

CHAPTER VI

DISCUSSION AND CONCLUSIONS

Discussion

An anticipated difficulty in using BEM was the possible occurrence of singular terms on the boundary at the point of application of the load . Although , these terms can be calculated as shown in chapter IV, but the numerical results are still effected from the singularity. The fluctuations of bending moments and shear forces in the domain near the corner and boundary are the effect of a singularity. From numerical experiments, this expected singular behavior may be improved further by introducing a larger number of nodal points on the boundary to a desired accuracy and also found that the effect of the radius of singularity is approximately one half of a mesh length, i.e. numerical results at interior points greater than one half of a mesh length from the boundary are excellent. singularity not only effects in the domain solutions but also effects on the boundary solution as well. Much insight into the effect of a singularity on the quality of a boundary solution is gained by considering the rectangular plate with two edges simply-supported and two edges free calculated by 20 and 40 boundary elements. The interesting point is that although the Kirchhoff shear forces of the different solutions oscillated considerably as shown in figure 50 and although their peaks differ enormously, the deflections and bending moments at the center of the plate calculated for these solutions differ only slightly as shown in table 4. Therefore, there is no singularity effect in an interior points greater than one half of a mesh length from the boundary.

Hartmann, F. and Zotemantel, R.[9] quoted that "The boundary values of a Kirchhoff plate become singular at corner points or points where the boundary conditions change if the internal angle (%) of the boundary point exceeds the values given in table 5 ". All of our numerical calculations for various cases corresponding to table 5 with the variation of skew angles are effected only the boundary values without any effect for the interior values.

Table 6 provides the numerical results of uniformly loaded rhombic plates for various values of skew angles with simply-supported edges calculated by 40 and 80 boundary elements and compares the results with those obtained by the variational approach as quoted by Maiti, M. and Chakrabarty, S.K. [8]. This table illustrates that the convergence of the present method is slow for the small skew angle and deteriorates with decreasing in the skew angles but, however, the accuracy of 40 boundary elements for any skew angles are acceptable in view of the numerical approximation. The characteristic of the distribution of deflections and bending moments in the interior points of plates show no dependence on the skew angles as illustrated in figure 14 through 49.

The convergence and accuracy of BEM not only depend on skew angles but also on the selected fundamental solution of virtual plates. Table 7 provides the numerical results of uniformly-loaded

rectangular plates with two opposite edges simply-supported and other two edges free and compares the results with those obtained by exact solutions and BEM but using different fundamental solution of virtual plates as quoted by Wu , B.C. and Altiero , N.J. [3]. This table indicates the advantage of using the present form of fundamental solution . From numerical experiments , it is seen that the convergence can be improved by using the constant " Z = the longest diagonal of skew plate " as shown in table 8 . It is most likely that they may be improved further by using more elegant numerical technique such as linear element.

Conclusions and recommendations

The direct formulations of BEM which makes use of Betti-maxwell reciprocal theorem based on energy consideration has been presented for solving plate problems. In concept, the method can be applied to plates of arbitrary loading, plan form and boundary conditions. In the present paper various boundary conditions of skew plates subjected to uniformly distributed loads have been considered but only four cases of boundary conditions as shown below have been treated numerically.

- 1) All Edges are Simply-Supported
- 2) All Edges are Clamped
- 3) Two Opposite Edges Simply-Supported and the Other Two Edges Clamped
 - 4) Two Opposite Edges Simply-Supported and the Other Two Edges
 Free

This proposed method started from the Betti-Maxwell reciprocal theorem which states for any two equilibrium states. This equation involving deflections, normal slopes, bending moments, Kirchhoff shear forces and load of the real and virtual plates. All of the virtual plate terms can be calculated from the fundamental solution of a unit singular load with the properties of dirac delta function. The proposed fundamental solution $\overset{\bullet}{W} = \{ \ r^2 \ln(r/Z) \ \}/8\pi D$ is different from the other investigators by introducing the constant " Z = the longest diagonal of plates " which can improve the convergence of the numerical solutions.

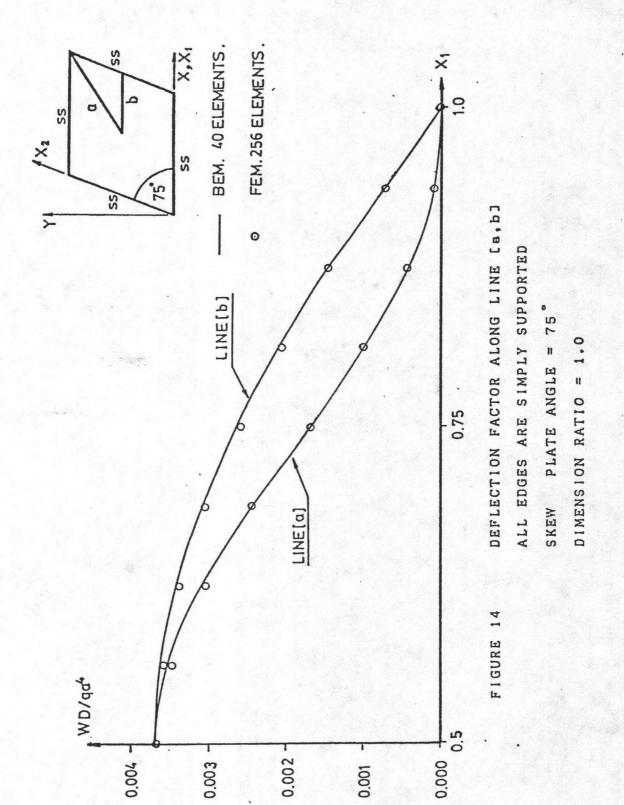
The set of boundary integral equations are established by the limiting process of those equations and their normal derivative. Since two of the four boundary variables W . N . M and V are prescribed by the boundary conditions , one can determine the remaining unknowns by simple discretization of the boundary functions into a series of elements which are assumed to be constant on each element (constant element). All of the integral terms are computed by numerical techniques. An anticipated difficulty in calculating singular terms on boundaries and corners are decreased by using the equilibrium equations on boundaries and corners. These sets of equations can be rewritten in the form of matrices and calculated by Gauss elimination method.

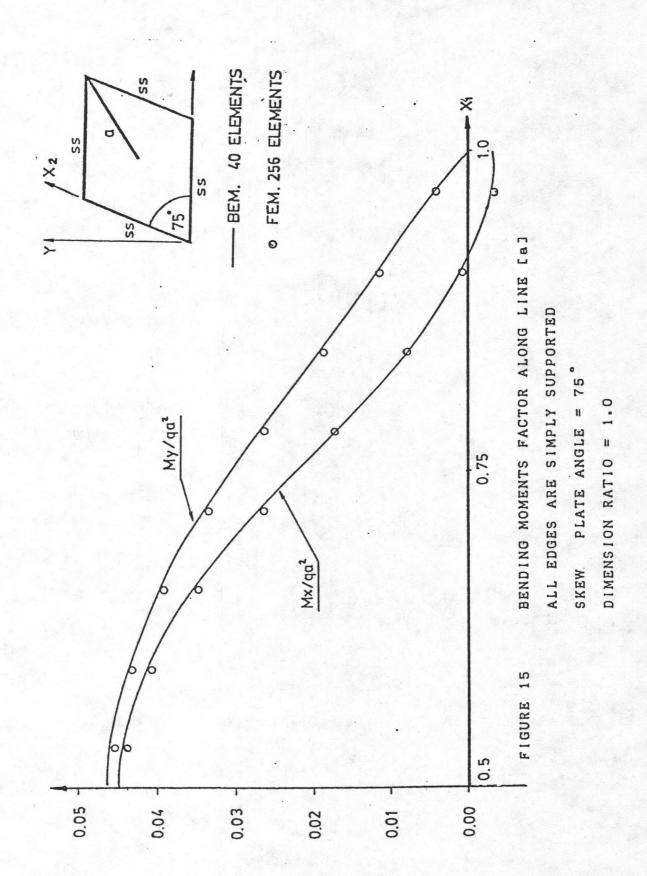
Numerical results obtained by this method are in excellent agreement with the known results of other investigators and those of the acceptable finite element program "SAP*4" except the results of bending moments and shear forces near the boundary and corner which

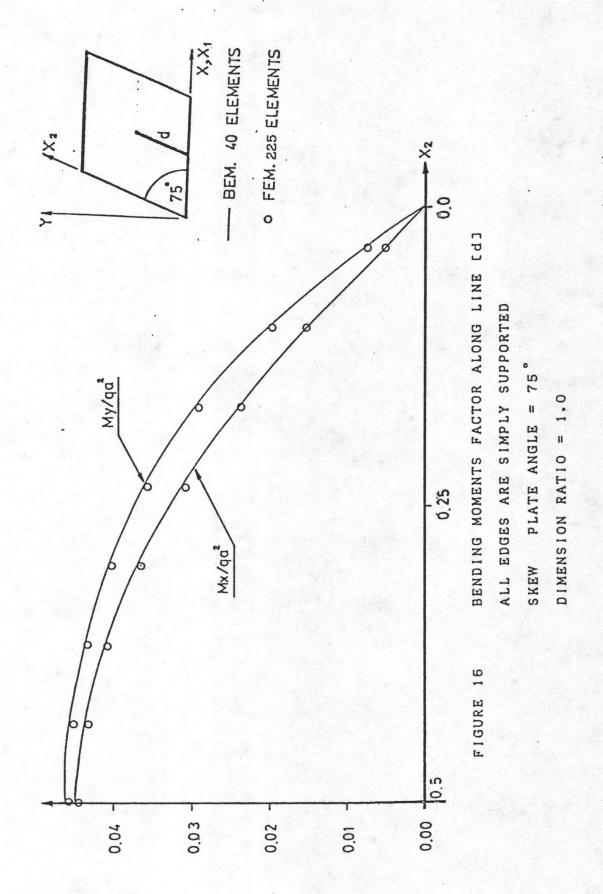
are the effect of a singularity. This results can be improved by introducing a larger number of nodal points on the boundary. The characteristic of distribution of deflections, bending moments and shear forces in the interior point of plate do not depend on the skew angle, but the convergence and accuracy of BEM deteriorate with decreasing in skew angles.

