the AII heuristic and improved by the I-VLSN algorithm is only 0.36 % over the optimal solution. However, with the same reason mentioned before, when the problem size gets larger, the AII heuristic is outperformed by the DR heuristic as shown in Tables 7.13-7.14 and Figures 7.15-7.18 but its performance in the stochastic model is better than in the deterministic model. For the improvement algorithms, the I-VLSN is still the best algorithm. It provides the solution with maximum average error of 5.07 %. The SE-OSM performs better than the OSM-SE but its performance is approximately as well as the S-VLSN. In contrast to the deterministic model, when the problem size increases, the gap between the lower bound and the solution obtained from the heuristics and improvement algorithms seem to increase.

Hueristic		Error bound (%)						
		Heuristic	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
DR	avg	3.71	2.63	2.60	1.48	1.11	0.51	0.64
	max	8.09	8.09	5.85	5.44	5.60	3.88	2.66
AII	avg	2.77	2.32	0.85	0.80	0.76	0.36	0.98
	max	6.68	6.68	2.10	2.10	2.10	0.95	2.37

Table 7.12 Error with respect to optimal solution of solutions obtained from the constructive heuristics and improvement algorithms for the stochastic case without minor ordering and stopover costs (n = 15, m = 3).

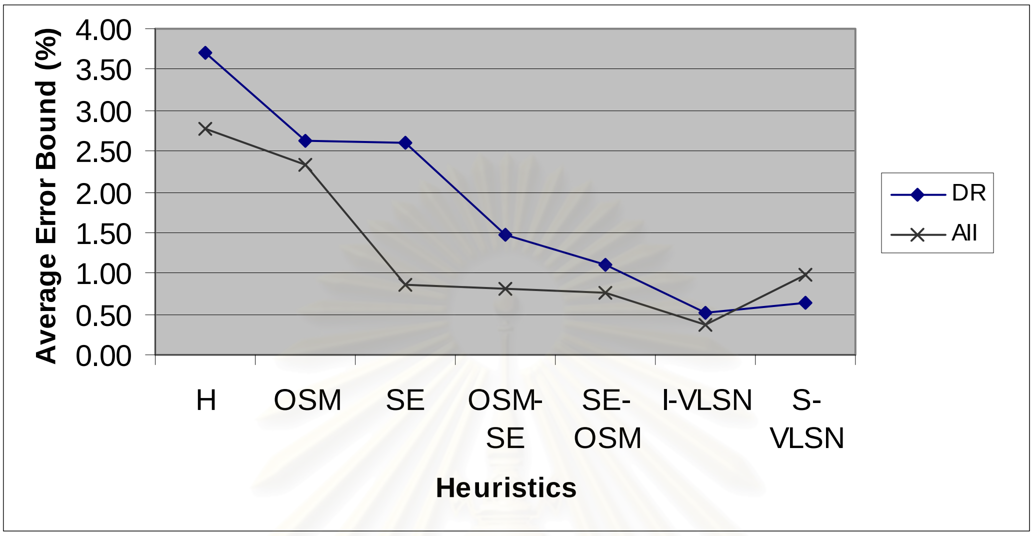

Figure 7.13 Average error with respect to optimal solution of solutions obtained from the constructive heuristics and improvement algorithms for the stochastic case without minor ordering and stopover costs (n = 15, m = 3).

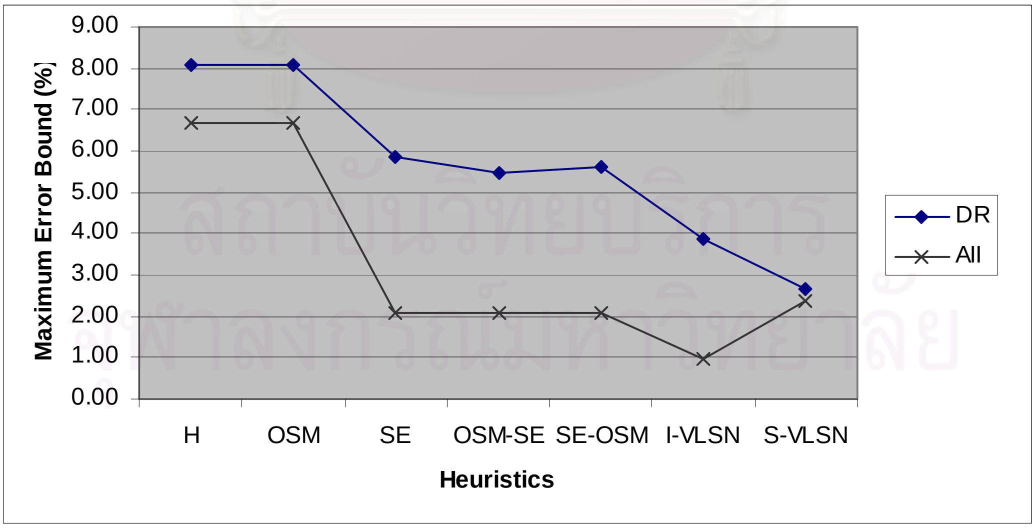

Figure 7.14 Maximum error with respect to optimal solution of solutions obtained from the constructive heuristics and improvement algorithms for the stochastic case without minor ordering and stopover costs (n = 15, m = 3).

Size			Error bound (%)						
n	m		DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
15	3	avg	5.45	4.36	4.31	3.19	2.82	2.21	2.36
		max	8.09	8.09	5.85	5.7	5.7	3.93	6.69
20	4	avg	3.59	3.51	2.9	2.77	2.73	2.14	2.39
		max	6.3	6.3	5.34	5.34	5.34	3.4	4.53
25	5	avg	4.26	4.03	3.64	3.48	3.52	3.19	3.42
		max	6.31	5.89	5.4	5.4	5.4	4.19	5.31
30	6	avg	4.99	4.44	3.78	3.62	3.58	3.27	3.96
		max	6.97	5.77	5.19	5.19	4.86	5.07	6.61

Table 7.13 Error bounds of solutions obtained from the DR heuristic and improvement algorithms for the stochastic case without minor ordering and stopover costs.

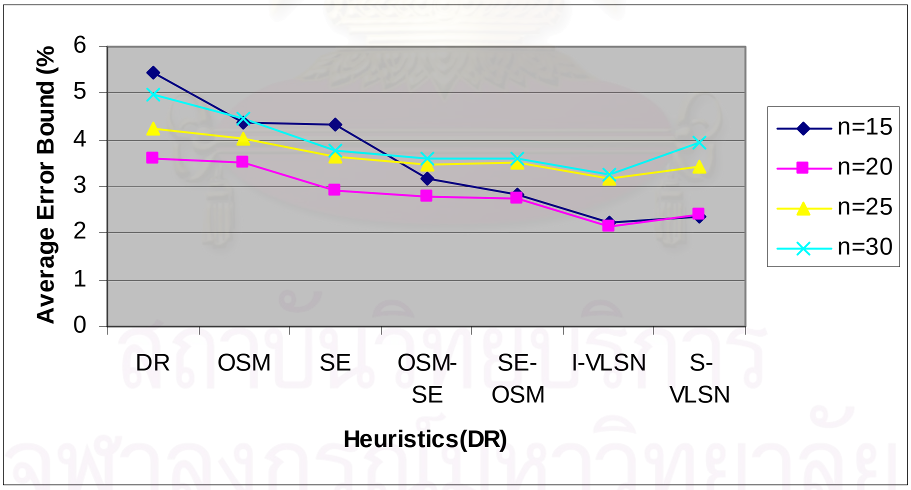

Figure 7.15 Average error bounds of solutions obtained from the DR heuristic and improvement algorithms for the stochastic case without minor ordering and stopover costs.

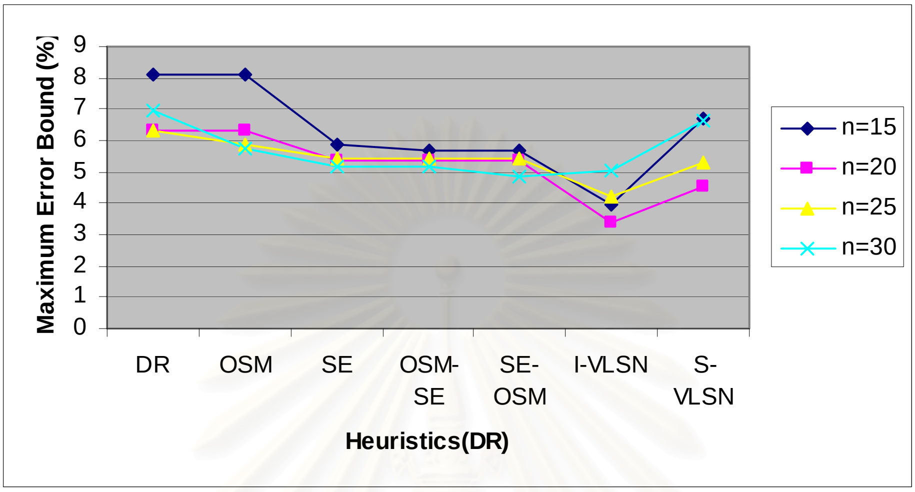

Figure 7.16 Maximum error bounds of solutions obtained from the DR heuristic and improvement algorithms for the stochastic case without minor ordering and stopover costs.

