CHAPTER 2

THEORETICAL INVESTIGATIONS

2.1 Reactor Equation (6)

Any calculations concerning nuclear reactor behaviors always involve the calculations of neutron distribution and diffusion within reactor core. In fact the neutron diffusion behavior is the most important basis for reactor physics calculations. Therefore, for this study, the diffusion of neutron in a finite medium is briefly reviewed providing sufficient background for core and fuel analysis by using computer models.

Typical neutron diffusion is described as follows

[Rate of change in number of neutron in volume V] = [Rate of production of neutron in Volume V] - [Rate of absorption of neutron in volume V] - [Rate of Leakage of neutron from Volume V]

It can be writen as in equation (2.1) as below

$$\frac{d}{dt}\int_V n(r,t)\,dV = \int_V S(r,t)\,dV - \int_V \Sigma_a(r)\phi\,(r,t)\,dV - \int_V \mathrm{div}\vec{J}(r,t)\,dV \qquad (2.1)$$

where $n(r,t)$ = density of neutrons at point r and time t

$S(r,t)$ = the rate at which neutrons are emitted from point r and time t

$\Sigma_a(r)$ = macroscopic absorption cross section at point r.

$\phi(r,t)$ = the neutron flux at point r time t

$\text{div}\vec{J}(r,t)$ = the divergence of neutron current density at point r time t

V = volume of the system

When the above equation is solved using steady-state boundary condition, the equation can be reduced to

$$D\nabla^2\phi(r)-\Sigma_a\phi(r) + S(r) = 0 \qquad (2.2)$$

Equation 2.2 is called"steady state diffusion equation" in fissioning medium, this equation can be written as

$$\nabla^2\phi(r) + \frac{k\infty-1}{L^2}\phi(r) = 0 \qquad (2.3)$$

by substituting $S= k\infty\Sigma_a(r)\,\phi(r)$ and $D = \Sigma_a(r)L^2$

it is often written as

$$\nabla^2\phi(r) + B^2\phi(r) = 0 \qquad (2.4)$$

This equation is called "Generalized Reactor Equation"

Where ∇^2 = div grad or Laplacian

$\phi(r)$ = neutron flux at point r

$B^2 = \frac{k\infty-1}{L^2}$ = Buckling

$k\infty$ = infinite multiplication factor

L^2 = diffusion area

2.2 Critical Equation (6,7)

By solving equation 2.4 criticality occurs when the first term eigenfunction becomes unity, i.e.

$$k_1 = \frac{k_\infty e^{-B^2\tau_T}}{1+B^2L^2_T} = 1$$

$$k_{eff} = \frac{(\eta_T f p \varepsilon) e^{-B^2\tau_T}}{1+B^2L^2_T} = 1 \qquad (2.5)$$

This is called "Critical equation". The equation is valid when reactor is bare and age theory is valid

For heterogeneous reactor all parameter of equation 2.5 must be modified to include the fact that fuel meat and moderator are separate homogeneous regions. Some important parameters of equation 2.5 are sumarized in table 2.1

Table 2.1 Summary of Some Parameters for Critical Equation of Heterogeneous Reactor System

Parameter	
η_T	$= \frac{\sum \nu_n \bar{\Sigma}_{fn}}{\bar{\Sigma}_a}$
f	$= \frac{\bar{\Sigma}_{aF} V_F}{\bar{\Sigma}_{aF} V_F + \bar{\Sigma}_{aM} V_M \zeta}$
p	$= \exp\left[-\frac{N_F V_F I}{\xi_F \Sigma_{pF} V_F + \xi_M \Sigma_{SM} V_m}\right]$
ε^*	$=$ ———

*The value of fast fission factor for heterogeneous reactor is not much different from in the homogeneous case.

Note : ν_n = the number of neutrons emitted per thermal fission in the n^{th} isotope (in the case of

the fuel consists of a mixture of fissile and nonfissile isotopes).