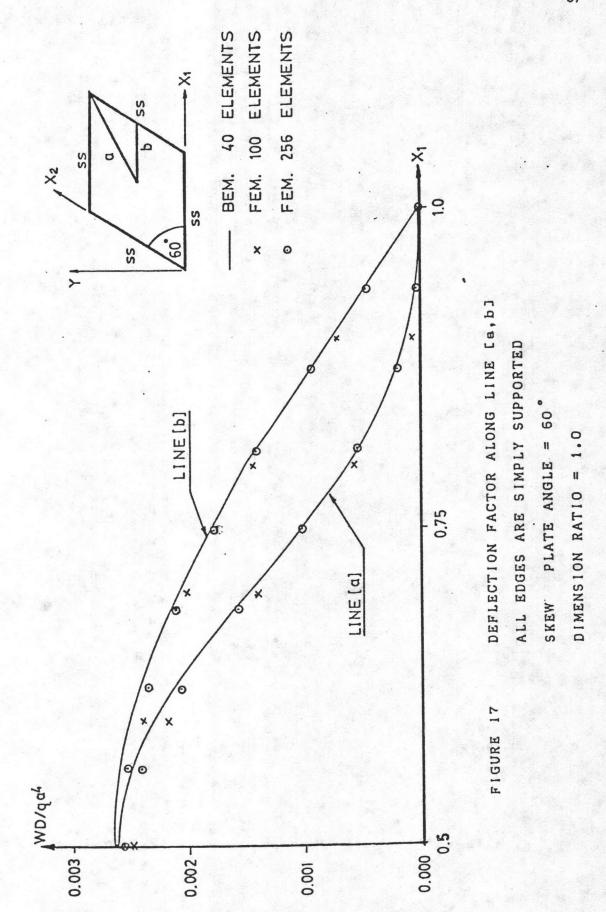
The computer program has been developed in fortran 77 language for any skew plate angles and aspect ratios with the poisson's ratio of 0.3. The computation has been performed on a PRIME 9750 computer using double precision arithmetics. On the PRIME computer the solution time for each example (40 boundary elements, 44 stress points) is approximately 5 minutes C.P.U. time. This time can be reduced by calculating all of the integral terms by analytical instead of numerical technique.

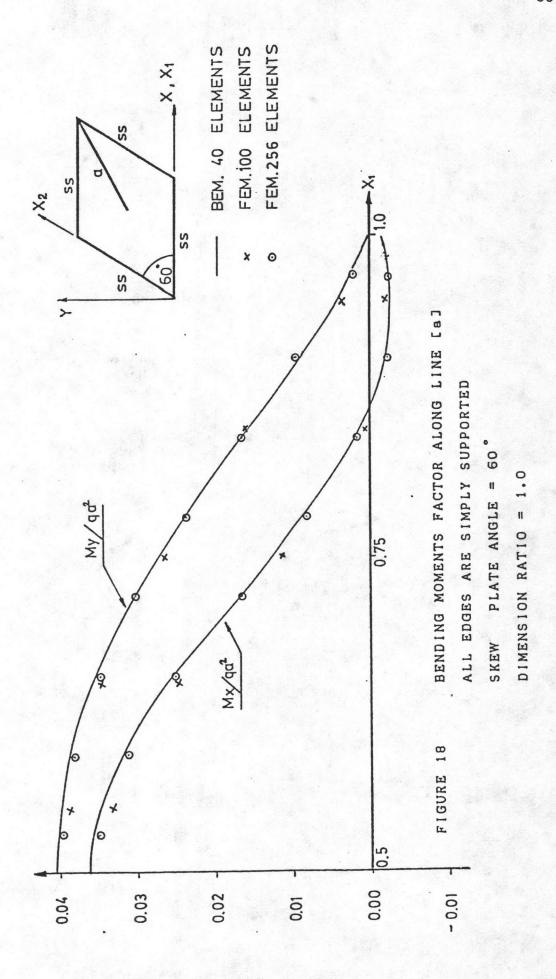
It would be worth while, however, to investigate further into the plates of arbitrary plan form and boundary conditions. Note also that the method may be improved further by using more elegent numerical techniques such as linear elements.