Size			Error bound (%)						
n	m		AII	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
15	3	avg	4.52	4.06	2.56	2.51	2.47	2.06	2.7
		max	9.6	9.6	5.28	5.28	5.28	3.93	5.56
20	4	avg	4.67	3.57	2.14	2.05	2.11	2.7	2.44
		max	8.19	5.88	4.38	4.68	4.38	4.82	5.39
25	5	avg	8.01	6.2	3.66	3.71	3.65	3.31	3.95
		max	10.47	9.77	5.52	5.52	5.52	4.55	7.56
30	6	avg	7.45	6.14	3.43	3.41	3.17	3.16	3.76
		max	8.92	7.74	4.95	5.68	4.75	4.48	6.37

Table 7.14 Error bounds of solutions obtained from the AII heuristic and improvement algorithms for the stochastic case without minor ordering and stopover costs.

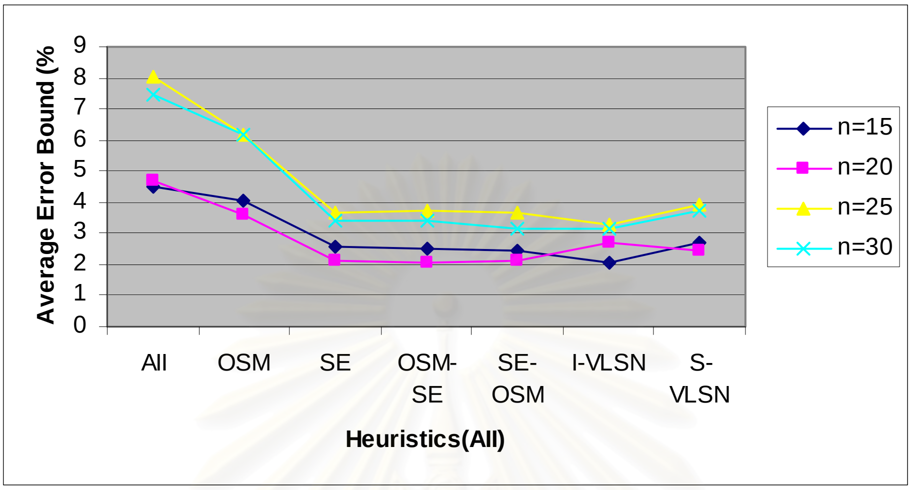

Figure 7.17 Average error bounds of solutions obtained from the AII heuristic and improvement algorithms for the stochastic case without minor ordering and stopover costs.

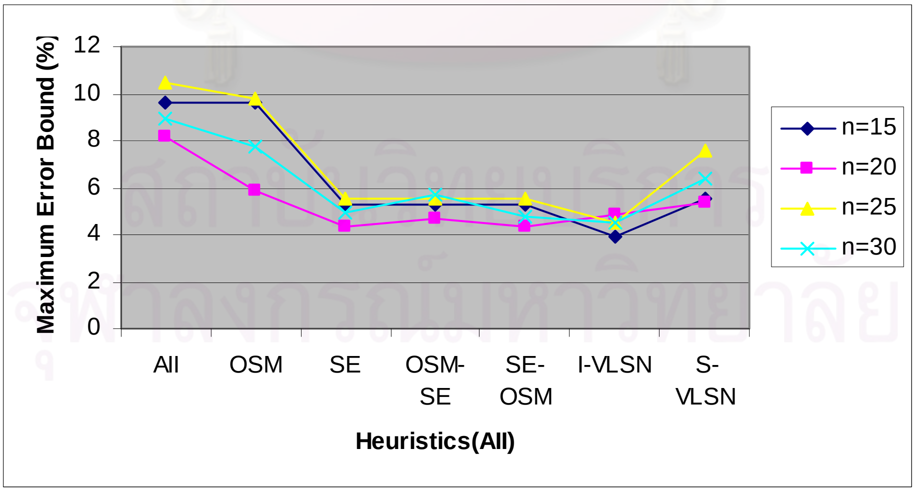

Figure 7.18 Maximum error bounds of solutions obtained from the AII heuristic and improvement algorithms for the stochastic case without minor ordering and stopover costs.

Tables 7.15-7.16 report the average computation time needed for finding the lower bound and for obtaining solutions by the heuristics and improvement algorithms. All the algorithms still perform very efficiently. The longest average time to compute a near-optimal solution by the I-VLSN is only 0.0842 second for instances with n =30 and m =6. However, the time needed for determining the lower bound is much longer than the one in the deterministic model, especially when the problem size increases.

Size		Average computational time (sec)							
n	m	LB	DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
15	3	14.9	0.0102	0.0007	0.0095	0.012	0.014	0.0111	0.0072
20	4	161.6	0.0423	0.0062	0.0095	0.0201	0.0126	0.0235	0.0104
25	5	3834.2	0.022	0.0045	0.019	0.0313	0.0221	0.0418	0.0188
30	6	11222.5	0.0155	0.0079	0.0191	0.0171	0.0222	0.0842	0.0153

Table 7.15 Average computational time for the column generation method (LB), the DR heuristic and the improvement algorithms for the stochastic case without minor ordering and stopover costs.

Size		Average computational time (sec)							
n	m	LB	AII	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
15	3	14.9	0.0116	0	0.0009	0.0011	0.0012	0.0106	0.0082
20	4	161.6	0.0141	0.0015	0.0032	0.0015	0.0032	0.0328	0.0281
25	5	3834.2	0.0156	0.0016	0.0077	0.0064	0.0077	0.0796	0.0532
30	6	11222.5	0.0469	0.0062	0.0031	0.0094	0.0047	0.1937	0.0767

Table 7.16 Average computational time for the column generation method (LB), the AII heuristic and the improvement algorithms for the stochastic case without minor ordering and stopover costs.

Now the results of the experiments on the stochastic model with the minor ordering and stopover costs included are discussed. From Table 7.17 and Figures 7.19-7.20 reporting average and maximum errors of the heuristic costs compared with the optimal costs for the instances with n =15 and m =3, the VLSN algorithms, especially the I-VLSN, work very well. The I-VLSN gives the smallest error of 0.34

% over the optimal cost when applied to the solutions obtained from the AII heuristic. Like the results in the model not considering the minor ordering and stopover costs, the DR heuristic outperforms the AII heuristic in most cases as shown in Tables 7.18-7.19 and Figures 7.21-7.24. In addition, the I-VLSN still performs better than any other improvement algorithms, with a maximum average error of 2.99 % when working with the AII heuristic. As for the combination of the one and two exchange algorithms, both the OSM-SE and the SE-OSM perform equally and reasonably well, with an average error of less than 3.00 % in almost every case. However, it cannot be concluded at this point that the gap between the lower bound and the solution obtained from the constructive heuristics and all improvement algorithms decreases when the number of items increases.

Hueristic		Error bound (%)						
		Heuristic	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
DR	avg	3.26	2.36	2.01	1.09	0.83	0.35	0.57
	max	7.58	7.58	4.30	4.28	4.08	2.38	2.17
AII	avg	5.49	2.50	1.43	0.93	1.42	0.34	0.60
	max	11.56	5.54	5.81	2.38	5.81	0.84	2.57

Table 7.17 Error with respect to optimal solution of solutions obtained from the constructive heuristics and improvement algorithms for the stochastic case with minor ordering and stopover costs (n = 15, m = 3).

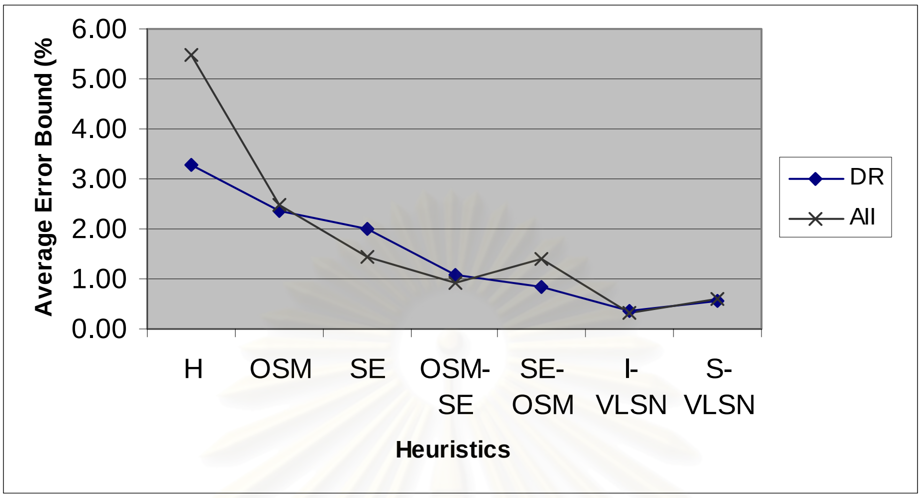

Figure 7.19 Average error with respect to optimal solution of solutions obtained from the constructive heuristics and improvement algorithms for the stochastic case with minor ordering and stopover costs (n = 15, m = 3).