$\bar{\Sigma}_{fn}$ = average thermal macroscopic fission cross section for the n^{th} isotope.

$\bar{\Sigma}_a$ = average thermal absorption cross section for the fuel mixture.

$\bar{\Sigma}_{aF}$ = average thermal absorption cross section for the fuel only.

$\bar{\Sigma}_{aM}$ = average thermal absorption cross section for moderator.

V_F, V_M = the volumes of fuel and moderator, respectively.

ζ = the ratio of average flux in the moderator ($\bar{\phi}M$) to the average flux in the fuel ($\bar{\phi}F$) is called thermal disadvantage factor.

N_F = the atom density of the fuel lump.

I = Resonance Integral = $\overline{\xi\Sigma}_p \int \sigma_{aF} \phi_F dE$, where the term $\overline{\xi\Sigma}_p$ is equal to $\frac{1}{V}(\xi_F \Sigma_{PF} V_F + \xi_M \Sigma_{SM} V_M)$, $V = V_F + V_M$, Σ_{PF} = Potential scattering of fuel, Σ_{SM} = Scattering cross section of moderator, at resonance energies, and ξ_F, ξ_M are the average increase in lethargy per collision in the fuel and moderator, respectively.

2.3 Method of Diffusion Theory in Heterogeneous System (7)

For thermal reactor, a reactor is said to be heterogeneous if the neutron mean free path at thermal energies is comparable to or less than the thickness of fuel element.

Consider, square lattice or hexagonal lattice in fig 2.1 below. By dividing the fuel-moderator lattice into unit cell,

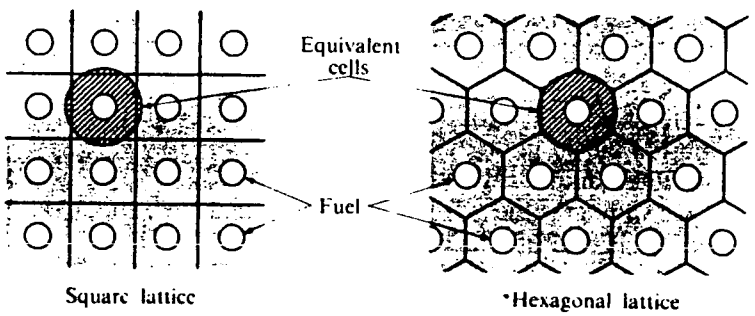

Fig.2.1 Two typical heterogeneous lattices and equivalent cells for each.

in individual unit cell, each containing one fuel lump at its center. Since all cells are identical in an infinite uniform lattice, there can be no net flow of neutron from one cell to another. It follows that the current density is zero along the boundary of each cell. This procedure, of using an equivalent cell with a zero current boundary condition, is known as the Wigner-Seitz Method. With this assumptions the thermal flux throughout the equivalent cell can easily be computed by diffusion theory. Consider, for example, a cylindrical fuel rod of radius a in a cylindrical cell of radius b as shown in fig 2.2 below

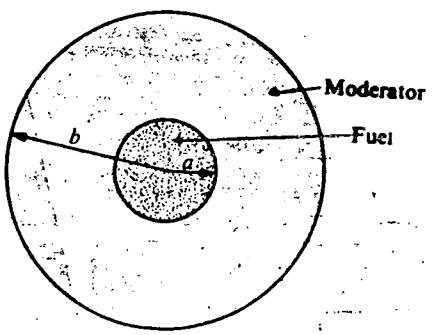

Fig.2.2 Cylindrical equivalent cell.

The diffusion equations for the fluxes in the fuel and moderator are, respectively

$$D_F \nabla^2 \phi_F - \Sigma_{aF} \phi_F = 0 \qquad o<r<a, \qquad (2.6)$$

and

$$D_M \nabla^2 \phi_M - \Sigma_{aM} \phi_M = q \qquad a<r<b \qquad (2.7)$$

Where D_F, D_M, are thermal diffusion coefficient of fuel and moderator, Σ_{aF}, Σ_{aM} are the thermal absorption cross section of the two regions, and q is the (constant) thermal slowing-down density in the moderator.