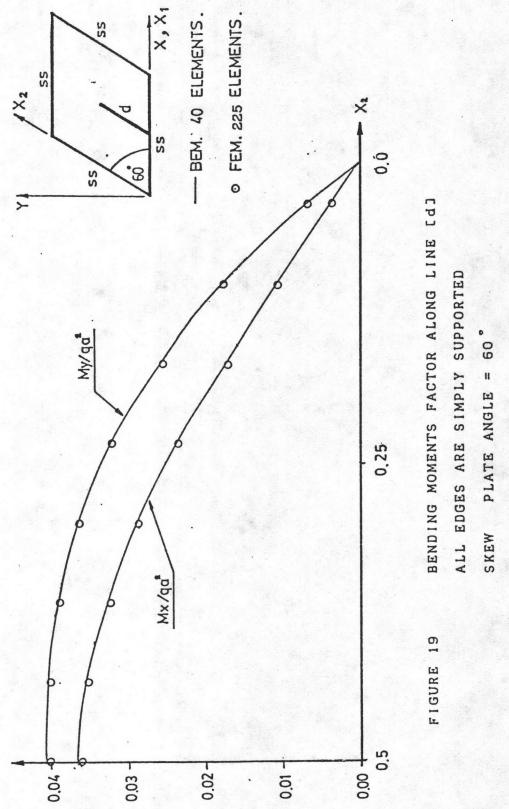




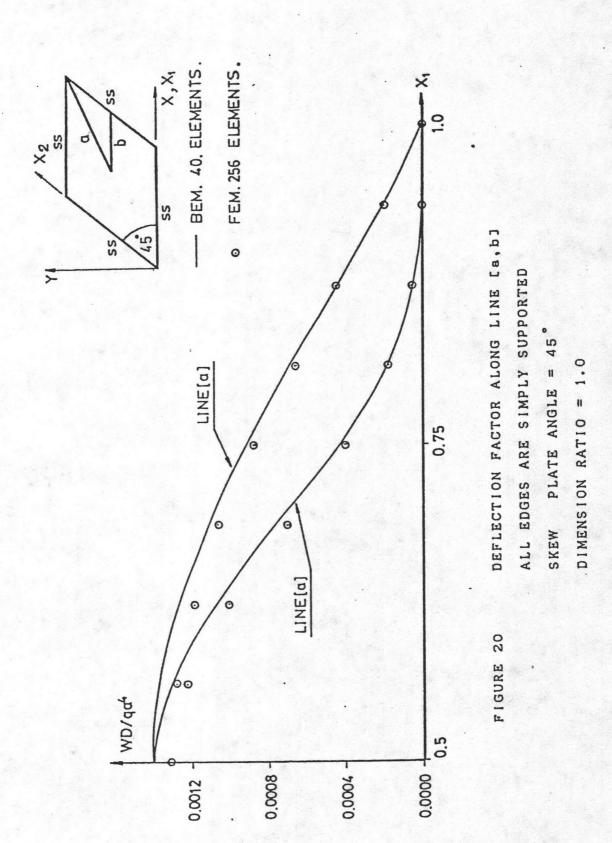


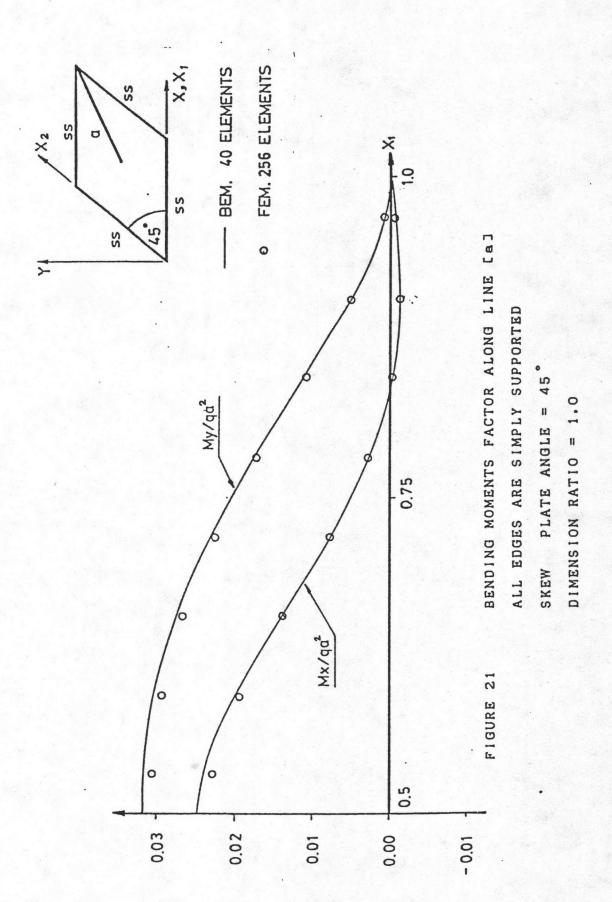


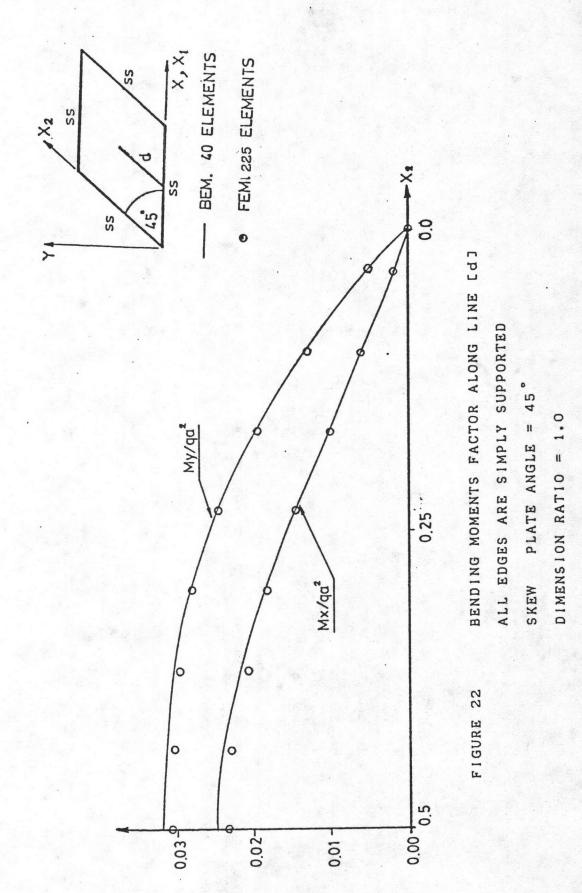


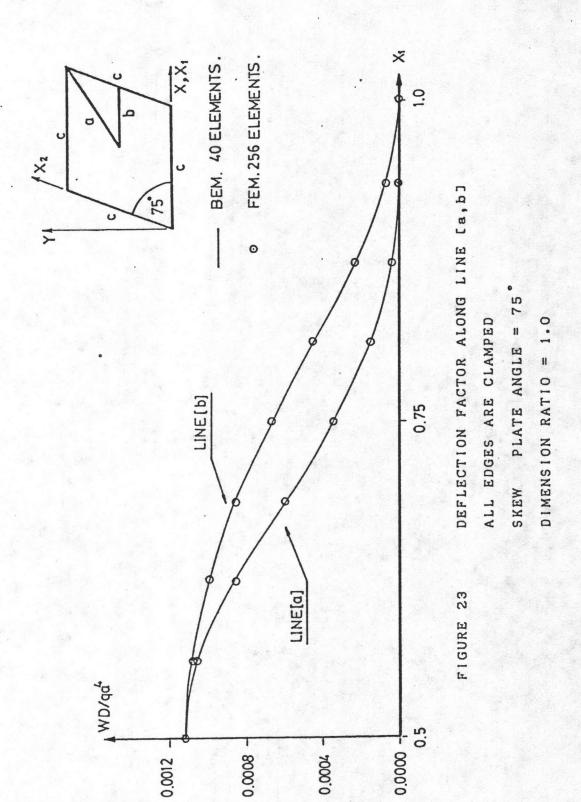


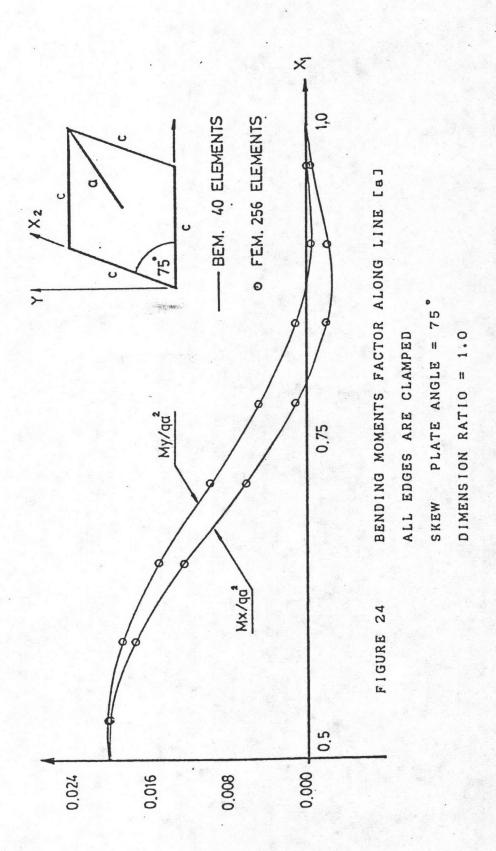
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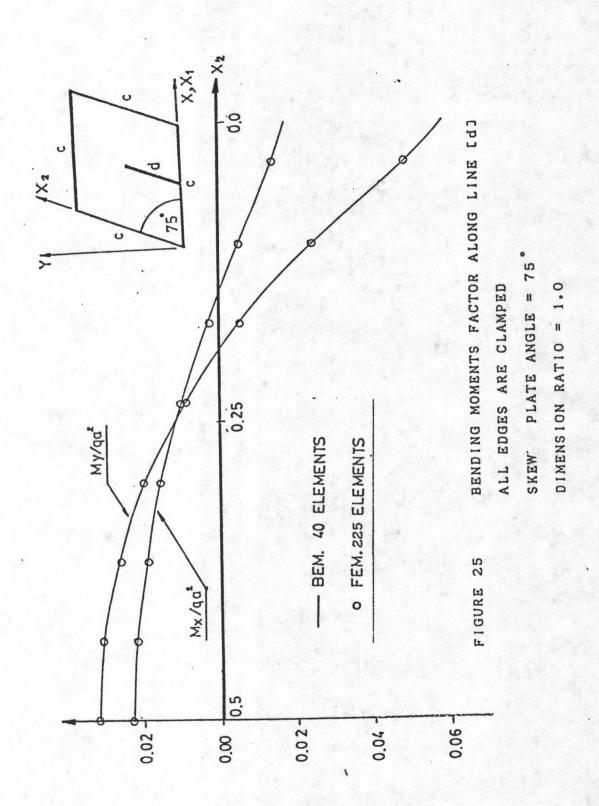