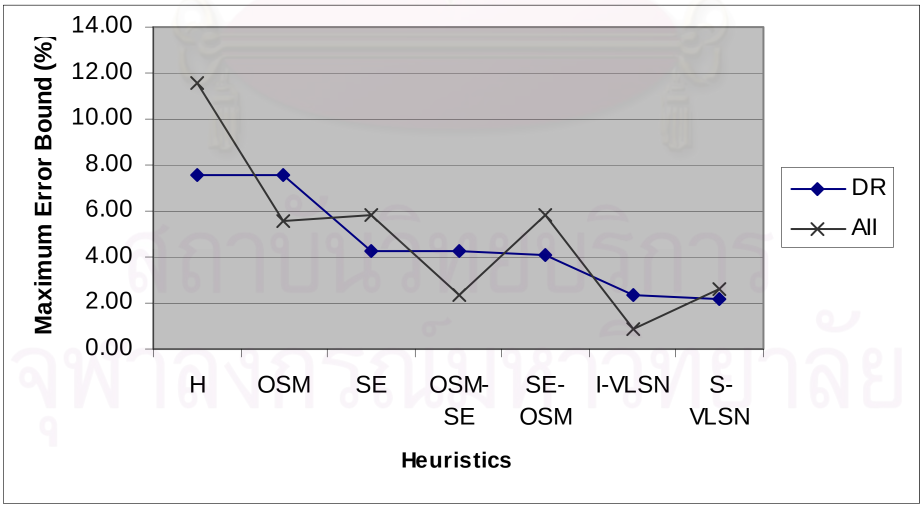

Figure 7.20 Maximum error with respect to optimal solution of solutions obtained from the constructive heuristics and improvement algorithms for the stochastic case with minor ordering and stopover costs (n = 15, m = 3).

Size			Error bound (%)						
n	m		DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
15	3	avg	4.59	3.69	3.33	2.4	2.14	1.66	1.88
		max	7.58	7.58	4.95	4.95	4.95	3.71	5.18
20	4	avg	3.68	2.44	2.6	1.97	2.16	1.57	1.79
		max	6.55	3.92	4.62	3.92	3.92	2.81	3.3
25	5	avg	3.38	3.12	2.62	2.5	2.48	2.16	2.47
		max	5.26	4.68	3.95	3.95	3.95	3.26	3.95
30	6	avg	4.62	3.57	2.91	2.97	2.75	2.62	2.83
		max	5.96	4.72	4.09	4.39	3.88	4.06	3.52

Table 7.18 Error bounds of solutions obtained from the DR heuristic and improvement algorithms for the stochastic case with minor ordering and stopover costs.

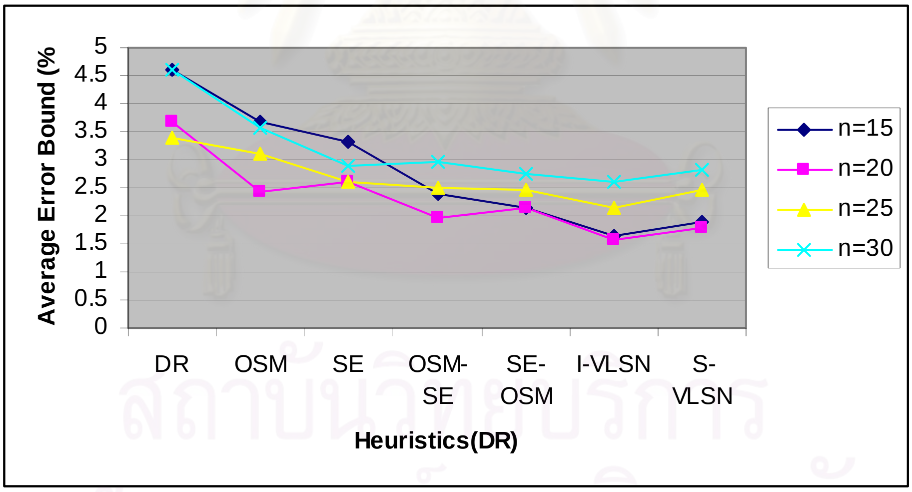

Figure 7.21 Average error bounds of solutions obtained from the DR heuristic and improvement algorithms for the stochastic case with minor ordering and stopover costs.

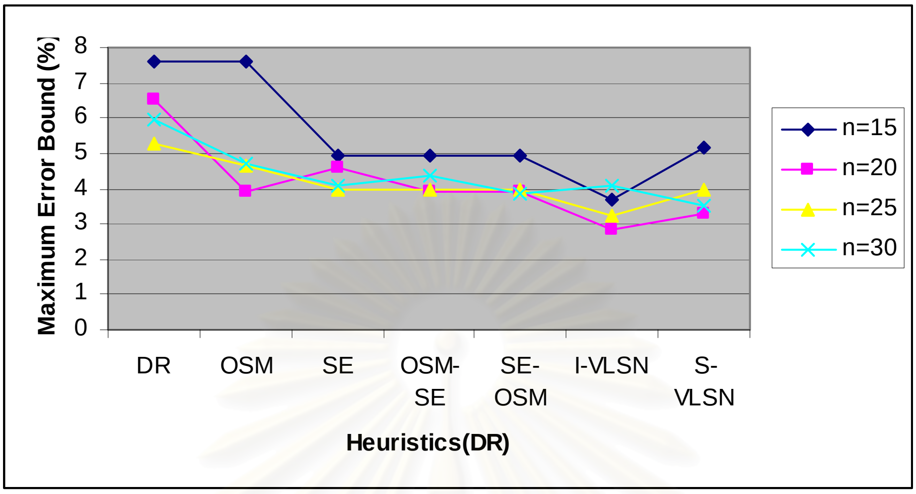

Figure 7.22 Maximum error bounds of solutions obtained from the DR heuristic and improvement algorithms for the stochastic case with minor ordering and stopover costs.

Size			Error bound (%)						
n	m		AII	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
15	3	avg	6.86	3.84	2.75	2.25	2.73	1.65	1.92
		max	11.56	9.11	5.81	4.55	5.81	3.73	4.93
20	4	avg	6.32	4.14	1.81	4.14	1.72	1.99	2.2
		max	8.82	7.29	4.22	7.29	4.22	4.16	7.58
25	5	avg	8.4	5.17	2.75	2.75	2.51	2.84	3.09
		max	10.81	7.16	5.03	4.51	5.03	3.86	8.4
30	6	avg	9.16	5.15	3.35	2.82	3.16	2.99	3.41
		max	10.43	6.2	4.16	3.88	3.76	4.49	5.57

Table 7.19 Error bounds of solutions obtained from the AII heuristic and improvement algorithms for the stochastic case with minor ordering and stopover costs.

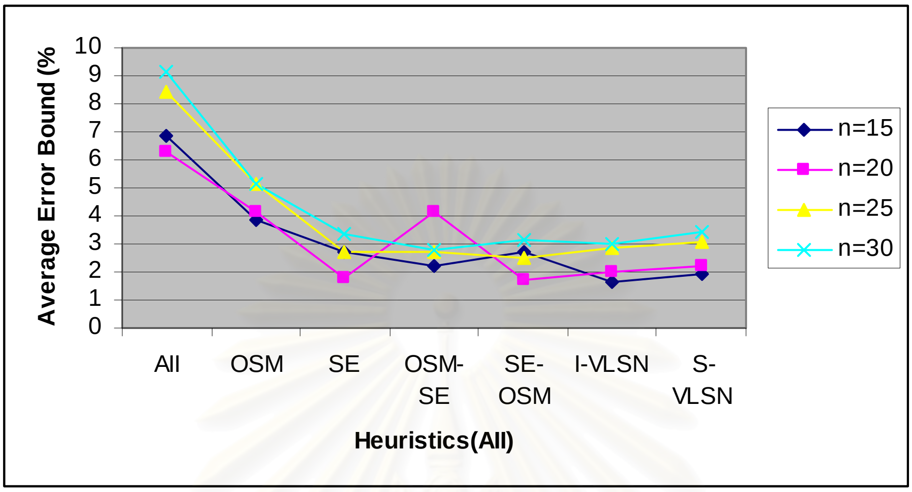

Figure 7.23 Average error bounds of solutions obtained from the AII heuristic and improvement algorithms for the stochastic case with minor ordering and stopover costs.