Note that sections 2.1 , 2.2 and 2.3 are important fundamental theory and equations for elaborate three dimensional reactor calculations. The actual data for the 3D calculations are very complicated such that computer models and codes have been developed to make the calculations more efficient and accurate. There were two major interest in this study requiring elaborate reactor calculations. They were the determination of critical mass, fuel burnup , excess reactivity of TRR-1/M1 so that core and fuel management strategy can be developed for future use.

2.4 Reactivity

Normal operation of nuclear reactor is at steady state. Its behavior can be predicted by diffusion equation, and its critical state is maintained at condition described in previous sections. However, as the fuel is fissioning away or absorbent is inserted in the core, its critical state is affected. Sufficient fuel must be available in the core to allow the critical condition to be maintained as long as needed by means of control machanism.

Reactivity is the measure of the departure of a reactor from critical condition. In the previous section, the effective multiplication factor (k_{eff}) determines changes of neutron density within a reactor. Since the power level is directly proportional to the neutron density. The power level, neutron density, etc, are constantly changing whenever k_{eff} is not equal to 1.000. The difference between a given value of multiplication factor (k_{eff}) and 1.000 is known as "change" of multiplication factor, δk, i.e.

$$\delta k = k_{eff} - 1.000 \qquad (2.6)$$

δk can be either positive or negative, depending on value of k_{eff}. A useful quantity, denoted by ρ and known as "reactivity", is given by

$$\rho = \frac{k_{eff} - 1.000}{k_{eff}} = \frac{\delta k}{k_{eff}} \qquad (2.7)$$

ρ in Eq. 2.7 has no unit. The value is in fraction. Reactivity can also be expressed in "dollar" and "cent" where

$$\text{Reactivity} = \rho / \beta_{eff} = \frac{\delta k}{k_{eff}} \cdot \frac{1}{\beta_{eff}} \qquad (2.8)$$

β_{eff} = Fraction of Delayed Neutron is equal to 0.007 for TRIGA reactors(8)

2.5 Loss of Reactivity (7)

In the field of core fuel management of research reactor it is necessary to load sufficient U-235 for a period of a steady state power that is to create certain excess reactivity in order to compensate the loss of reactivity caused by the depletion of fuel due to burnup, the temp rature coefficients, fission product poisoning or even the isotope production, these effects will be briefly described later.

As the preceding description of the multiplication factor, k_{eff}, is the function of four factors, i.e., Eta (n) , thermal utilization (f), resonance escape probability (p) and the fast fission factor (ε) these factors and their changes cause changes in reactivity of the system. In fact the major direct effects to each of four factor are caused by changes in macroscopic cross section. Details consideration of phenomena causing loss of reactivity are as follows.

2.5.1 Fuel Burnup and Conversion (7)

The fuel consumed in the reactor core is not at constant rate, nor fertile material converted uniformly. If the reactor is fueled with a single fissile isotope. and no fertile material is present. The depletion of this isotope is

$$N_F(r,t) = N_F(r,o) \exp\left[-\bar{\sigma}_{aF}\int_o^t \phi_T(r,t)dt\right] \qquad (2.9)$$

If the reactor is fueled with uranium, the rate of concentration of Pu^{240} and Pu^{241} can be found in the following equation.

$$\frac{dN_{40}}{dt} = (N_{49}\bar{\sigma}_{a49} - N_{40}\bar{\sigma}_{a40})\ \phi_T$$

$$\frac{dN_{41}}{dt} = (N_{40}\bar{\sigma}_{a40} - N_{41}\bar{\sigma}_{a41})\ \phi_T$$

where $N_F(r,t)$ = the atom density of ^{235}U at point r time t

$N_F(r,o)$ = the atom density of ^{235}U at point r time o

$\bar{\sigma}_{aF}$ = thermal absorption cross section

$\phi_T(r,t)$ = thermal flux at point r time t

N_{40}, N_{49}, N_{41} = the concentrations of Pu^{240} , Pu^{239} and Pu^{241} respectively.