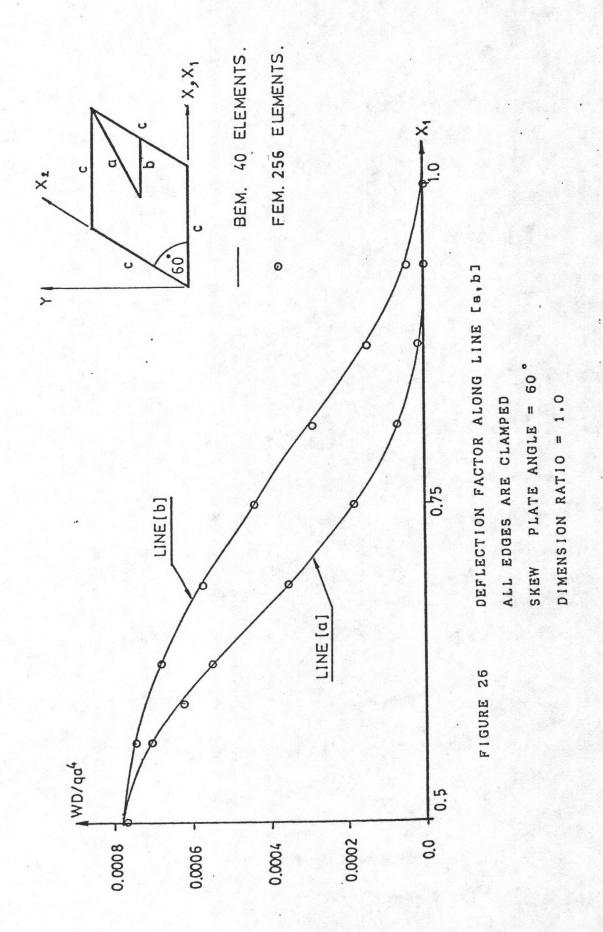


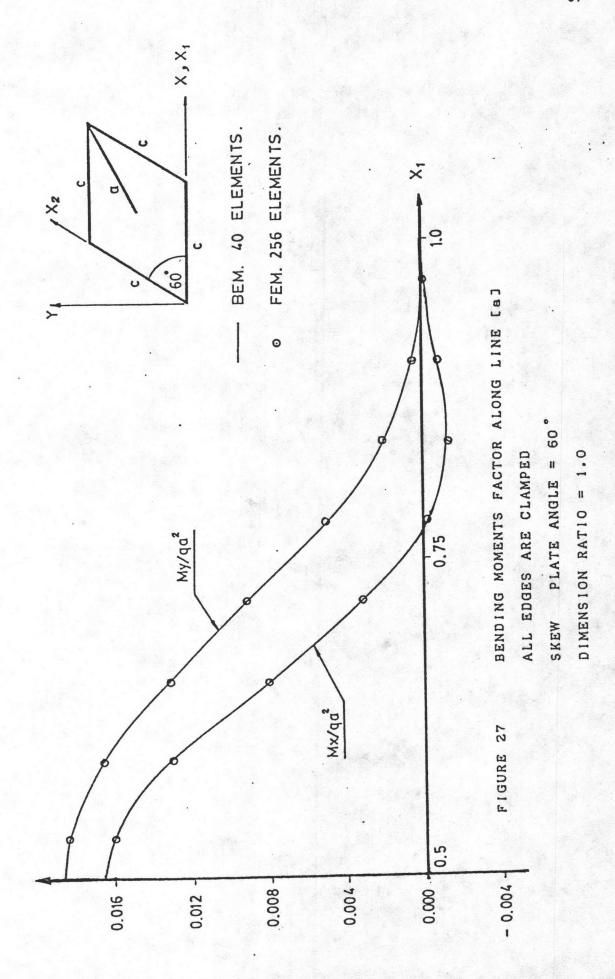


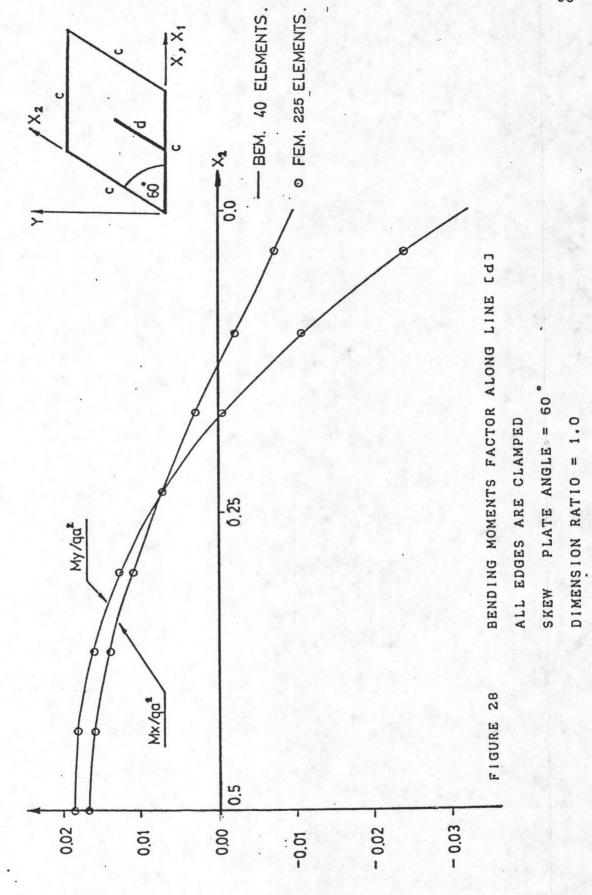


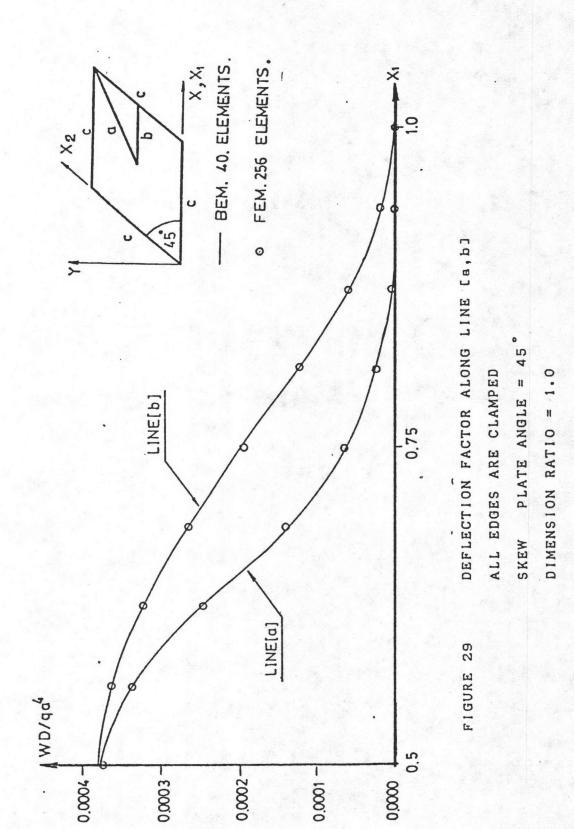


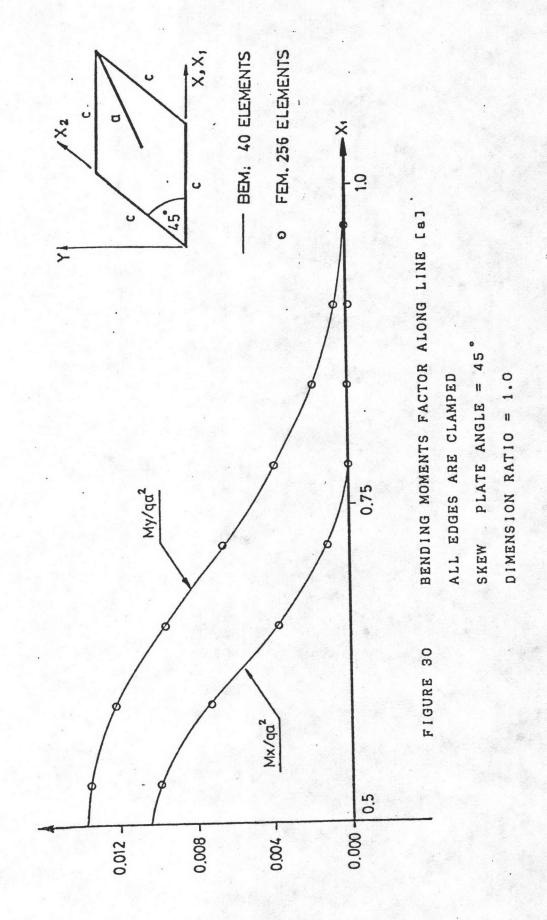


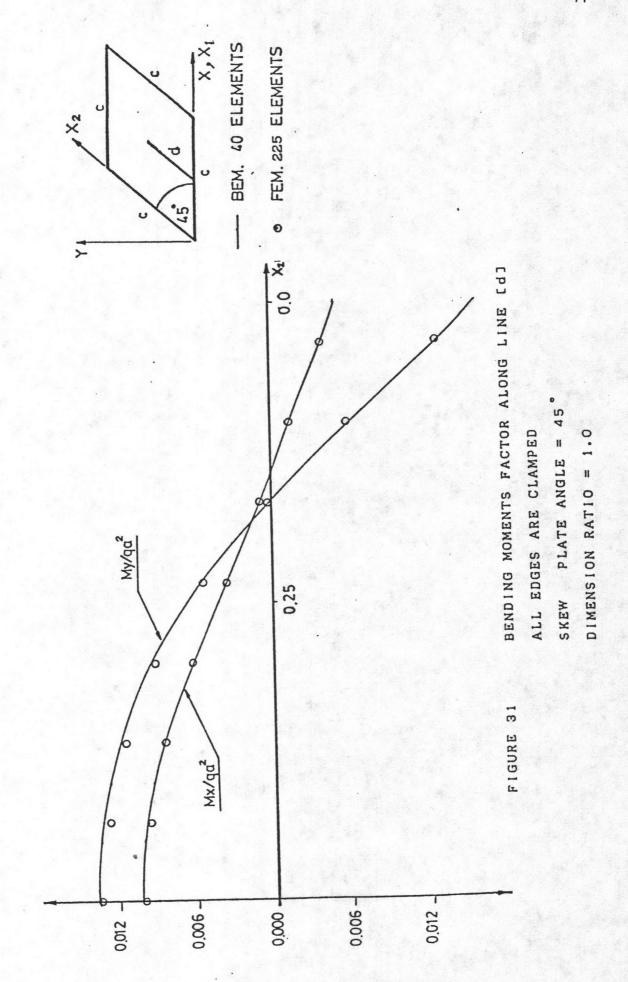


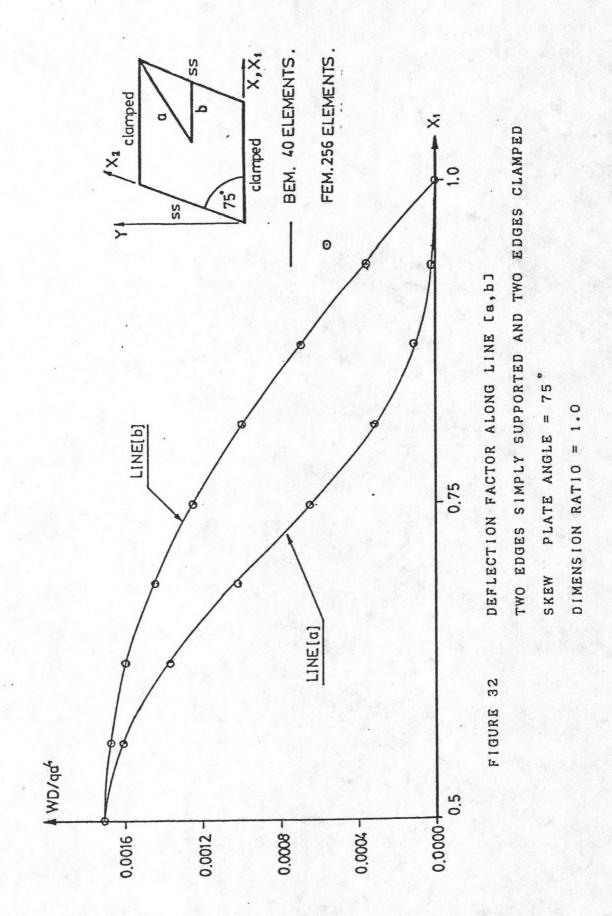