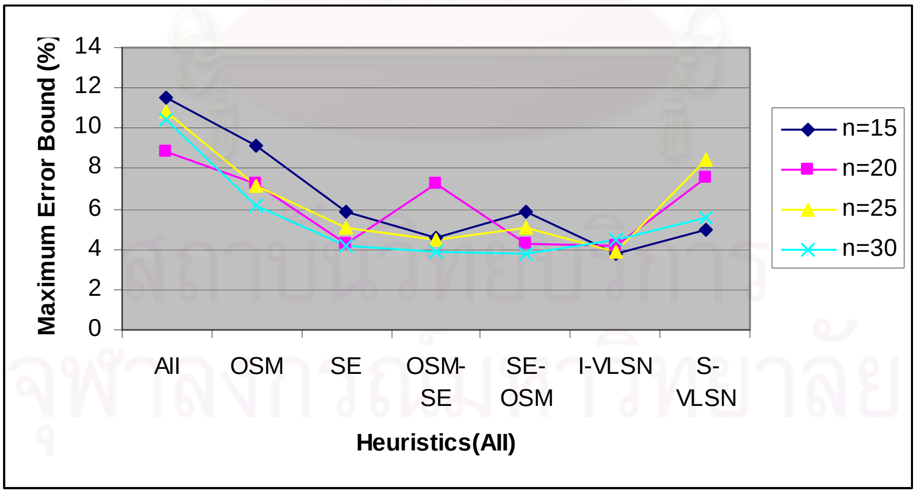

Figure 7.24 Maximum error bounds of solutions obtained from the AII heuristic and improvement algorithms for the stochastic case with minor ordering and stopover costs.

The average computation times corresponding to the tests on the stochastic problems that the minor ordering and stopover costs are included are given in Tables 7.20-7.21. It appears that the average computation time for all heuristics is still small, with less than 0.2 second for all cases. However, for this model, finding the lower bound by the column generation approach takes longer time than in the deterministic model and the stochastic model without these costs included.

Size		Average computational time (sec)							
n	m	LB	DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
15	3	36.9	0.0126	0.0016	0.0156	0.014	0.022	0.0116	0.0078
20	4	254.7	0.0187	0.0031	0.0171	0.0156	0.0202	0.0485	0.05577
25	5	1579.2	0.0141	0.0063	0.0158	0.0233	0.0173	0.0398	0.0095
30	6	16258.2	0.0251	0.0046	0.0328	0.0203	0.0376	0.0905	0.0124

Table 7.20 Average computational time for the column generation method (LB), the DR heuristic and the improvement algorithms for the stochastic case with minor ordering and stopover costs.

Size		Average computational time (sec)							
n	m	LB	AII	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
15	3	36.9	0.0158	0.0359	0	0.0374	0	0.0176	0.0121
20	4	254.7	0.0127	0	0.0093	0.0062	0.0109	0.0391	0.0374
25	5	1579.2	0.0591	0	0.0032	0.0015	0.0032	0.0624	0.0579
30	6	16258.2	0.0547	0.0048	0.0047	0.0108	0.0063	0.1515	0.1124

Table 7.21 Average computational time for the column generation method (LB), the AII heuristic and the improvement algorithms for the stochastic case with minor ordering and stopover costs.

7.3.3 Sensitivity analysis

In the sensitivity analysis, only the stochastic model with the minor ordering and stopover costs included is considered. The 30-item instances are used as a base case. In addition to changing the vehicle capacity C, the maximum number of trips per vehicle F and the fixed costs K as in the deterministic case, the impact of varying the service level p and the standard deviation σ_j of demands for item j on the performance of the proposed heuristics and improvement algorithms is also investigated. The service level and the percent deviation of demands for the standard deviation have been varied from 0.90 to 0.975 and 10% to 30% respectively. The DR heuristic and all the improvement algorithms have been chosen for conducting the sensitivity analysis.

The results in Table 7.22 and Figures 7.25-7.26 reveal that varying the service level has small impact on the performance of all the heuristics. The error bound of the solution obtained from the heuristics slightly decreases when the service level increases. Based on the results in Table 7.23 and Figures 7.27-7.28, it is implied that increasing the standard deviation of demand seems to decrease the gap between the lower bound and the solution obtained from the heuristics. It may be said that the proposed heuristics and improvement algorithms will perform better in a situation that demand uncertainty is high.

The results in Tables 7.24-7.26 and Figures 7.29-7.34 indicate that the gap between the lower bound and the solution obtained from the DR heuristic and improvement algorithms decreases as the vehicle capacity, maximum number of trips per time unit, or fixed transportation cost decreases. These results are the same as the ones in the deterministic model. It can be observed further that the changes of the maximum number of trips allowed and the vehicle capacity have less impact on the error bound than the changes of the fixed cost K does. In addition, the I-VLSN approach is outperformed by the OSM-SE, SE-OSM and S-VLSN algorithms as the fixed cost K decreases.

service level p		Error bound (%)						
		DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
0.9	avg	5.15	3.84	3.46	3.11	3.19	2.52	2.81
	max	6.47	5.40	4.81	4.81	4.67	4.35	3.69
0.95	avg	4.86	3.52	3.32	3.08	3.04	2.58	2.83
	max	6.21	5.15	4.63	4.63	4.48	4.22	3.62
0.975	avg	4.62	3.57	2.91	2.97	2.75	2.62	2.83
	max	5.96	4.72	4.09	4.39	3.88	4.06	3.52

Table 7.22 Error bounds for the stochastic case with minor ordering and stopover costs when the service level p is varied.

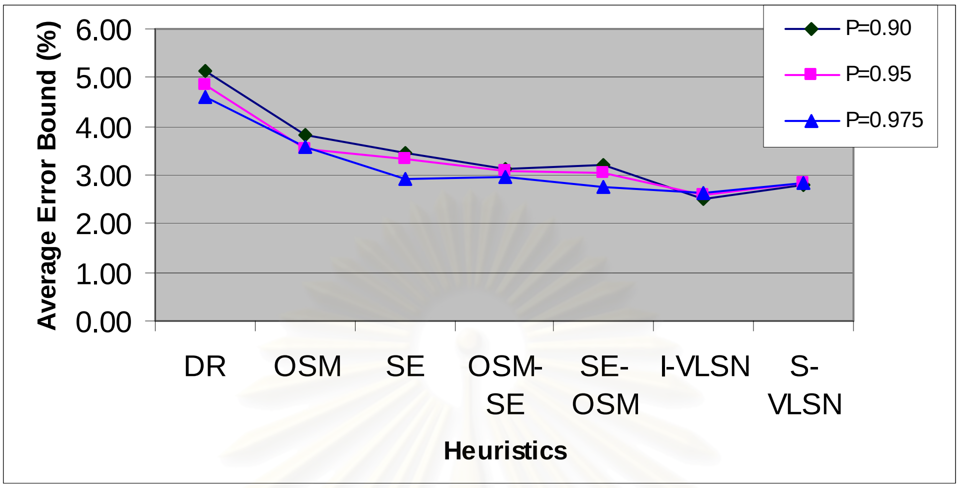

Figure 7.25 Average error bounds for the stochastic case with minor ordering and stopover costs when the service level p is varied.

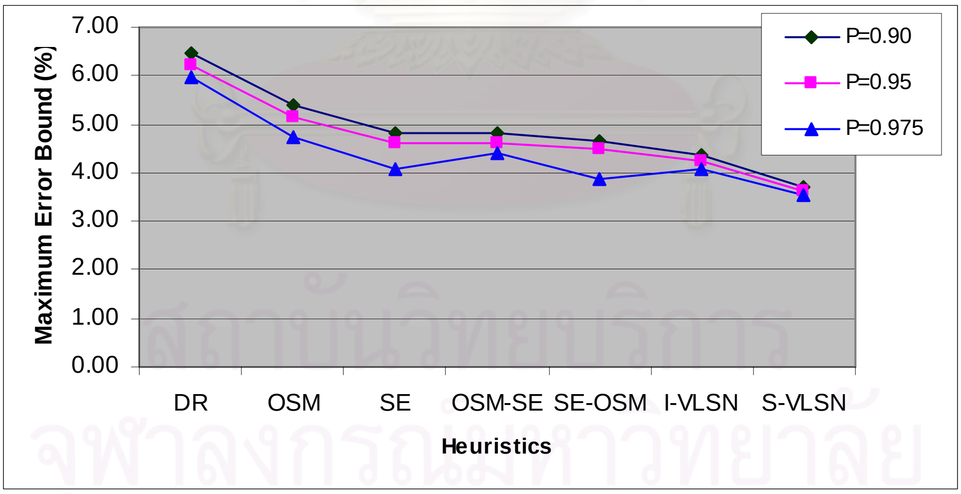

Figure 7.26 Maximum error bounds for the stochastic case with minor ordering and stopover costs when the service level p is varied.

% of demand for standard deviation		Error bound (%)						
		DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
10	avg	5.38	3.95	3.11	3.28	2.83	2.81	3.03
	max	6.61	5.59	4.98	4.98	4.10	4.44	4.08
20	avg	4.62	3.57	2.91	2.97	2.75	2.62	2.83
	max	5.96	4.72	4.09	4.39	3.88	4.06	3.52
30	avg	3.96	3.03	2.62	2.54	2.38	2.28	2.66
	max	5.17	4.13	3.68	3.47	3.46	3.67	4.51

Table 7.23 Error bounds for the stochastic case with minor ordering and stopover costs when the standard deviation is varied.

