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นายที เวย์น

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แผนกวิชาวิศวกรรมเครื่องกล  
มหาวิทยาลัย จุฬาลงกรณ์มหาวิทยาลัย  
พ.ศ. ๒๕๑๓

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PLASTIC BEHAVIOUR OF SIMPLY SUPPORTED BEAMS  
AND PORTAL FRAMES

Mr. Tee Wian

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in partial fulfillment of the requirements for the degree of  
Master of Engineering.

*T. Nilanidhi*  
.....

Dean of the Graduate School.

Thesis Committee

*P. Pattabongse* ..... Chairman.  
*U. Chakibhanu* .....  
*W. Khan* .....

Thesis Supervisor .....

*W. Khan*

Date <sup>15<sup>th</sup></sup> May 1969: .....

## ABSTRACT

This thesis involves the plastic behaviour of simply supported beams and single bay portal frames subjected to concentrated load at mid-span.

In the former case emphasis is placed on the relationship between load and transverse deflexion. Rolled black mild steel and rolled brass are chosen for investigation and their characteristics as obtained from a simple tensile test are idealized so that the slope-deflexion equation can be analysed in comparison with that obtained from experiment.

In the latter case, study of the collapse load is the main purpose. The bases of the frames have two different forms, namely pinned-bases with special fittings and fixed-bases. The experimental values of collapse load are compared with values obtained from plastic bending theory and reasons for any divergence are suggested.

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## TABLE OF CONTENTS

<u>CHAPTER</u>	<u>TITLE</u>	<u>PAGE</u>
	Title Page . . . . .	i
	Thesis Approval . . . . .	ii
	Acknowledgment . . . . .	iii
	Abstract . . . . .	iv
	Table of Contents . . . . .	v
	List of Tables . . . . .	vii
	List of Figures . . . . .	viii
	List of Symbols . . . . .	xi
	Abbreviation . . . . .	xiii
I	INTRODUCTION . . . . .	1
	Purpose of research . . . . .	2
	Scope of research . . . . .	2
II	LITERATURE REVIEW AND THEORETICAL CONSIDERATIONS . . . . .	6
	The conventional stress strain curves in simple tension . . . . .	7
	Idealization of stress strain curves, theirs empirical formulas and dynamic models. . .	10
	Theory of elastic-plastic bending . . . .	13
	Analysis of the elastic-plastic slope- deflexion equation of a simply- supported beam subjected to a concentrated load at mid-span . . . . .	20
	Rectangular portal frame . . . . .	24
	Analysis of collapse loads of portal frame with pinned bases . . . . .	25
	Analysis of collapse loads of portal frame with fixed bases . . . . .	27

<u>CHAPTER</u>	<u>TITLE</u>	<u>PAGE</u>
III	EXPERIMENTS ON SIMPLY SUPPORTED BEAMS.	28
	Preparation of simply supported beams.	28
	Description of tests . . . . .	29
	Discussion of results . . . . .	29
IV	EXPERIMENTS ON THREE RECTANGULAR PORTAL FRAMES WITH SPECIAL FITTINGS ATTACHING THEM TO PINNED BASES . . . . .	37
	Description of apparatus . . . . .	38
	Discussion of results . . . . .	38
V	EXPERIMENTS ON TWO RECTANGULAR PORTAL FRAMES WITH FIXED BASES, CENTRAL POINT LOAD . . . . .	47
	Description of apparatus . . . . .	47
	Description of tests . . . . .	50
	Description of results . . . . .	50
VI	GENERAL CONCLUSIONS . . . . .	54
	SUGGESTIONS FOR THE MODIFICATION OF THE APPARATUS . . . . .	55
	SUGGESTIONS FOR FURTHER WORK . . . . .	56
	LIST OF REFERENCES . . . . .	57
	APPENDIX . . . . .	58
	VITA . . . . .	65

LIST OF TABLES

<u>TABLE</u>	<u>TITLE</u>	<u>PAGE</u>
1	Examples of simple structural collapse .	4
4	Results of experiment 4 . . . . .	39
5	Results of experiment 5 . . . . .	51
AI	Results of rolled brass . . . . .	58
AII	Results of black mild steel . . . . .	58