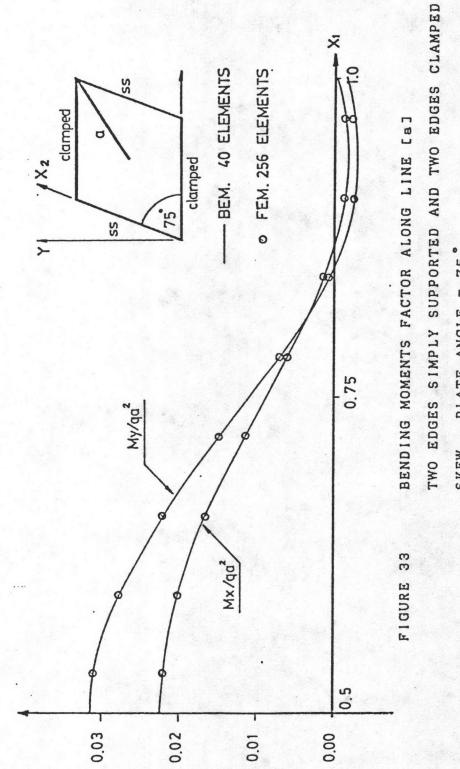




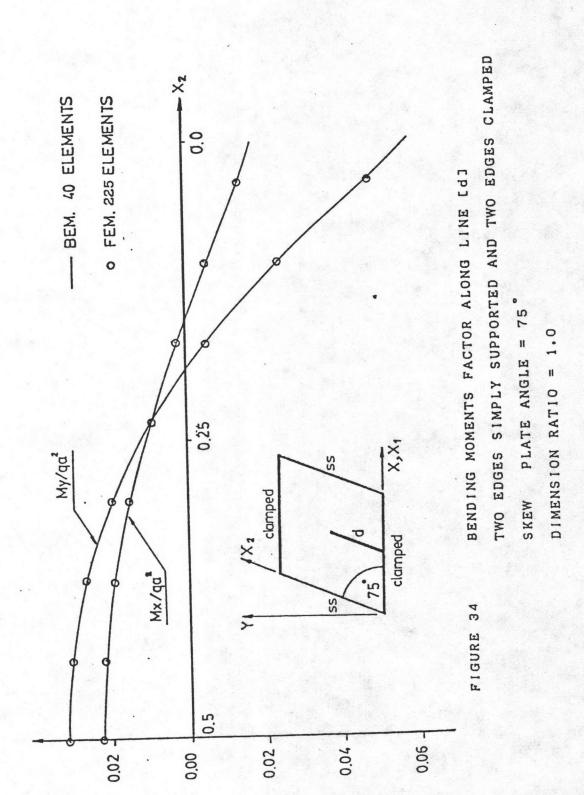


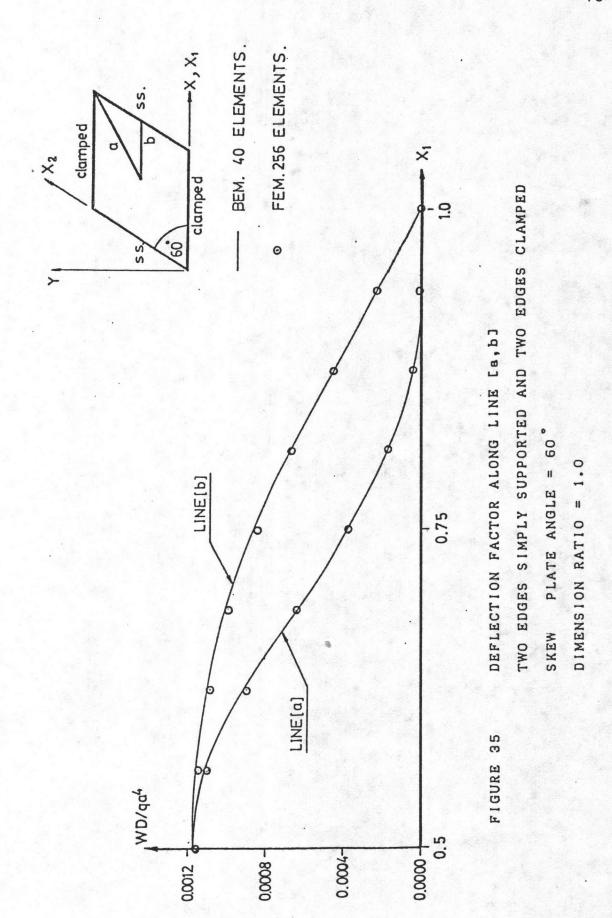


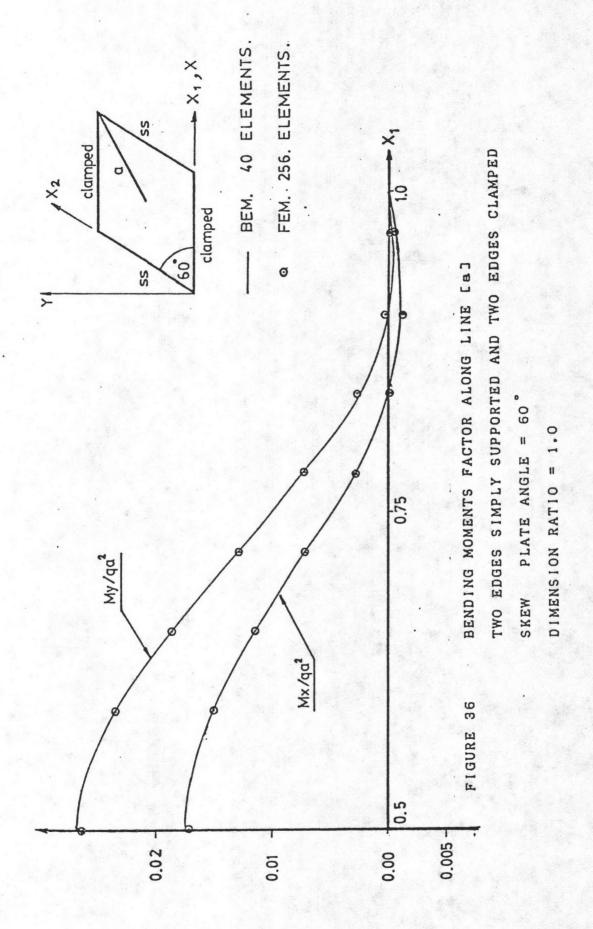


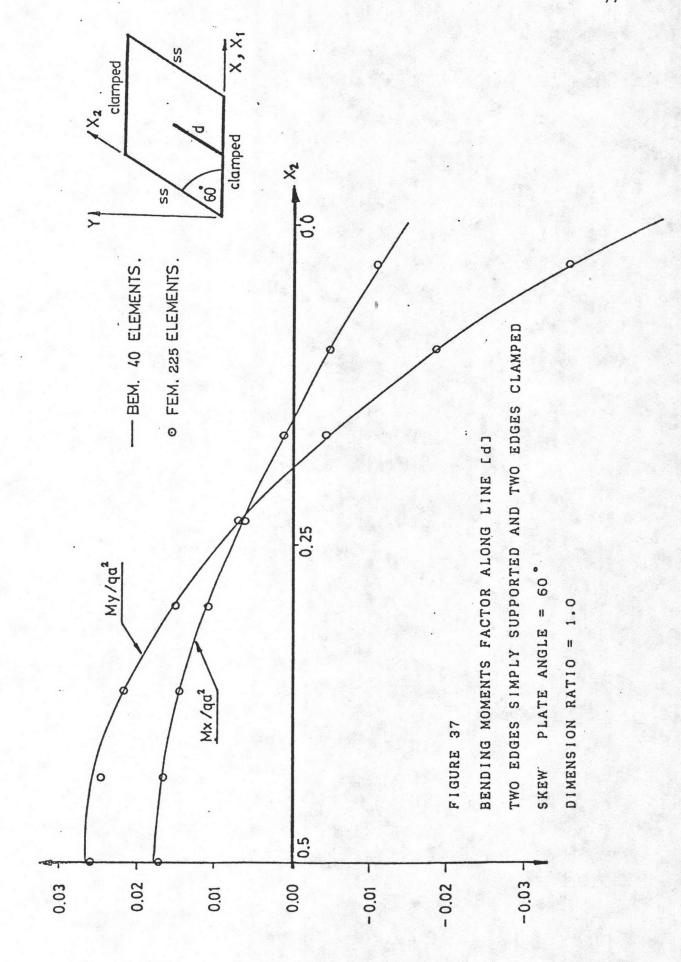


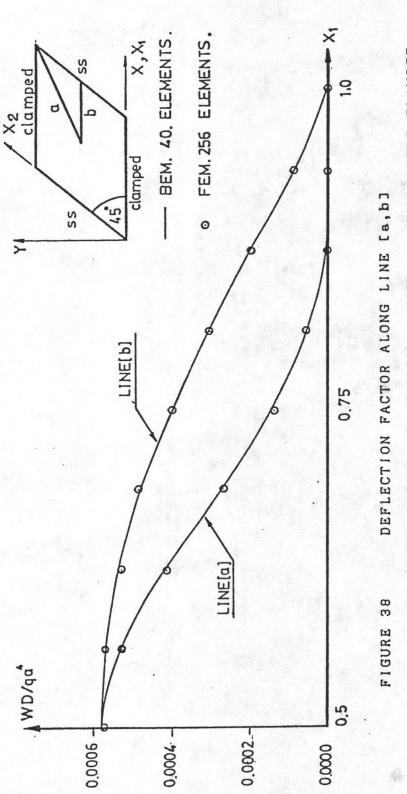
SKEW PLATE ANGLE = 75° DIMENSION RATIO = 1.0







