CHAPTER I

INTRODUCTION

The purpose of this thesis is to study a generalization of the binomial coefficients using gamma functions, and to investigate uses of the function for constructing generalizations of the binomial theorem.

We first start with the binomial coefficients, which we can write in the form ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$, defining a function on the lattice points in the 1st quadrant of the (r, n) plane. These coefficients form Pascal's triangle. By using the well-known rule for constructing Pascal's triangle, we can extend the values of the function to the lattice points of the 2nd quadrant.

From the binomial series

$$(1 + a)^{n} = {}^{n}C_{0}\cdot 1 + {}^{n}C_{1}a + {}^{n}C_{2}a^{2} + \ldots\ldots ,$$

(1) with n a negative integer and $|a| < 1$, we can extend the function to the lattice points of the 3rd and 4th quadrants, and

(2) with n a negative integer, $|a| > 1$ and using the binomial series in the form of $(a + 1)^{n}$, we obtain an alternating set of values of the function on the lattice points in the 3rd and 4th quadrants. Thus, we have two sets of values of the binomial coefficient function on the lattice points of the (r, n) plane.

Replacing factorials in the formula for ${}^{n}C_{r}$ by gamma functions, we have

$$ {}^{n}C_{r} = \frac{\Gamma(n + 1)}{\Gamma(r + 1)\,\Gamma(n - r + 1)} \; . \qquad \ldots\ldots\ldots(1) $$

Then using (1) , we can define values of the binomial coefficient function on almost all of the other points of the (r, n) plane. The properties of this function and its relation to the extensions of Pascal's triangle over all the lattice points of the plane are discussed in the following chapters.