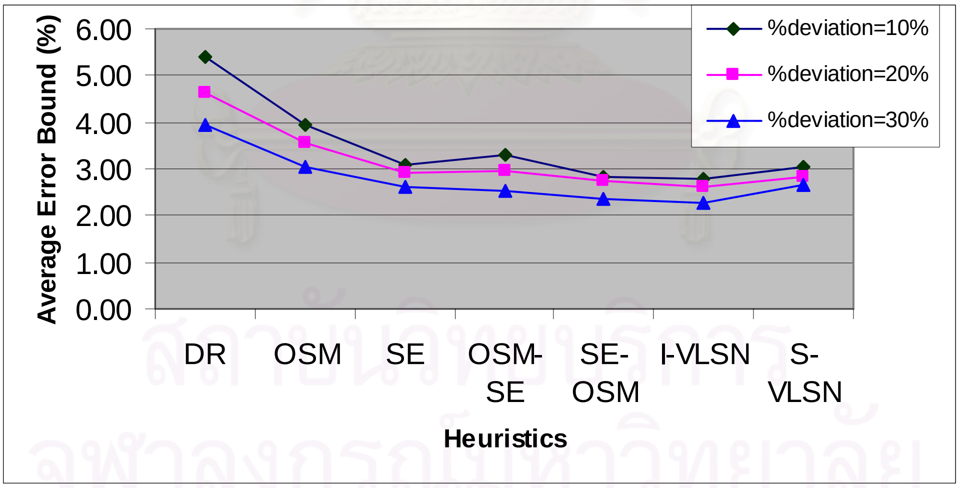

Figure 7.27 Average error bounds for the stochastic case with minor ordering and stopover costs when the standard deviation is varied.

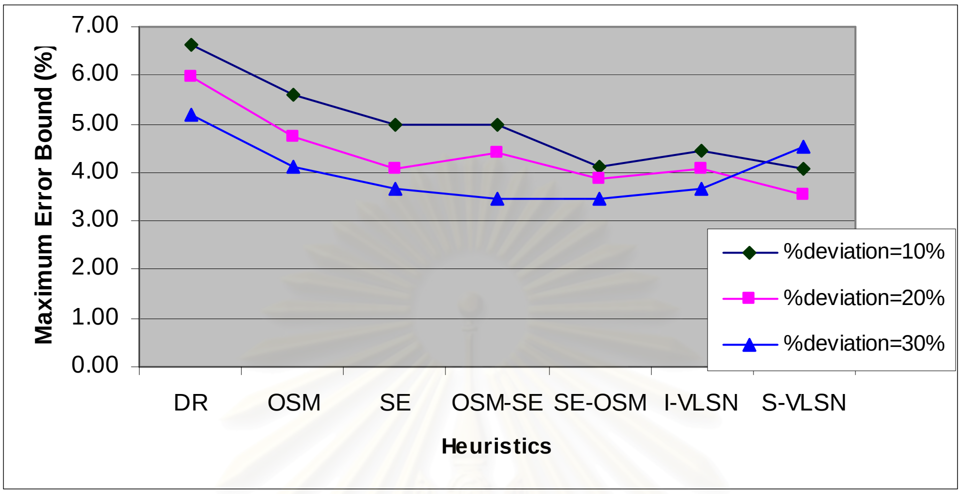

Figure 7.28 Maximum error bounds for the stochastic case with minor ordering and stopover costs when the standard deviation is varied.

C	m		Error bound (%)						
			DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
100	9	avg	3.48	2.55	2.64	2.13	2.21	1.97	2.10
		max	5.25	3.41	4.55	3.05	2.97	2.57	2.84
150	6	avg	4.62	3.57	2.91	2.97	2.75	2.62	2.83
		max	5.96	4.72	4.09	4.39	3.88	4.06	3.52
200	5	avg	3.90	3.46	2.80	2.92	2.64	2.26	2.66
		max	6.49	5.59	5.14	5.14	5.14	3.29	4.25

Table 7.24 Error bounds for the stochastic case with minor ordering and stopover costs when the vehicle capacity C is varied.

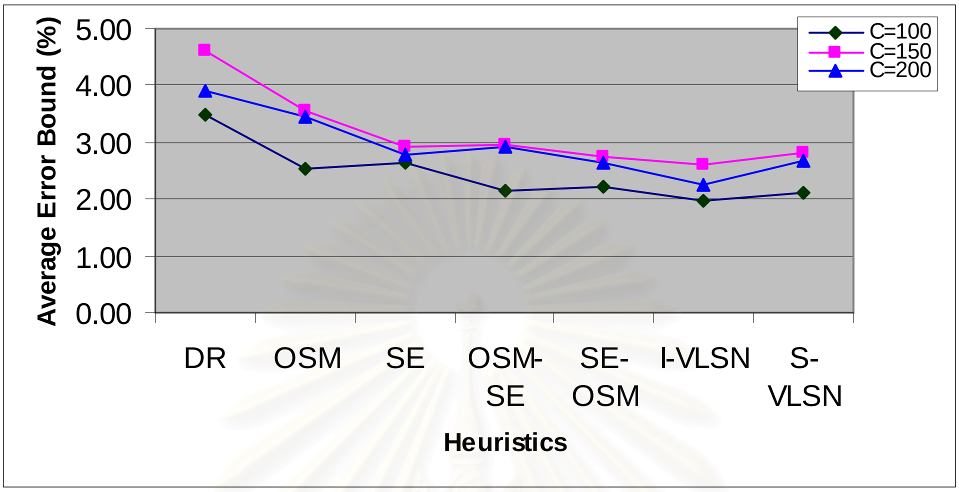

Figure 7.29 Average error bounds for the stochastic case with minor ordering and stopover costs when the vehicle capacity C is varied.

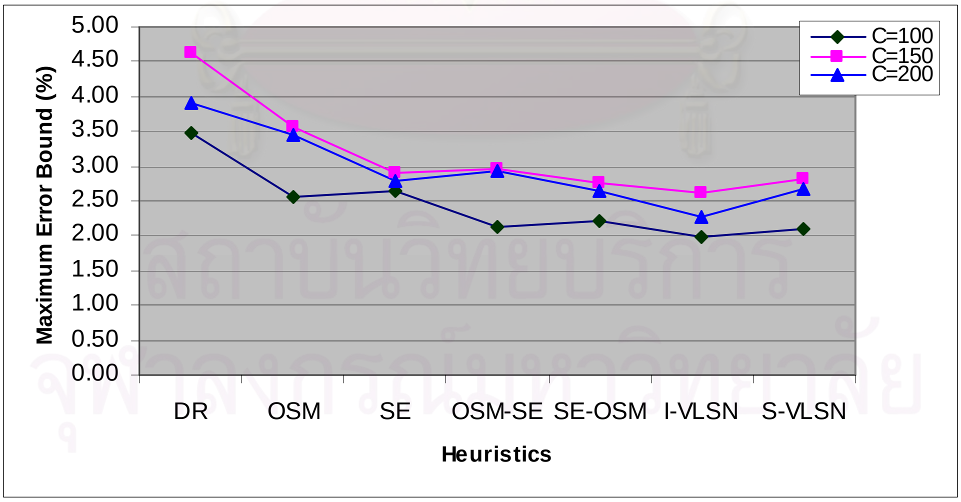

Figure 7.30 Maximum error bounds for the stochastic case with minor ordering and stopover costs when the vehicle capacity C is varied.

F	m		Error bound (%)						
			DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
8	8	avg	3.32	2.59	2.46	2.21	2.11	2.36	2.25
		max	4.54	3.56	3.21	3.21	3.06	3.60	3.52
10	6	avg	4.62	3.57	2.91	2.97	2.75	2.62	2.83
		max	5.96	4.72	4.09	4.39	3.88	4.06	3.52
12	5	avg	4.19	3.52	3.03	2.75	2.76	2.29	2.49
		max	5.05	4.72	3.86	3.79	3.63	3.47	2.82

Table 7.25 Error bounds for the stochastic case with minor ordering and stopover costs when the maximum number of trips allowed F is varied.

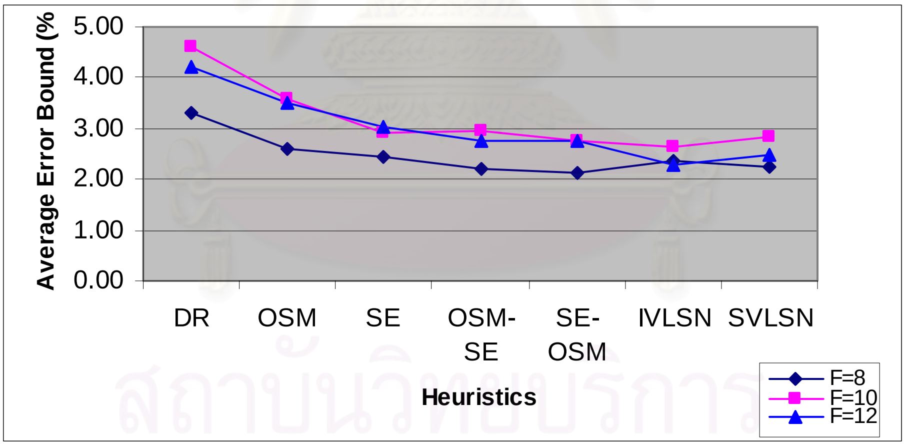

Figure 7.31 Average error bounds for the stochastic case with minor ordering and stopover costs when the maximum number of trips allowed F is varied.

