CHAPTER II

Literature Review

2.1 Concepts and Theoretical Background

The term structure of interest rates is the structure of spot rates in each time period. There are two main approaches to modeling the term structure of interest rates. The first approach is based on models of evolution of state variables and asset prices using equilibrium argument. On the other hand, the second approach is to measure the term structure using statistical techniques or fitting the data. This approach basically is to fit the data obtained from asset prices to describe the yield curve without considering the factors determining it. This study focuses on the former approach and provides brief discussion of this area of studies

An equilibrium model of term structure is a model that is derived from a general equilibrium model of the economy. They use generally constant parameters including a constant volatility and the actual parameters used are often calculated from the historical data. There are some popular examples of models of the term structure that have been constructed for a given choice of state price process such as Vasicek (1977) and CIR (1985).

These models are the traditional one factor equilibrium models. Both models incorporate a mean reversion feature that means that the behavior of short term interest rate differs from stock price. The behavior of interest rate appears to be pulled back to the long run average level overtime. This phenomenon is known as mean reversion. When interest is high, mean reversion tend to cause it to have negative drift; when interest rate is low, mean reversion tends to cause it to have a positive drift. There are compelling economic arguments in favor of mean reversion. When interest rates are high, the economy tends to slow down, causing low demand for funds from borrowers. As a result, interest rates decline. When interest rates are low, there tends to be high demand for funds from borrowers, thus increasing the level of interest rate.

This paper looks at two leading single factor interest rate models, those of Vasicek (1977) and of CIR (1985). Both models assume that the risk neutral process for the instantaneous short rate r is stochastic, with one source of uncertainty. The stochastic process includes drift and volatility parameter, which depend only on the short rate r, and not on time. The short rate models involve number of variables, and different parameter choices for these variables will lead to different shapes for the term structure generated from the model.

Both of the interest rate models features called mean reversion of the short rate, that is, a tendency for the short rate to drift back to some underlying rate. The two models differ in handling of volatility. I start with Vasicek model and then consider CIR model.

2.1.1 Vasicek Model

As proposed by Vasicek model (1977), the model assumes that the term structure of interest rates at time t is given by instantaneous interest rate r(t), which follows the mean- reverting process of the form.

$$dr(t) = \kappa(\theta - r(t))dt + \sigma dz(t) \qquad (1)$$

From (1), a small change (dr) in the short rate in time increment dt includes a drift back to mean level θ at rate κ for the short rate. The second volatility term involves uncertainty, $dz(t)$ representing a standard Brownian motion. The short rate r is assumed to be the instantaneous rate at time t appropriate for continuous compounding.

In this one-factor model, the values of bonds and options on bond are solely function of short rate. As derived in Harrison and Kreps (1979) and extended by Harrison and Pliska (1981), the value of a discount bond $P_\tau[r(t)]$ at time t with maturity τ and face value 1 to be expressed discount factor in a risk -neutral economy:

$$P_\tau[r(t)] = \widetilde{E}_t\left[\exp\left(-\int_t^{t+\tau} r(s)ds\right)\right] \tag{2}$$

The expectation $\widetilde{E}_t$ has to be taken with respect to the risk –neutral stochastic process of spot rate:

$$dr(t) = \kappa\left[\widetilde{\theta} - r(t)\right]dt + \sigma d\widetilde{Z}(t) \tag{3}$$

where $\widetilde{\theta} = \theta + \lambda$ with λ is the market price of risk and where $\widetilde{Z}(t)$ is another standard Brownian motion.