2.5.2 Temperature Coefficients (7)

Many of the parameters determining the reactivity, i.e. thermal utilization, resonance escape probability, diffusion length and so on, are functions of the temperature of the fuel, moderator and coolant The temperature coefficient denoted by α_T, and defined by

$$\alpha_T = \frac{d\rho}{dT} \qquad (2.10)$$

where ρ = the reactivity of the system
T = the temperature of specific component

Substitute $\rho = 1-k^{-1}$ in equation (2.10)

Then $\alpha_T = \frac{1}{k^2}\frac{dk}{dT}$

In all cases $k \simeq 1$

Therefore, $\alpha_T = \frac{1}{k}\frac{dk}{dT}$ (2.11)

2.5.3 Fission Product Poisoning (7)

Due to substantial absorption cross sections and amount of the fission products. They are neutron absorber which their appearance in the reactor reduce value of the multiplication factor. This is why these nuclei are called "Fission Product poisons"

The only effect of fission product poisons on the multiplication factor is on the thermal utilization. The reactivity equivalent of the poison is

$$\rho = \frac{f' - f}{f'} = \frac{-\bar{\Sigma}_{ap}}{\bar{\Sigma}_{aF} + \bar{\Sigma}_{aM}} \qquad (2.12)$$

where the primed parameters refer to the poisoned reactor.

Again $k^\infty = 1 = \eta_T f p \varepsilon$

$$= \frac{\eta_T p \varepsilon \bar{\Sigma}_{aF}}{\bar{\Sigma}_{aF} + \bar{\Sigma}_{aM}}$$

$$= \frac{\nu p \varepsilon \bar{\Sigma}_f}{\bar{\Sigma}_{aF} + \bar{\Sigma}_{aM}} \qquad (2.13)$$

Finally, from equations (2.12) and (2.13)

$$\rho = \frac{-\bar{\Sigma}ap/\bar{\Sigma}_f}{\nu p\epsilon} \qquad (2.14)$$

Equation (2.14) is suitable for calculations of fission-product poisoning. The most important fission-products should be taken into account in the consideration of reactivity loss are xenon-135 and samarium-149.

In the calculation of reactivity rundown caused by burnup of the fuel in the core, it is very important to calculate the cross section first, in order to be used in the subsequent calculations for fuel burnup of each core. The cross section in the following calculation will be the function of time of burnup of fuel in the core. This can be accomplished by using the computer codes such as EPRI-CELL/RERTR(9) This code was originally generated by Nuclear Associates International Corporation (NAI) as an account of work sponsored by Electric Power Research Institute, Inc. (EPRI). This code has been modified extensively for use in the calculation of research and test reactors in the Reduced Enrichment Research and Test Reactor (RERTR) Program at the Argonne National Laboratory. The code computes the space, energy and burnup dependence of the neutron spectrum within cylindrical cell of Light Water Reactor fuel rods. Its primary output consists of broad group, microscopic, exposure dependent cross sections for subsequent use in multidimensional diffusion theory depletion analysis.

In preparing the inputs for the EPRI-CELL/RERTR (9) computer code in order to generate cross section for the TRR-1/M1 fuel rod, it is necessary to specify the pure number density of constituent nuclides

in compositions (i.e. UZrH, H_2O etc), the volume fraction in each zone. Volume fractions allow easy input of the atom densities of zone containing more than one composition.

These inputs were prepared by using the Unit-Cell that is equivalent to the hexagonal cell of the TRR-1/M1 fuel rod, which will be discussed in the next chapter. In order to obtain the accurate results from the calculations the computer codes DIF3D (10) and REBUS-3 (11) (Three Dimensional Diffusion Theory and Reactor Burnup Systems, respectively) were used for diffusion and burnup calculations.