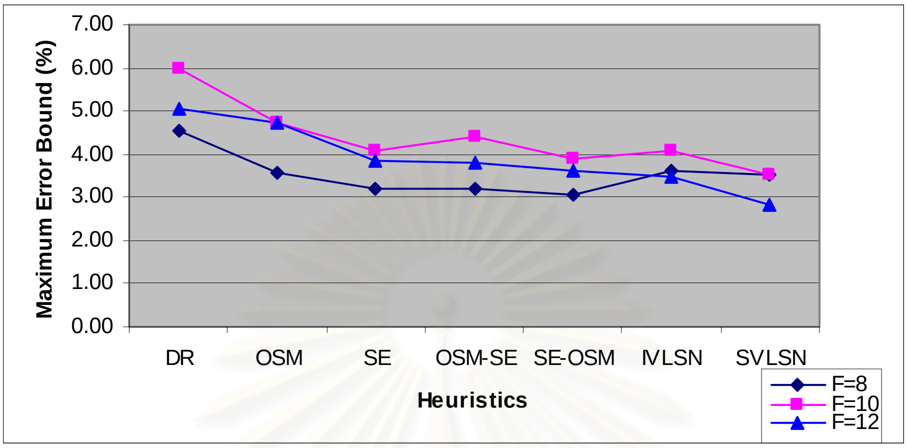

Figure 7.32 Maximum error bounds for the stochastic case with minor ordering and stopover costs when the maximum number of trips allowed F is varied.

K		Error bound (%)						
		DR	OSM	SE	OSM-SE	SE-OSM	I-VLSN	S-VLSN
0	avg	2.25	1.32	0.97	0.77	0.71	1.34	0.57
	max	3.35	2.23	1.49	1.48	1.39	1.93	1.53
20	avg	3.26	2.25	2.25	2.07	1.85	1.97	1.64
	max	4.29	3.17	3.32	2.47	2.39	3.50	2.24
50	avg	4.62	3.57	2.91	2.97	2.75	2.62	2.83
	max	5.96	4.72	4.09	4.39	3.88	4.06	3.52
100	avg	5.26	4.24	3.10	3.16	3.07	2.31	2.93
	max	6.47	5.66	4.65	4.65	4.65	4.02	4.04

Table 7.26 Error bounds for the stochastic case with minor ordering and stopover costs when the fixed costs K is varied.

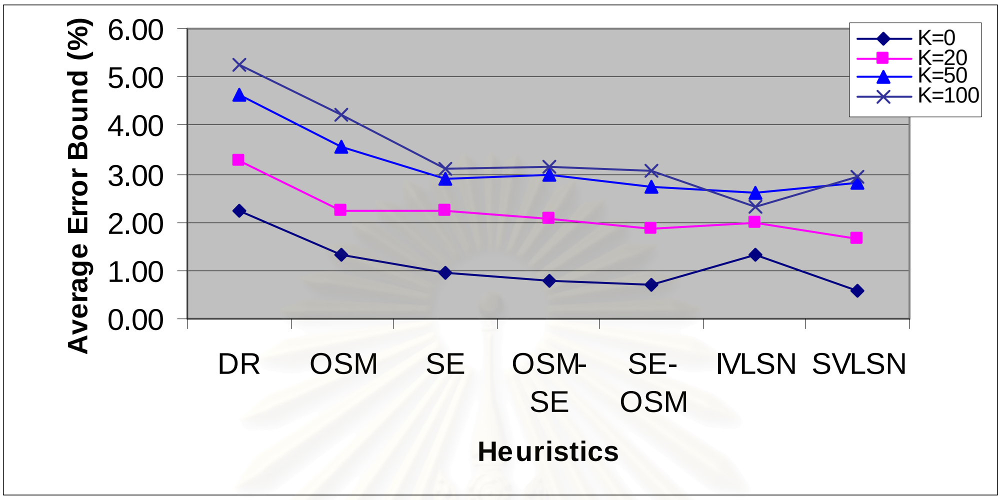

Figure 7.33 Average error bounds for the stochastic case with minor ordering and stopover costs when the fixed costs K is varied.

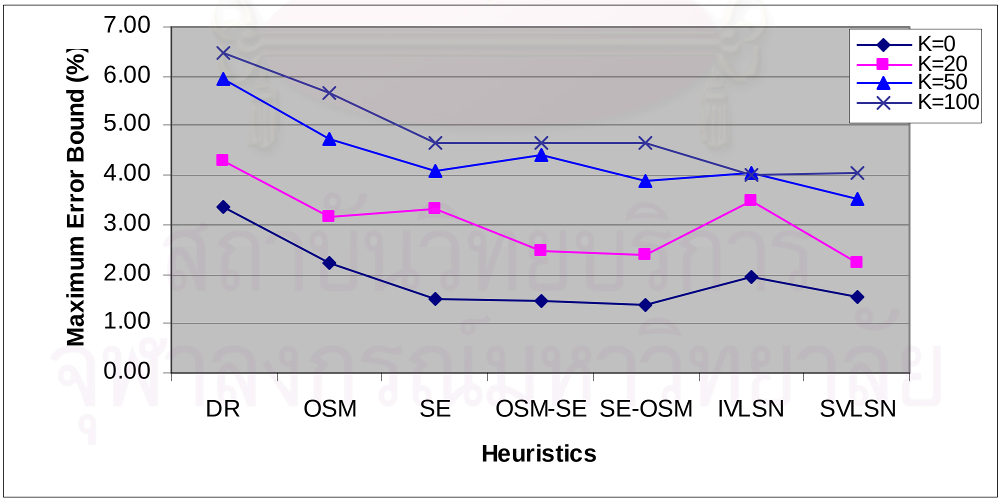

Figure 7.34 Maximum error bounds for the stochastic case with minor ordering and stopover costs when the fixed costs K is varied.

CHAPTER 8

CONCLUSION AND FUTURE RESEARCH

8.1 Conclusion

In this research, the integration of the inventory replenishment and transportation decisions for an inbound commodity collection system with one warehouse, multiple suppliers, and multiple items has been studied. In this system, a fleet of capacitated vehicles are dispatched from the central warehouse to collect a set of items at suppliers' locations and then return to the central warehouse. Each vehicle also faces a frequency constraint. The problems in both deterministic and stochastic settings are considered. For each problem, a mathematical formulation model has been developed.

In the deterministic model, the central warehouse faces constant and deterministic demands for its items. It is assumed that the items are jointly replenished according to an economic order quantity policy. In order to find the optimal operating parameters for this assumed policy, a set partitioning formulation for the problem is developed and a column generation approach that can be used to obtain a lower bound on the objective function value is proposed. In order to solve the small size instances, a branch-and-price algorithm is also developed. Since the branch-and-price algorithm is not scalable, i.e., the solution time requirement increases very quickly as the size of the instance increases, constructive as well as improving heuristics that efficiently find near-optimal solutions for the problems are proposed.

The computational analysis of this model indicates that the constructive heuristics used in conjunction with one of the proposed VLSN algorithms, the I-VLSN, can find near-optimal solutions very efficiently. The sensitivity analysis has shown that this behavior is robust under changes in various key problem parameters. In addition, the OSM-SE and SE-OSM algorithms that are based on the one and two exchange heuristics for the VRP also perform reasonably well. For smaller instances or with some more time investment, the column generation algorithm may be used to provide a bound on the deviation of the cost of the heuristic solution from the optimal cost.

In the stochastic model, it is assumed that demands at the central warehouse from outside retailers are assumed to be independent and identically distributed. A periodic review fixed order quantity policy is adopted to ensure that the vehicle capacity is not exceeded for each collection. In addition to inventory holding, joint fixed ordering, vehicle dispatching and routing costs that are incorporated in the deterministic model, the minor ordering and stopover costs are also taken into account to make the problem more realistic. The mathematical model is formulated by adding to the deterministic model the inventory holding cost due to the safety stock for a specific service level. The constructive heuristics and the improvement algorithms as well as the branch-and-price algorithm developed for the deterministic model can still be employed to solve the problem in the stochastic setting with satisfactory performance even when one of problem parameters has been varied.