From (2) and (3), $r(t)$ is normal, therefore $\int_t^{t+\tau} r(\tau)d\tau$ is also normal. This makes the calculation particularly simple, because we can use the following result: when χ is a normal random variable, $E(e^{\chi}) = e^{E(\chi)+\frac{1}{2}Var(x)}$. Thus, the mean and variance of $\int_t^{t+1} r(\tau)d\tau$ is needed to calculate. Finally, the closed form solution of discount bond value can expressed as:

$$P_\tau[r(t)] = \exp\left(\frac{1-e^{-\kappa\tau}}{\kappa}[R_\infty - r(t)] - \tau R_\infty - \frac{\left(1-e^{-\kappa\tau}\right)^2}{4\kappa^3}\sigma^2\right) \tag{4}$$

The yield to maturity of discount bond is defined as $R_t(t) = -\frac{-\ln P_\tau[r(t)]}{\tau}$, which implies:

$$R_\tau(t) = R_\infty + \frac{1-e^{-\kappa\tau}}{\kappa\tau}[r(t) - R_\infty] + \sigma^2\frac{\left(1-e^{-\kappa\tau}\right)^2}{4\kappa^3\tau} \tag{5}$$

As the maturity $\tau \longrightarrow \infty$, the yield to maturity converges to:

$$\lim_{\tau \longrightarrow \infty} R_\tau(t) = R_\infty = \widetilde{\theta} - \frac{\sigma^2}{2\kappa^2} \tag{6}$$

Notice that the process that exhibits a mean-reverting feature from Vasicek model is the parameter θ may be regarded as the equilibrium level of the short term interest rate, around which it stochastically evolves. When interest rate falls below (above) its long term value θ, the expected instantaneous variation of interest rate is positive (negative). In this case, the short term interest rate will tend to move up (down). It will move toward its long-term value quickly when it is far from it and when the parameter κ (speed of return to the long term mean value) is high. Also, the volatilities of short rate have normal distribution. One drawback of the Vasicek model is that interest rate can become negative, although with a small probability in practice.

The purpose of the parameterization (5) is that positive interest rates at all maturities can be guaranteed by the simple parameter restrictions $\kappa > 0, R_{\infty} > 0, \sigma > 0$ and $r(t) > 0$.

While the Vasicek model assumes that the volatility of the short rate is independent of the level of the short rate. This is almost certainly not true at extreme level of the short rate. Periods of high inflation and high short-term interest rates are inherently unstable and, as a result, the volatility of the short rate tends to be high. Also, when the short-term rate is very low, its volatility is limited by the fact that interest rates cannot decline much below zero[1].

[1] See Tuckman (2002, p.254-258)

2.1.2 CIR Model

The previous model shows it assumes the short rate to follow the normal distribution, which can result in negative rate. The CIR model is proposed the modified process in order to overcome the negative interest rate problem. Under the CIR model, the risk-neutral dynamics of short rate are described by

$$dr(t) = \kappa(\theta - r(t))dt + \sigma\sqrt{r}dz(t) \tag{7}$$

In equation (7) κ is the mean reversion coefficient, θ is the mean of process, σ is the proportional to the square root of level of interest rate and dz is a Wiener process. This model has the same mean reverting drift as the Vasicek model, but the standard deviation of change in the short rate in a short period of time is proportion to the square root of level of interest rate. It means that, as the short term interest rate increase, its standard deviation increases. This model overcomes the negative interest rate problem.

Under their assumptions, CIR (1985) show that the value P(r, t) at time t of all interest rate dependent claims in the economy satisfy to following version of the fundamental valuation equation for price of a discount bond:

$$rP + \lambda rP_r = P_r\kappa(\theta - r) + P_t + \tfrac{1}{2}P_{rr}\sigma^2 r \tag{8}$$

and the conditional variance is given by

$$V_t[r(t+s)] = \sigma^2\frac{1-e^{-\kappa s}}{2\kappa}\left[2r(t)e^{-\kappa s} + \theta(1-e^{-\kappa s})\right] \tag{9}$$

With the boundary condition that

$$P(r, T, T) = 1 \tag{10}$$

By taking the relevant expectation, we obtain the bond price as[2]:

$$P_\tau\left[r(t)\right] = A(\tau)^{R_\infty} e^{-B(\tau)r(t)}, \tag{11}$$

where for $\tau = T - t$ (12)

$$A(\tau) = \left[\frac{2\gamma e^{(\tilde{\kappa}+\gamma)\tau/2}}{(\tilde{\kappa}+\gamma)(e^{\gamma\tau}-1)+2\gamma}\right]^{\frac{\tilde{\kappa}+\gamma}{\sigma^2}}$$