## LIST OF FIGURES

<u>FIGURE</u>	<u>TITLE</u>	<u>PAGE</u>
1	Examples of simple structural collapse .	3
2.1	Typical stress-strain curves . . . . .	8
2.2	Idealized stress-strain curves . . . . .	9
2.3	Representation of stress-strain curves .	14
2.4	Distribution of stress and strain across the beam section which is subjected by a couple $M$ . . . . .	14
2.5	Deformation of beam subjected to an end couple . . . . .	17
2.6	Simply supported beam subjected to a concentrated load at mid-span . . . . .	20
2.7	Portal frame subjected to central vertical load . . . . .	24
2.8	Bending moment diagram at collapse of portal frame with pinned bases . . . . .	25
2.9	Collapse moment diagram of portal frame with fixed bases . . . . .	27
3.1	General arrangement of simply supported beam . . . . .	28
3.2	Details of mild steel test bench . . . . .	32
3.3	The test bench and loads . . . . .	33

<u>FIGURE</u>	<u>TITLE</u>	<u>PAGE</u>
3.4	Dimensionless moment-depth of elastic core.	34
3.5	Central load-deflexion curves of brass beam	35
3.3	Central load-deflexion curves of black mild steel beam . . . . .	36
4.1	The arrangement of experiment 4 . . . . .	37
4.2	Details of frame and stand . . . . .	40
4.3	Details of clamps . . . . .	41
4.4	Details of pinned bases . . . . .	42
4.5	Dimensions of frame, mode of collapse and collapse moment diagram . . . . .	43
4.6	Central load-deflexion curve of rectangular portal frame with pinned bases, special fittings at bases of stanchions. Frame AI.	44
4.7	. . . . . Frame AII	45
4.8	. . . . . Frame AIII	46
5.1	General arrangement of the apparatus . . .	47
5.2	Details of frame and base fittings . . . .	48
5.3	Dimensions of frame and collapse bending moment diagram . . . . .	49
5.4	Mode of failure . . . . .	51
5.5	Central load-deflexion curve of rectangular portal frame with fixed bases . Frame BI .	52
5.6	. . . . . Frame BII .	53

<u>FIGURE</u>	<u>TITLE</u>	<u>PAGE</u>
6.1	Portal frame with vertical and horizontal loads . . . . .	55
6.2	Representation of stress-strain curve . .	56
A1	Stress-strain curves of rolled brass . .	59
A2	Stress-strain curve of black mild steel .	60
A3	Stress-strain curve of black mild steel .	61
A4	Stress-strain curve of black mild steel .	62
A5	Stress-strain curve of black mild steel .	63
A6	The Avery testing machine . . . . .	64
A7	Method of testing specimen in tension . .	64



## LIST OF SYMBOLS

$a, b$	Distances
$A$	Cross sectional area, numerical coefficient
$B, C, K, Y$	Numerical coefficient
$b$	Width of beam
$e$	Base of natural logarithm
$E, E'$	Elastic modulus, plastic modulus
$h$	Thickness of beam
$H$	Horizontal load, numerical coefficient
$I$	Second moment of plane area
$L$	Length, span
$M$	Bending moment, couple
$M_P, M_W, M_O, M_C$	Plastic moment, working moment, moment corresponds to lower yield stress and moment at collapse
$M_P^B, M_P^S$	Full plastic moment of beam and stanchion
$N$	Factor of safety
$n$	Power coefficient
$P, P_O$	Concentrated load, load corresponds to lower yield stress
$R$	Radius of curvature
$S$	Shape factor, the ratio between the plastic moment to the moment at first yield
$V$	Vertical load

$w$	Load per unit length
$w_w, w_c$	Working load, collapse load
$x, y$	Rectangular coordinates
$y_e, y_p$	Deflexion in elastic zone and plastic zone
$y_o$	Distance of elastic core from neutral axis
$\alpha$	Ratio of depth of elastic core to full depth
$\beta$	Ratio of plastic modulus to elastic modulus
$\gamma$	Numerical power coefficient
$\delta$	Central deflexion
$\epsilon$	Unit conventional strain
$\epsilon_n$	Natural unit strain
$\epsilon_o$	Unit strain at yield
$\sigma$	Unit normal stress
$\sigma_u, \sigma_l$	Upper and lower yield stress
$\sigma_w, \sigma_o$	Working stress and represented lower yield stress
$\sigma_e, \sigma_p$	Unit stress in elastic range and plastic range

## ABBREVIATIONS

British Standard Whitworth	B.S.W.
centimetre	cm.
feet, foot	ft., (')
inch	in., (")
minute	min.
neutral axis	N.A.
pound	lb.
pound-inch	lb-in.
pounds per square inch	psi.
kips per square inch	ksi.