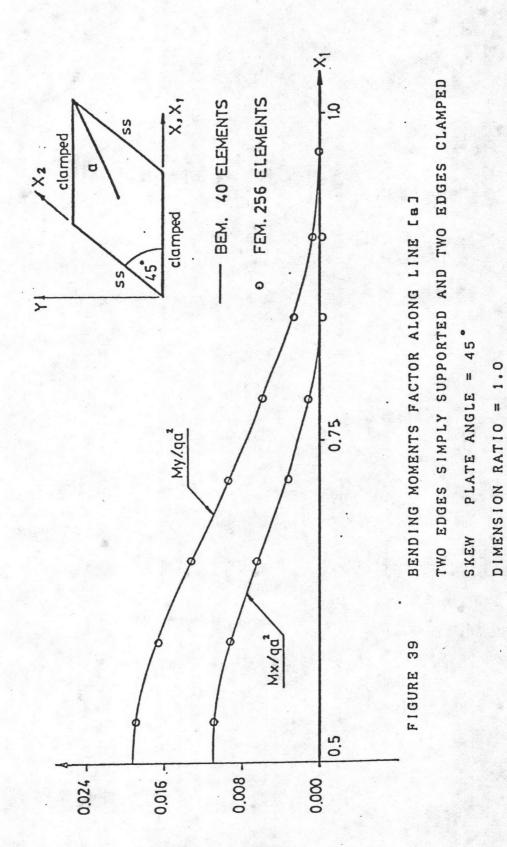


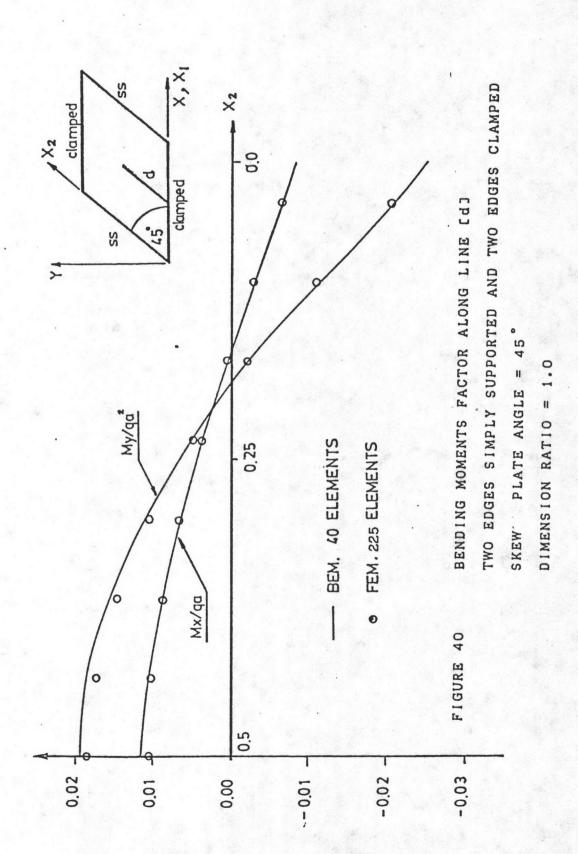


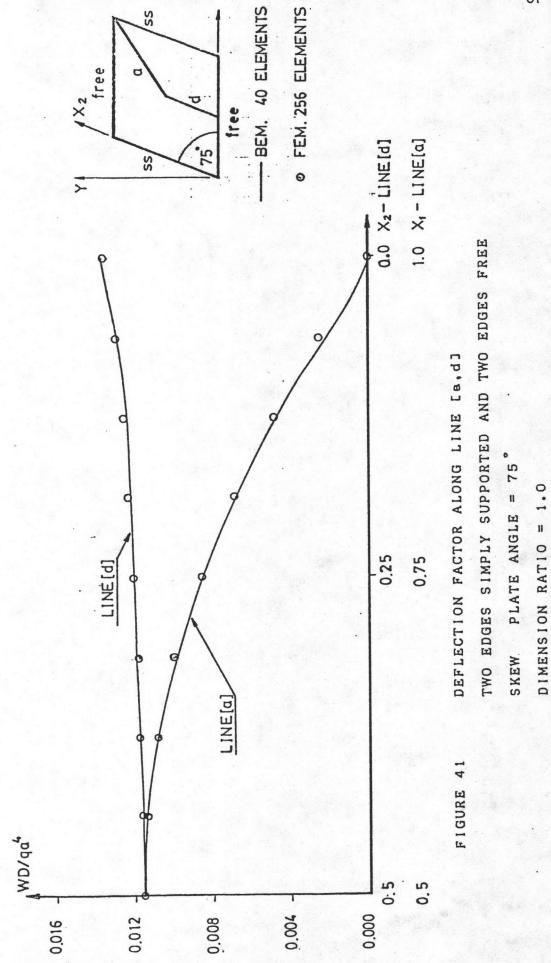
DEFLECTION FACTOR ALONG LINE (8, D.)

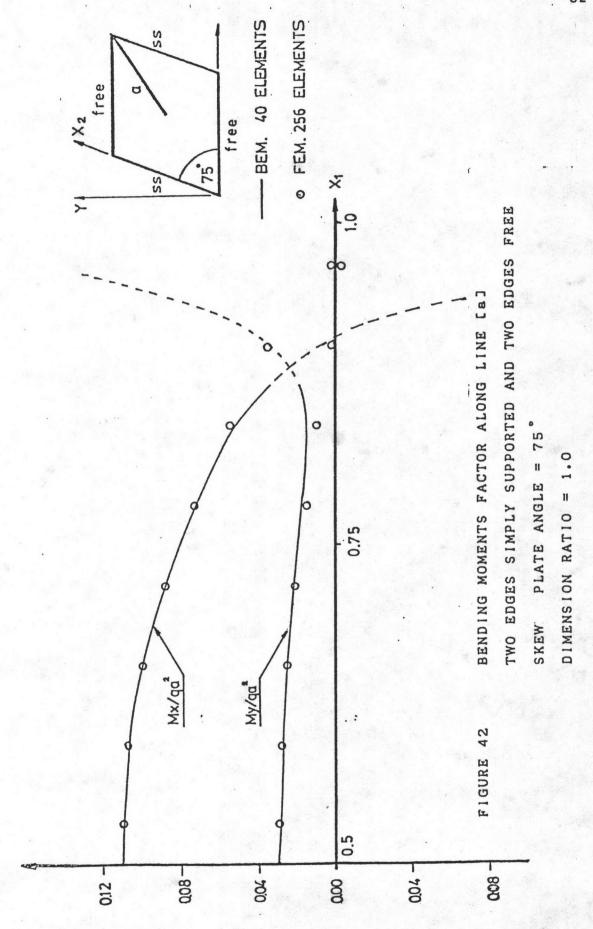
TWO EDGES SIMPLY SUPPORTED AND TWO EDGES CLAMPED SKEW PLATE ANGLE = 45°

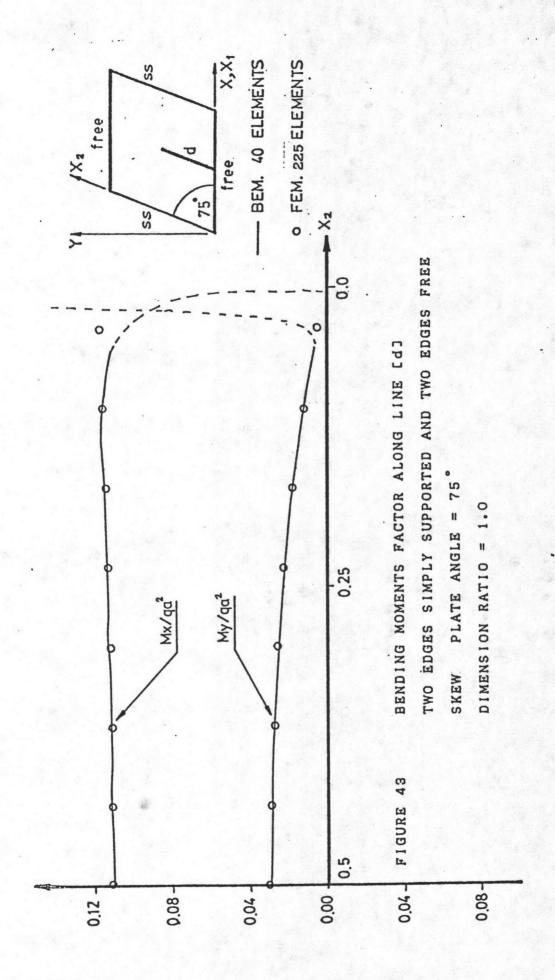
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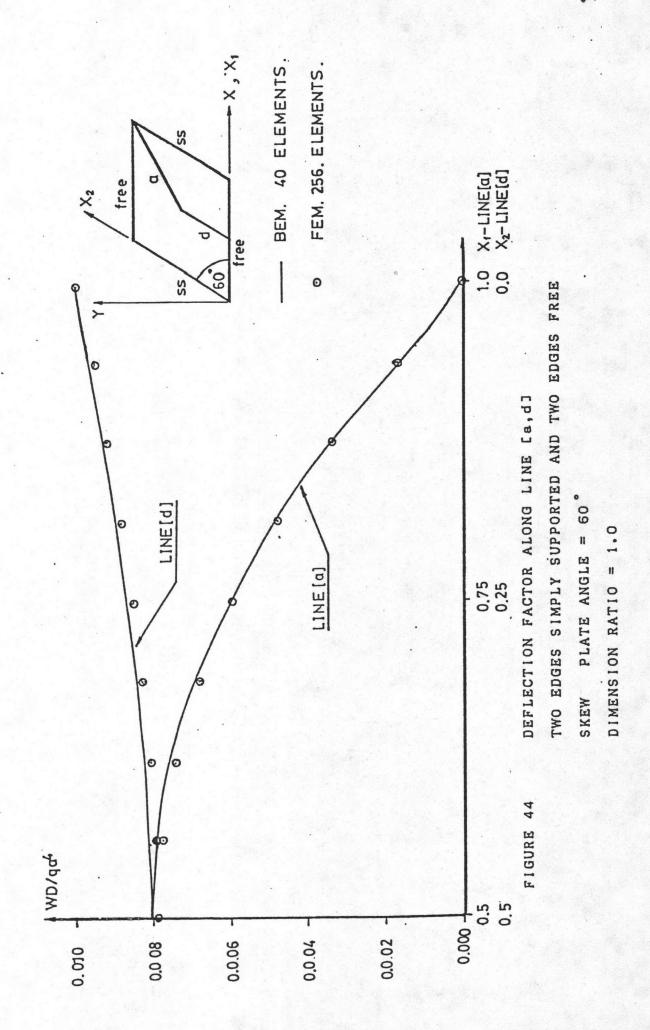