$$B(\tau) = \frac{2(e^{\gamma\tau}-1)}{(\tilde{\kappa}+\gamma)(e^{\gamma\tau}-1)+2\gamma}$$

$$\gamma = \sqrt{\tilde{\kappa}^2 + 2\sigma^2}$$

$$R_\infty = \frac{2\kappa\theta}{\tilde{\kappa}+\gamma}$$

and

$$\tilde{\kappa} = \kappa - \lambda$$

The bond is commonly quoted in term of yields rather than price. For the discount bond we are now considering, yield to maturity on a τ - period zero-coupon bond, $R(r,t,T)$ from equation (9) is

$$R(r,\tau) = \frac{-\ln(P)}{\tau} = \frac{-\ln(A(\tau)) + B(\tau)r}{\tau} \tag{13}$$

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[2] The notations differ slightly from the usual representation of the CIR model in order to facilitate comparison with the Vasicek model.

Both the CIR model and the Vasicek model are is a single factor equilibrium model of the term structure that is consistent with asset pricing equilibrium and free of arbitrage opportunities. Specially, the CIR model retains the feature that the interest rate must be non - negative. Therefore it is interesting to study how these two models fitted with the term structure of Thailand.

2.2 Empirical Studies

The study involves on the empirical test of the term structure of interest rates issue, which estimated by the Vasicek model and CIR model. The study also testing trading strategy based on each yield curve and the studies of the Thai bond market. Thus, I divide the literature, which involves the term structure of interest rates into four groups.

2.2.1 Empirical Test of CIR Model

The first extensive empirical analysis using the CIR model is due to Brown and Dybvig (1986), who interprets a model of the nominal term structure and analyze US treasury bills, notes, and bonds over the period 1952-83. Their approach, which we broadly follow, estimates the (risk-adjusted) parameters of the model from cross-sections prices. They take the market price of every treasury bills and bonds and estimate the model parameters that produce the best least squares fit of model to observed market prices of treasury bills better than it does other treasury issues. However, the Brown and Dybvig approach has also been by Barone, Cuoco, and Zautzik (1991) and Brown and Schaefer (1994).

While Gibbons and Ramaswamy (1993) use the CIR model to describe the real rather than the nominal term structure. In the absence of traded index bond in the US, they test the model by imposing the further assumption of monetary neutrality which allows them to use the model to characterize the real return on a nominal bond up to its maturity in terms of the model price of the corresponding real bond and the realized rate of inflation over its life. Gibbons and Ramaswamy generate a number of restrictions on the moments of this real return and use the generalized method of moments to estimate the CIR parameters. Testing the restrictions on the moments also provide a test of the model. The result are encouraging, with the data failing to reject the model since CIR model performs reasonably well in explaining short-term treasury bills returns.

In addition, Brown and Schaefer (1994) estimate real term structures from Cross-sections of British government index-linked bond prices by using CIR model. The model closely approximate the shapes of the directly-estimated term structures and as Brown and Dybvig (1986) and Barone, Cuoco, and Zautzik (1991) have found, the values of interest rate volatility estimated from cross-sections correspond well to estimates obtained from time series. Also their results differ from those obtained by others; they found that the implied real long rate shows a high degree of stability, particularly when estimates are averaged over sub period. The other parameters, however, are often highly correlated and intertemporal parameter stability is rejected.

2.2.2 Comparable Empirical Tests between Vasicek and CIR Model

Other tests of the CIR model include Longstaff (1989), Longstaff and Schwartz (1992). In these empirical analyses the CIR model is quite often superior to Vasicek model, according to comparisons based only on time series data without considering cross-sectional data. Since the parameter estimated on cross-sectional data are often nonsensical (i.e. negative variance) and another problem is that the parameters are not very stable over time.[3] In contrast, Pearson and Sun (1994) conclude that the CIR model fails to provide a good description of the treasury market. However, the cross-sectional estimated model will provide the best possible fit and it not needs more data than just for one day.

De Munnik and Schotman (1994) compare time series and cross-section estimates of Vasicek and CIR model for Dutch bond market. They find that there is greatly similarity between the two models with respect to the cross sectional

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[3] See Brown and Dybvig (1986) and also other empirical papers

parameter estimates and implied bond option values. From time series estimation, they find that for some maturity the data reject the constant volatility Vasicek model and indicate the presence of the CIR volatility effects.