The conclusion of this research is depicted in Figures 8.1-8.2

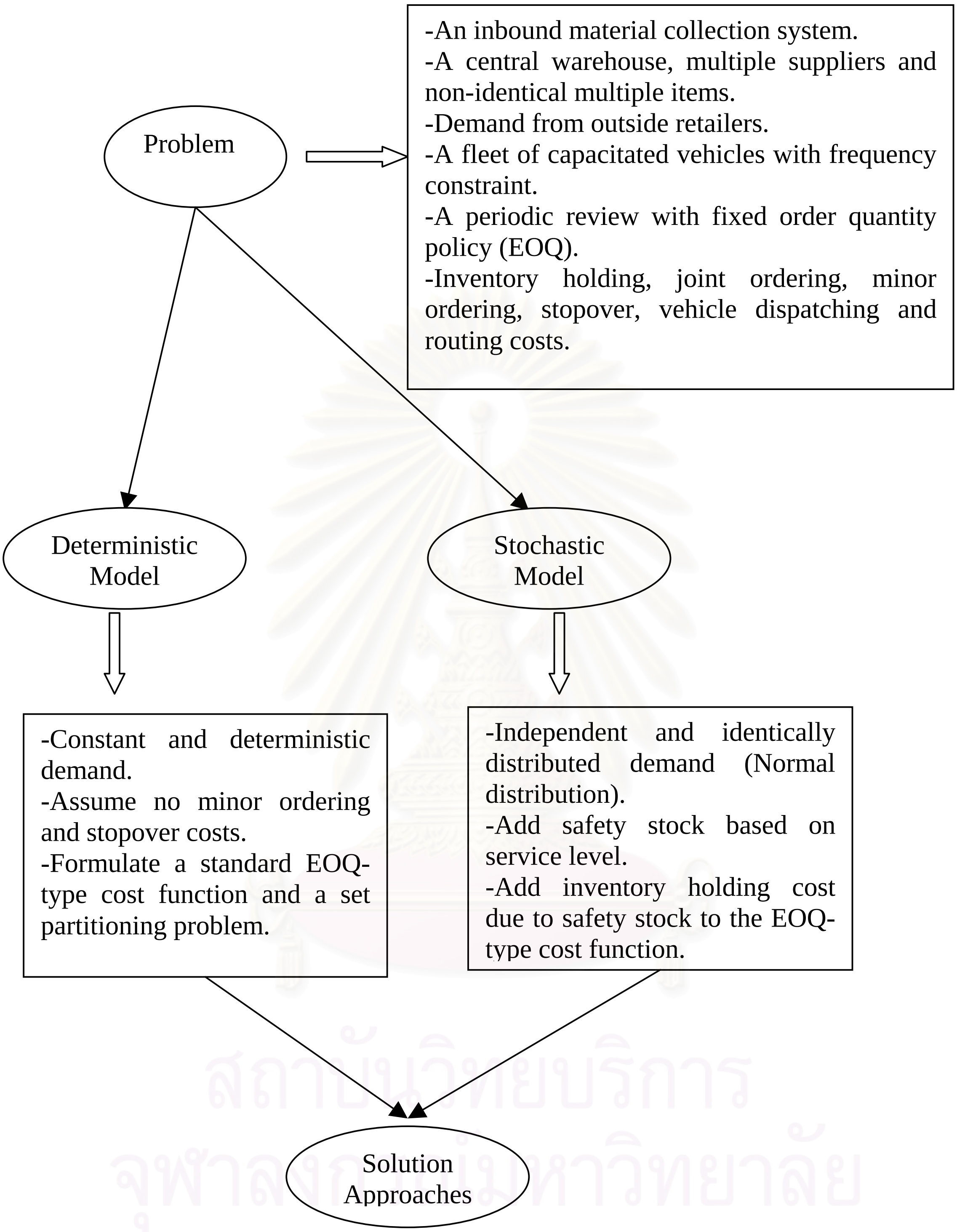

Figure 8.1 Research Conclusion

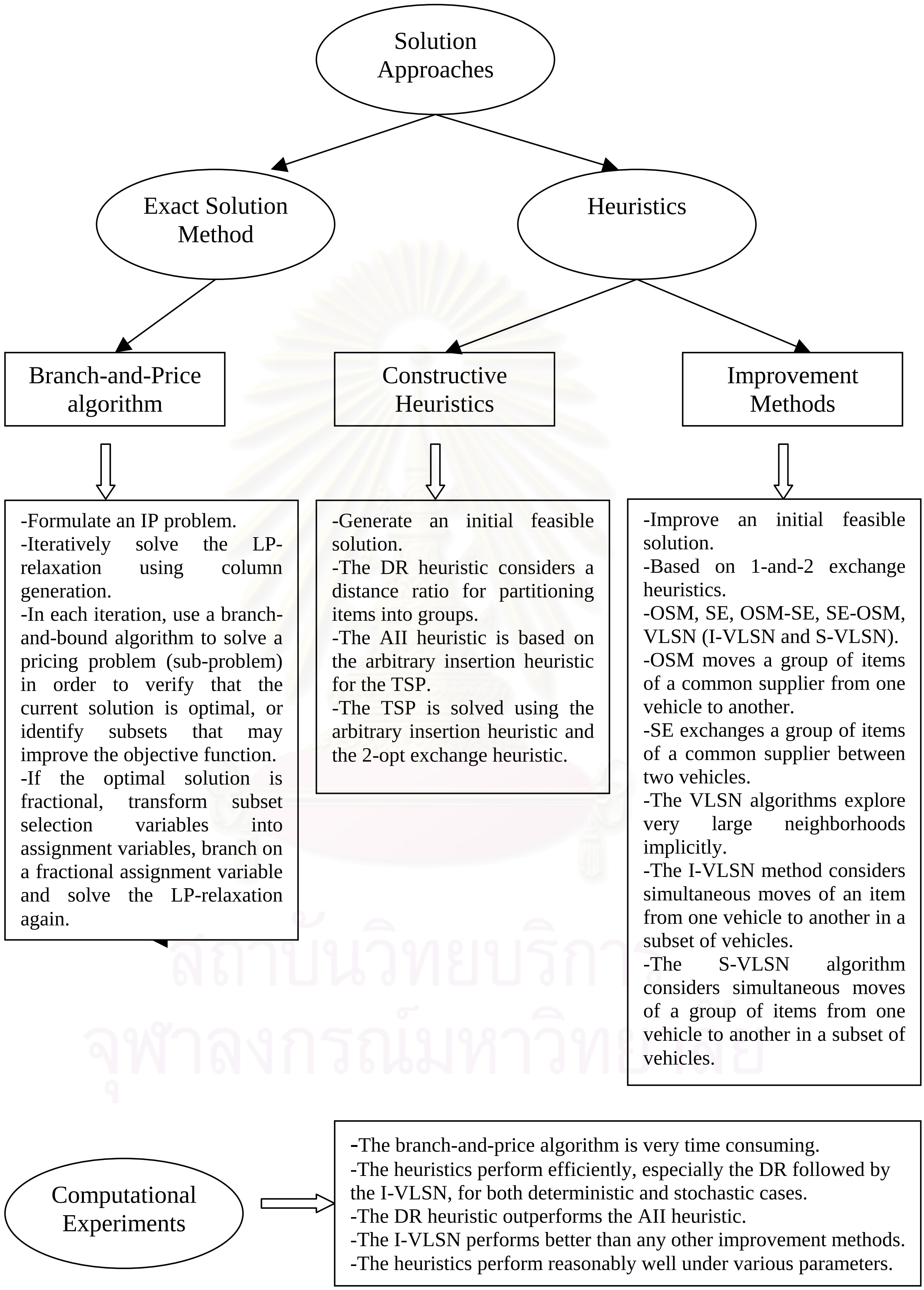

Figure 8.2 Research Conclusion (continued)

8.2 Future Research

This research studies an inbound commodity collection system which is only one segment of the supply chain. A multi-echelon system can be focused where a manager of the central warehouse sends a fleet of vehicles to collect items at suppliers and also dispatches the same fleet of vehicles to distribute items stored at the central warehouse to retailers. In this case, each segment could be optimized separately first and then linked together to achieve a whole minimum costs.

Another interesting scenario is that the distribution of items to retailers is instantly followed by the item collection. That is after a vehicle sent from the central warehouse has finished distributing items to a set of retailers, it visits a set of suppliers for item collection and then returns to the central warehouse where the items are stored. In this case, the replenishment at the central warehouse and at retailers could be assumed to occur simultaneously.

The models studied in this research may be extended to the case where the central warehouse and suppliers belong to the same organization. Therefore, inventory holding costs incurred at each supplier must be considered as well. In the case where the vehicle capacity is very large, compared to the aggregate demand rate of all items, it would be interesting to study if the optimal replenishment strategy is to collect all the items by using a single vehicle under the policy studied.

For the problem in the stochastic setting, a periodic review fixed order quantity policy is selected for inventory control. This policy has the advantage that the quantities of items collected at suppliers are deterministic so the order quantities of each subset of items can be determined in such a way that the vehicle capacity constraint is not violated. Alternatively, the (Q, r, T) policy could be considered. This is a periodic review with flexible order quantity policy. Q and r of items may not be the same. In this policy, for each subset of items the inventory of each item in the subset is reviewed every T unit time. If its inventory level is below the re-order point r, the replenishment quantity is Q. In the other case, if its inventory level is at or above the re-order point r, say I, the replenishment quantity is $(Q+r)$- I .To satisfy the vehicle capacity constraint, the sum of Q for all items in the subset must not be more than the vehicle capacity. As a result, under this policy the total replenishment quantities of all items in the subsets in any period will never exceed the vehicle capacity. The problem is to determine Q, r and T of each item as well as the vehicle route that minimize the average integrated inventory-transportation costs.

In the real world, it is unlikely that the vehicle can visit a set of suppliers any time. It may be more realistic to include a time window constraint in the models studied in this research. A constraint that limits the total distance traveled by a vehicle could be added to the model. This constraint reflects a maximum amount of time that the vehicle can travel. In addition, asymmetric TSP may be considered. This will affect the solution to the TSP. Moreover, the vehicles may have different sizes of capacity. With these changes, the problem can still be formulated as a set partitioning problem. The constructive heuristic and improvement algorithms may need some modification to deal with a more realistic problem.