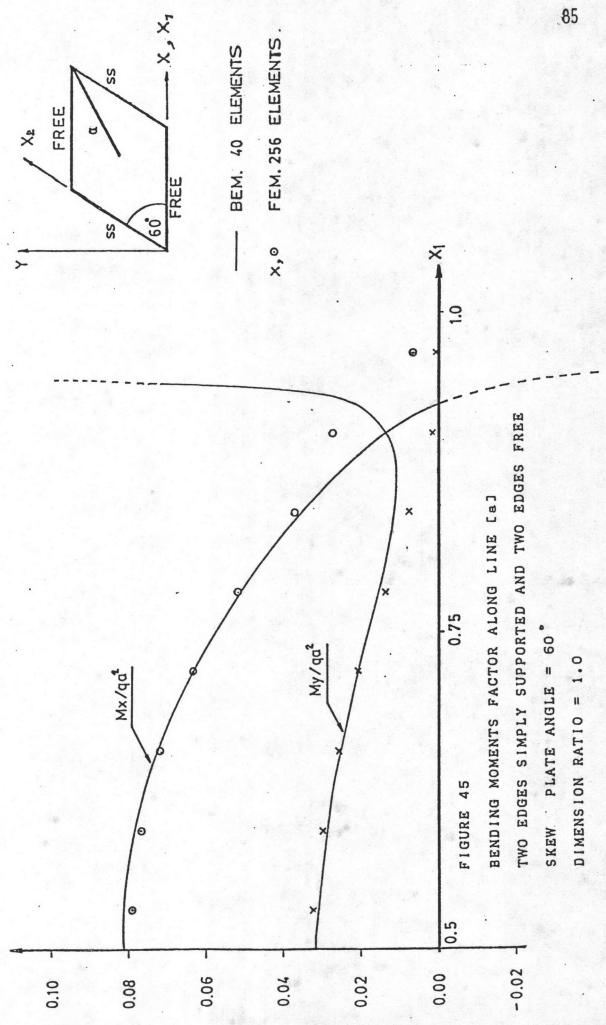


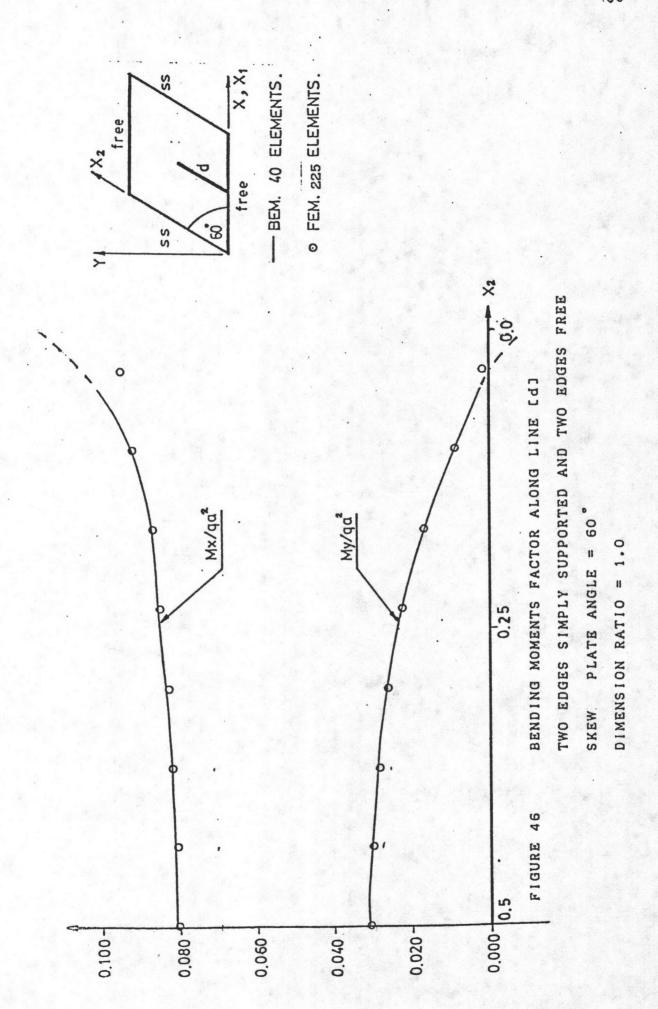


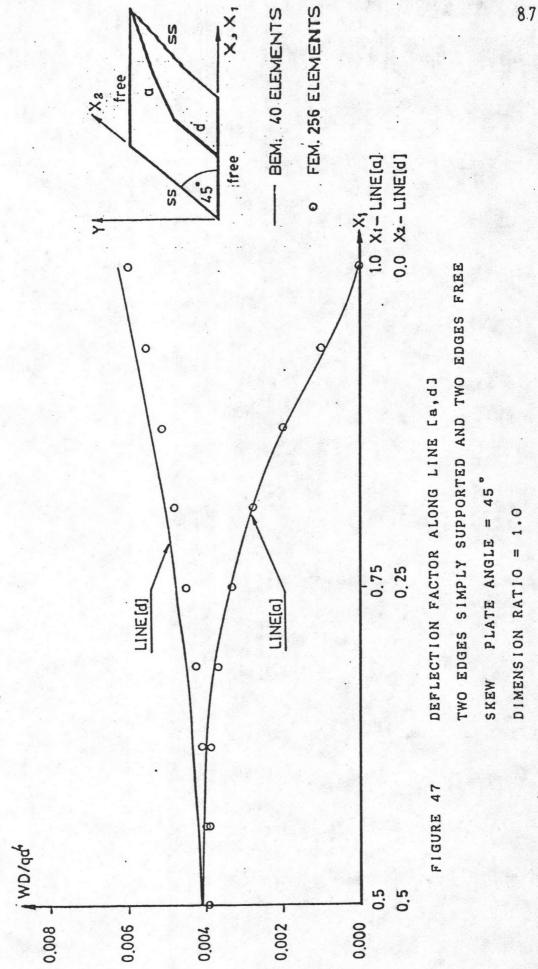


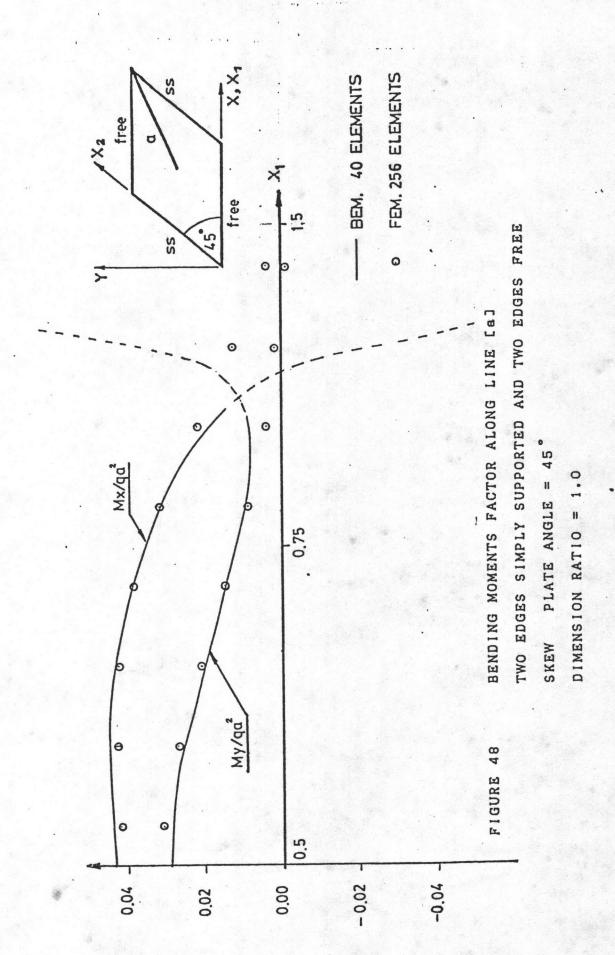


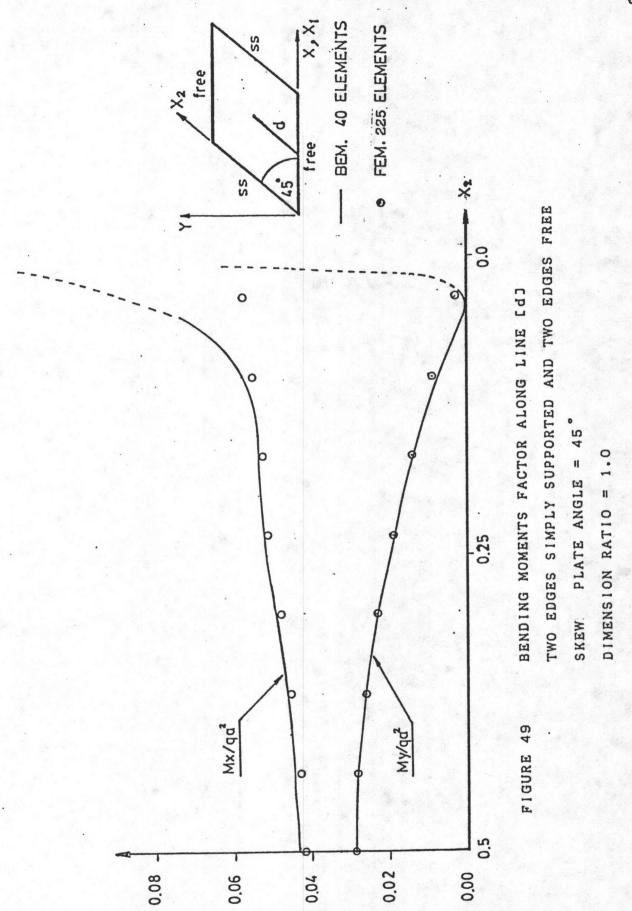












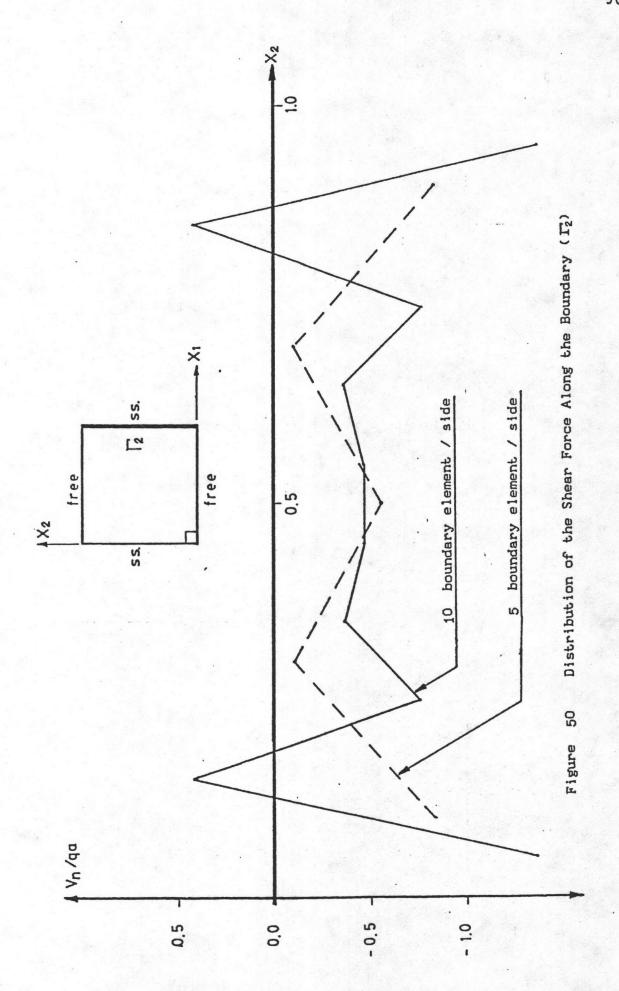


Table 1. Numerical Factor at Center of Rectangular Plate

with Simply-Supported Edges

Number of Boundary Elements = 40

Poisson's Ratio = 0.3

b/e	DEFLECTION WD/qa ⁴				OING MON	MENT	BENDING MOMENT My/qa²		
	BEM. x10 ⁻²	Ref.7	ERROR	BEM.	Ref.7	ERROR	BEM. x10 ⁻²	Ref.7	ERROR
1.0	0.4071	0.4060	0.2818	4.786	4.790	0.0828	4.786	4.790	0.0827
1.1	0.4879	0.4850	0.6002	5.544	5.540	0.0707	4.941	4.930	0.2221
1.2	0.5660	0.5640	0.3344	6.267	6.270	0.0540	5.001	5.010	0.1642
1.3	0.6400	0.6380	0.3263	6.933	6.940	0.9890	5.038	5.030	0.1699
1.4	0.7091	0.7050	0.5873	7.551	7.550	0.0118	5.012	5.020	0.1565
1.5	0.7728	0.7720	0.1067	8.110	8.120	0.1244	4.973	4.980	0.1413
1.6	0.8310	0.8300	0.1189	8.611	8.620	0.1025	4.929	4.920	0.1878
1.7	0.8837	0.8830	0.0787	9.067	9.080	0.1448	4.849	4.860	0.2266
1.8	0.9312	0.9310	0.0259	9.468	9.480	0.1269	4.788	4.790	0.0358
1.9	0.9739	0.9740	0.1115	9.831	9.850	0.1938	4.699	4.710	0.2230
2.0	1.0121	1.0130	0.0941	10.150	10.170	0.1934	4.623	4.640	0.3549
3.0	1.2220	1.2230	0.0813	11.864	11.890	0.2195	4.060	4.060	0.0011
4.0	1.2816	1.2820	0.0289	12.333	12.350	0.1352	3.841	3.840	0.0180
5.0	1.2976	1.2970	0.0444	12.457	12.460	0.0270	3.772	3.750	0.5779

