

การวิเคราะห์ค่าความเค้นของแผ่นสี่เหลี่ยมด้านขนานโดยวิธี บาวตารี เอเลเมนต์



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STRESS ANALYSIS OF SKEW PLATES

BY

BOUNDARY ELEMENT METHOD

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The boundary element method has been developed in this paper to calculate the deflections and stress resultants of uniformly loaded skewed plates with arbitrary supports along the boundary. The direct formulation which makes use of Betti's reciprocal theorem based on energy consideration is employed to obtain the integral equations. Thereafter, a numerical scheme for computation is used to calculate the resulting integral equations approximately by simple discretization of the boundary functions into a series of elements which are assumed to be constant on each element (constant element). Some numerical results are given in illustrative curves which are found to be in excellent agreement with those of the finite element method and the results quoted by Timoshenko (7.). The tables of the solutions for various cases with practical accuracy are also included in the appendix.

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ลายมือชื่ออาจารย์ที่ปรึกษา
Variddhi Ungbhakorn



วิรัช บุญบำรุงชัย : การวิเคราะห์ค่าความเค้นของแผ่นสี่เหลี่ยมด้านขนานโดยวิธีบาวคาร์
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ต่าง ๆ ของแผ่นสี่เหลี่ยมด้านขนานที่ถูกกระทำด้วยแรงสม่ำเสมอ และรองรับที่ขอบในลักษณะต่าง ๆ การ
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LIST OF SYMBOLS

- b/a = aspect ratio
 D = plate rigidity
 E = young's modulus
 H = plate thickness
 (n,t) = normal co-ordinates
 q = applied distributed load
 Q_x = shear force in x direction
 r = distance between source point (ξ,η) and field point (x_1,x_2)
 ν = poisson's ratio
 W,N,M,V,R = deflection , normal slope , bending moment , kirchhoff shear and corner reaction on boundary per unit length of real plate
 $\bar{W},\bar{N},\bar{M},\bar{V},\bar{R}$ = deflection , normal slope , bending moment , kirchhoff shear and corner reaction on boundary per unit length of virtual plate
 $\bar{\bar{W}},\bar{\bar{N}},\bar{\bar{M}},\bar{\bar{V}},\bar{\bar{R}}$ = normal derivative of deflection , normal slope , bending moment , kirchhoff shear and corner reaction on boundary per unit length of virtual plate
 (X_1,X_2) = skew or oblique co-ordinate
 (X,Y,Z) = cartisian co-ordinates
 Γ = boundary of plate
 Ω = domain of plate
 ϕ = skew angle
 α = angle formed between co-ordinate axis-x and normal on the curve at field point (x_1,x_2) under consideration

- β = angle formed between co-ordinate axis-x and normal on the curve at source point (ξ, η) under consideration
- γ = interior angle of the boundary point
- Δ = dirac delta function
- (ξ, η) = source point in domain
- $(\bar{\xi}, \bar{\eta})$ = source point on boundary
- ϵ = distance
- θ = angle