As for the improvement algorithms, one of meta-heuristics could be proposed to solve the problem. An interesting one is a greedy randomized adaptive search procedure (GRASP) [see Resende(1998)]. GRASP is an iterative process that has been applied to solve a wide range of combinatorial optimization problems. This meta-heuristic consists of two phases, a construction phase and a local search phase. The I-VLSN could be incorporated in GRASP in the local search phase. The solution that is obtained from this combined algorithm should be better than the one obtained from the I-VLSN alone. However, it would require more computational effort.

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APPENDICES

APPENDIX A

Proof of the Global Optimum

Consider the total cost function in the range of 0 to the cost at the inflection point. From (6.4),

$$f(Q) = \frac{A}{Q} + BQ + C\sqrt{Q}$$

After taking the first and second derivatives, $df(Q)/dQ$ and $d^2 f(Q)/dQ$ respectively, of $f(Q)$, the result is

$$f'(Q) = -\frac{A}{Q^2} + B + \frac{C}{2\sqrt{Q}} \qquad \text{(A.1)}$$

$$f''(Q) = \frac{2A}{Q^3} - \frac{C}{4Q\sqrt{Q}} \qquad \text{(A.2)}$$

It is needed to prove that the term $f''(Q)$ is greater than zero at Q^* that makes $f'(Q) = 0$. From (A.1) and at optimal Q^*,

$$f'(Q^*) = -\frac{A}{Q^{*2}} + B + \frac{C}{2\sqrt{Q^*}} = 0$$

Therefore,

$$\frac{C}{2\sqrt{Q^*}} = \frac{A}{Q^{*2}} - B$$

Multiplying both sides by $\frac{1}{2Q^*}$, it would be

$$\frac{C}{4Q^*\sqrt{Q^*}} = \frac{A}{2Q^{*3}} - \frac{B}{2Q^*}$$

Then, substitute $\frac{C}{4Q^*\sqrt{Q^*}}$ into (A.2) and get

$$f^{//}(Q) = \frac{2A}{Q^{*3}} - \frac{A}{2Q^{*3}} + \frac{B}{2Q^{*}} = \frac{3A}{2Q^{*3}} + \frac{B}{2Q^{*}} > 0$$

The term $f^{//}(Q)$ is definitely greater than zero because A, B and Q^* all are positive. Therefore, the total integrated cost $f(Q)$ is convex in the range of 0 to the cost at the inflection point.

Now the question is "Will the value of $f(Q)$ never decrease for every Q that is greater than Q^* ? " To answer this question, it is necessary to prove that after the inflection point where $Q = Q^{/}$, the slope of $f(Q)$ for all $Q > Q^{/}$ is positive. That is $f^{/}(Q)$ is greater than zero for all $Q > Q^{/}$ and the local optimum is also the global optimum.

At the inflection point where $Q = Q^{/}$, the second derivative $f^{//}(Q)$ becomes zero. Therefore, the equation (A.2) is set to zero and $Q^{/}$ can be solved at the inflection point in term of A and C.

$$f^{//}(Q^{/}) = \frac{2A}{Q^{/3}} - \frac{C}{4Q^{/}\sqrt{Q^{/}}} = 0$$

$$\frac{2A}{Q^{/3}} = \frac{C}{4Q^{/}\sqrt{Q^{/}}}$$

$$2A = \frac{C}{4} Q^{/}\sqrt{Q^{/}}$$

$$Q^{/} = \left(\frac{8A}{C}\right)^{\frac{2}{3}} = 4\left(\frac{A}{C}\right)^{\frac{2}{3}}$$

To prove $f^{/}((1+\alpha)Q^{/})$>0 for all α >0, from equation (A.1), replace Q by $(1+\alpha)Q^{/}$ and substitute $Q^{/}$ to obtain

$$f^{/}((1+\alpha)Q^{/}) = f^{/}((1+\alpha)4(\frac{A}{C})^{\frac{2}{3}})$$

$$= -\frac{A}{16(1+\alpha)^2\left(\frac{A}{C}\right)^{\frac{4}{3}}} + B + \frac{C}{4\sqrt{(1+\alpha)}\left(\frac{A}{C}\right)^{\frac{1}{3}}}$$

$$= -\frac{1}{(1+\alpha)^2}\frac{A}{16}\left(\frac{C}{A}\right)^{\frac{4}{3}} + B + \frac{1}{\sqrt{(1+\alpha)}}\frac{C}{4}\left(\frac{C}{A}\right)^{\frac{1}{3}}$$

$$= -\frac{1}{4(1+\alpha)^2}\frac{C}{4}\left(\frac{C}{A}\right)^{\frac{1}{3}} + B + \frac{1}{\sqrt{(1+\alpha)}}\frac{C}{4}\left(\frac{C}{A}\right)^{\frac{1}{3}}$$

Because $(1+\alpha)^2$ is greater than $\sqrt{(1+\alpha)}$ for all $\alpha > 0$, this makes the term $\frac{1}{4(1+\alpha)^2}$ is less than $\frac{1}{\sqrt{(1+\alpha)}}$. Therefore, the term $\frac{1}{4(1+\alpha)^2}\frac{C}{4}\left(\frac{C}{A}\right)^{\frac{1}{3}}$ is always smaller than $\frac{1}{\sqrt{(1+\alpha)}}\frac{C}{4}\left(\frac{C}{A}\right)^{\frac{1}{3}}$ for all $\alpha > 0$ because both A and C are positive values. Consequently, it can be said that the negative term which is $-\frac{1}{4(1+\alpha)^2}\frac{C}{4}\left(\frac{C}{A}\right)^{\frac{1}{3}}$ will never be larger than the positive term which is $\frac{1}{\sqrt{(1+\alpha)}}\frac{C}{4}\left(\frac{C}{A}\right)^{\frac{1}{3}}$. Hence, it can be concluded that

$$f'((1+\alpha)Q') = -\frac{1}{4(1+\alpha)^2}\frac{C}{4}\left(\frac{C}{A}\right)^{\frac{1}{3}} + B + \frac{1}{\sqrt{(1+\alpha)}}\frac{C}{4}\left(\frac{C}{A}\right)^{\frac{1}{3}} > B > 0 \text{ , for all } \alpha > 0 .$$

This means that the slope of the function $f(Q)$ for all $Q > Q'$ after the inflection point is always positive. It can be concluded that the local optima is also the global optima.

APPENDIX B

Replenishment Quantity Comparison with EOQ for the stochastic model

In this part, the order quantity obtained from the stochastic model is compared with the EOQ. For the simple EOQ model, the total cost function is as follows.

$$f(Q) = \frac{A}{Q} + BQ \qquad \text{(C.1)}$$

where $A = L(S)D(S)$ and $B = \frac{h(S)}{2}$

Because the EOQ model considers deterministic demands, there is no cost related to the safety stock in term of C.

To compare the optimal order quantity with the EOQ, the slope of the total cost function at Q equal to the EOQ will be determined. If the slope at that point is negative, the optimal order quantity obtained from the proposed model is larger than the EOQ. If the slope is positive, the optimal order quantity is smaller than the EOQ. In the case that the slope is equal to zero, it can be concluded that the optimal order quantity is equal to the EOQ.

From (C.1), take the first derivative, set it equal to zero and solve for the optimal order quantity EOQ. That is

$$f'(Q) = -\frac{A}{Q^2} + B = 0$$

Therefore, $$EOQ = \sqrt{\frac{A}{B}}$$

From equation (A.1) which is the slope of the total cost function, substitute Q with EOQ to obtain

$$f'(Q) = -\frac{A}{Q^2} + B + \frac{C}{2\sqrt{Q}} = -A\frac{B}{A} + B + \frac{C\sqrt{B}}{2\sqrt{A}} = \frac{C\sqrt{B}}{2\sqrt{A}} > 0$$

The slope of the total cost function at Q equal to EOQ is positive because A, B and C are all non-negative. This means that from Figure 6.1 the EOQ is on the right side of the optimal order quantity obtained from the model studied. Therefore, it is concluded that the optimal order quantity from the stochastic model studied is always smaller than the EOQ.

BIOGRAPHY

I was born on July 5th, 1968 in Ubonratchathani, Thailand. I have 4 sisters and 2 brothers. I graduated from the Faculty of Engineering, Chulalongkorn University with a Bachelor's Degree in Industrial Engineering. I started my career with Thai Stanley Ltd. and then moved to work for Bangkok Bank Ltd. I got my third job with Seagate Technology (Thailand) Ltd. before furthering my study in the USA. I obtained my Master's Degree in Operations Research from University of New Haven, USA. After returning to Thailand, I started my new career as a lecturer at Ubonratchathani University. Two and a half year later, I got a scholarship from the Thai government to study for a Ph.D. in Industrial Engineering at Chulalongkorn University. After completing my study, I will return to work as a lecturer at Ubonratchathani University in my hometown.