Table 2. Numerical Factor at Center of Rectangular Plate
with Clamped Edges

Number of Boundary Elements = 20

Poisson's Ratio = 0.3

b/a	DEFLECTION WD/qa4			BEN	Mx/qa ²		BENDING MOMENT My/qa ²		
	BEM. ×10 ⁻²	Ref.7	ERROR	BEM. x10 ⁻²	Ref.7	ERROR (%)	BEM. x10 ⁻²	Ref.7	ERROR
1.0	0.1269	0.126	0.7528	2.285	2.31	1.0985	2.285	2.31	1.0985
1.1	0.1513	0.150	0.8701	2.660	2.64	0.7632	2.322	2.31	0.5232
1.2	0.1731	0.172	0.6264	2.992	2.99	0.0831	2.278	2.28	0.0825
1.3	0.1918	0.191	0.4379	3.265	3.27	0.1435	2.223	2.22	0.1355
1.4	0.2076	0.207	0.2789	3.494	3.49	0.1232	2.120	2.12	0.0000
1.5	0.2205	0.220	0.2245	3.675	3.68	0.1385	2.019	2.03	0.5409
1.6	0.2309	0.230	0.3865	3.813	3.81	0.0892	1.929	1.93	0.0624
1.7	0.2391	0.238	0.4776	3.926	3.92	0.1557	1.816	1.82	0.2024
1.8	0.2456	0.245	0.2256	4.005	4.01	0.1073	1.736	1.74	0.2224
1.9	0.2505	0.249	0.5941	4.070	4.07	0.0000	1.639	1.65	0.6502
2.0	0.2542	0.254	0.0702	4.115	4.12	0.1200	1.565	1.58	0.9639

Table 3. Numerical Factor at Center of Rectangular Plate

Two Edges Simply-Supported and Two Edges Clamped

Number of Boundary Elements = 20

Poisson's Ratio = 0.3

b/a	DEFLECTION WD/qa			BEN	DING MO	MENT	BENDING MOMENT My/qe ²		
	BEM. x10 ⁻²	Ref.7	ERROR (%)	BEM. ×10 ⁻²	Ref.7	ERROR (%)	BEM.	Ref.7	ERROR
1.0	0.1919	0.192	0.0342	2.430	2.440	0.4057	3.313	3.320	0.1925
1.1	0.2529	0.251	0.7894	3.074	3.070	0.1191	3.695	3.710	0.4127
1.2	0.3196	0.319	0.1812	3.760	3.760	0.0000	3.991	4.000	0.2376
1.3	0.3879	0.388	0.4333	4.456	4.460	0.0993	4.252	4.260	0.1856
1.4	0.4612	0.460	0.2727	5.155	5.140	0.2921	4.429	4.480	1.1359
1.5	0.5325	0.531	0.2838	5.833	5.850	0.2879	4.566	4.600	0.7409
1.6	0.6020	0.603	0.1720	6.478	6.500	0.3332	4.669	4.690	0.4441
1.7	0.6685	0.668	0.0695	7.092	7.120	0.3942	4.707	4.750	0.8925
1.8	0.7313	0.732	0.1024	7.656	7.680	0.3113	4.740	4.770	0.6232
1.9	0.7898	0.790	0.0256	8.181	8.210	0.3510	4.722	4.760	0.7843
2.0	0.8439	0.844	0.0156	8.657	8.690	0.3784	4.699	4.740	0.8583

Table 4. Numerical Factor at Center of Rectangular Plate with

Two Edges Simply-Supported and Two Edges Free

Poisson's Ratio = 0.3

b/a	DEFLECTION WD/qa ⁴			BENDING MOMENT Mx/qa ²			BENDING MOMENT My/qa²		
	BEM.	Ref.7	ERROR	BEM. x10 ⁻²	Ref.7	ERROR (%)	BEM.	Ref.7	ERROR (%)
			NUMBER	OF BOUNI	DARY EL	EMENTS =	20		
0.5	1.2575	1.377	8.6805	11.887	12.35	3.7456	1.1739	1.020	15.0878
1.0	1.2627	1.309	3.5309	11.908	12.25	2.7935	2.9184	2.710	7.6916
2.0	1.2896	1.289	0.0453	12.336	12.35	0.1081	3.7069	3.640	1.8382
			NUMBER	OF BOUNI	DARY ELI	EMENTS =	40		
0.5	1.3578	1.377	1.3928	12.297	12.35	0.4247	1.061	1.020	4.0329
1.0	1.2957	1.309	1.0101	12.152	12.25	0.7927	2.759	2.710	1.8108
2.0	1.2871	1.289	0.1453	12.321	12.35	0.2342	3.659	3.640	0.5198
			NUMBER	OF BOUNI	DARY EL	EMENTS =	60		
0.5	1.3793	1.377	0.1673	12.408	12.35	0.4711	1.026	1.020	0.5945
1.0	1.3036	1.309	0.4108	12.208	12.25	0.3438	2.716	2.710	0.2288
2.0	1.2898	1.289	0.0617	12.346	10.05	0.0300	3.644	3.640	0.1148

Table 5. Critical Angles for Moments and Shear Forces

b.c.	Moments	Shear forces
c/c	180.00	126.283
c/h	128.73	90.000
c/f	95•35	52.054
h/h	90.00	60.000
h/f	90.00	51 • 123
f/f	180.00	77•753

c = clamped , f = free , h = hinged

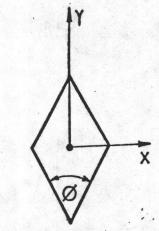


Fig. 51 Rhombic Plate

Table 6 Numerical Factor at Center of Rhombic Plate for Various Values of Skew Angle with Simply-Supported Edges
Poisson's Ratio = 0.3

ø	DEFLECTION WD/qa ⁴			BENI	OING MOM	IENT	BENDING MOMENT My/qe ²		
	BEM. x10 ⁻²	Ref.8 x10 ⁻²	Diff.	BEM. x10 ⁻²	Ref.8 x10 ⁻²	Diff.	REM. x10 ⁻²	Ref.8 x10 ⁻²	Diff.
			NUMBER	OF BOUN	DARY ELE	MENTS =	40		
90 °	0.407	0.406	0.350	4.787	4.80	0.263	4.787	4.80	0.261
85°	0.403	0.401	0.530	4.852	4.86	0.166	4.673	4.66	0.285
80 *	0.390	0.387	0.751	4.864	4.86	0.083	4.513	4.48	0.739
60°	0.263	0.256	2.664	4.298	4.25	1.122	3.432	3.33	3.073
50 *	0.179	0.172	4.118	3.688	3.62	1.889	2.722	2.58	5.514
40°	0.102	0.096	5.828	2.896	2.81	3.059	1.958	1.80	8.762
30°	0.044	0.041	6.326	1.984	1.91	3.878	1.206	1.08	11.67
			NUMBER	OF BOUN	DARY ELE	EMENTS =	80		
90 *	0.407	0.406	0.239	4.786	4.80	0.301	4.786	4.80	0.299
85°	0.403	0.401	0.429	4.852	4.86	0.161	4.668	4.66	0.180
08	0.393	0.387	1.603	4.899	4.86	0.807	4.534	4.48	1.216
60 °	0.260	0.256	1.336	4.273	4.25	0.538	3.379	3.33	1.465
50°	0.176	0.172	2.245	3.655	3.62	0.962	2.660	2.58	3.095
40°	0.010	0.096	3.738	2.864	2.81	1.925	1.905	1.80	5.806
30 °	0.043	0.041	4.537	1.963	1.91	2.757	1.176	1.08	8.891

Table 7. Convergence Comparison with Other Investigator

Numerical Factor at Center of Rectangular Plate with

Two Edges Simply-Supported and Two Edges Free

Number of Boundary Elements = 60

Poisson's Ratio = 0.3

NUMERICAL	EXACT	OUR PROP	OSED	Ref.[3]		
FACTORS	Ref.[7]	RESULTS	% Diff.	RESULTS	% Diff.	
DEFLECTIONS WD/qa	0.01289	0.012898	0.0617	0.013012	0.94647	
BENDING MOMENTS M _x /qa ²	0.1235	0.12346	0.0300	0.12412	0.50202	
BENDING MOMENTS My/qe²	0.0364	0.03644	0.1148	0.03596	1.20879	

Table 8. Convergence Comparison of Two Fundamental Solutions

Numerical Factor at Center of Rectangular Plate with

Two Edges Simply-Supported and Two Edges Free

Number of Boundary Elements = 60

Poisson's Ratio = 0.3

NUMERICAL	EXACT	{r²ln(r/	/Z)3/8MD	(r ² ln(r)3/8MD		
FACTORS	Ref.[7]	RESULTS	% Diff.	RESULTS	% Diff.	
DEFLECTIONS WD/qe	0.01289	0.012898	0.0617	0.012870	0.15515	
BENDING MOMENTS M _x /qa ²	0.1235	0.12346	0.0300	0.12323	0.21862	
BENDING MOMENTS My/qa ²	0.0364	0.03644	0.1148	0.036476	0.20879	