They also comment that both procedures have their merit and drawbacks. Estimation of a mean reversion parameter for the short- term interest rate will be very difficult with time series data since one require any observations spanning a large number of years in order to estimate mean reversion parameter.[4] But a long time series creates its own problems, since the empirical model has to cope with the structural breaks.

For the empirical analysis based on panel data, which consists of both time series and cross sectional data, includes Chen and Scott (1993), Pearson and Sun (1994), Babbs and Nowman (1999), Geyer and Pichler (1999), Jong and Santa-Claa (1999) and Miyazaki and Tsubaki (2001).

Chen and Scott (1993) estimate parameters of a multifactor CIR model using the maximum-likelihood (ML) method including observation error in their estimation equations. They estimate parameters of one-, two-and three-factor models based on time series data of four bond prices. Assuming there is no observation errors in 13-week T-bills yield data, they directly adopt it as the first factor, so their first factor is not the one derived implicitly from a time series of all maturity bond yield data.

Pearson and Sun (1994) estimate parameters of a two-factor CIR model based on time series and cross-sectional data using the ML method and assuming that the observation error is zero. In their estimation technique, the number of factors should match the number of cross-sectional data point, and it is difficult to estimate parameters of a one-factor model based on time series and cross-sectional data.

The most famous of the parameter estimation techniques based on panel data of bond yields is the Kalman filter approach. Babbs and Nowman (1999) estimate the

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[4] See Enders (1995,p.254-258)

parameters of the generalized Vasicek model based on panel data using the Kalman filter approach. Because the transition density function of the Vasicek model follows a normal distribution, it is not necessary to use the normal approximation in application of Kalman filter.

Geyer and Pichler (1999) adopt the Kalman filter approach to estimate the parameters of a K-factor CIR model. In their application, they substitute the non-central chi square density with a normal density, which has the same first two moments.

Miyazaki and Tsubaki (2001) refer that Geyer and Pichler (1999) do not mention anything about the impact of the substitution on the transition density. So they provide a comparison of the CIR model and the Vasicek model based on panel data for the Japanese government bond market. They provide a semi parametric estimation technique that does not require assumption of a distribution density function. The technique can be implemented using generalized method of moment (GMM). They find that average absolute differences in each specific-maturity time data between estimated yield based on panel data and actual yields in the sample period are generally smaller in CIR model than in the Vasicek model.

Although, the parameter estimation techniques based on panel data is the new method to fitted bond data but it is more complexity method. However, the famous methodology that referred in many papers is related to Brown and Dybvig (1986), who represented empirical result for the U.S. market and also De Munnik and Schotman (1994) who shown the empirical result in Dutch bond market. Hence, I will estimate term structure in Thailand by closely their method.

2.2.3 Trading Strategy Based on Estimated Yield Curve

There are two recent papers which differ from earlier work on bond pricing model since the researchers want to know whether term structure models contain information about future bond returns and which model seem to be best at identifying mispriced bond. Hence, they focus on how useful the model residuals are to a bond trader and whether the economic models outperform other models for the purpose of identifying mispriced bonds. It involves a simple trading strategy which one can realize abnormal returns by buying (shortselling) bonds that were classified as undervalued (overvalued) relative to a particular estimated term structure. Then abnormal returns from bond trading are measured relative to a particular estimated term structure.

Sercu and Wu (1997) estimate daily Vasicek, CIR and Spline models on Belgian data and compare the trading profit that can be made on the basis of their residuals. Abnormal returns measured using three difference benchmarks. One is the duration ratio, the second benchmark is the return on a portfolio that matches the bond in term of both duration and convexity and the last benchmark is the conditional expected bond return implied by the change in the fitted model prices. They found that the best results are obtained if trading is based on the Vasicek and CIR models, while the traditional spline model overfit the data.

Ioannides (2003) compares difference spline methods of estimating the term structure of interest rate on a daily UK treasury bills and gilts data since these models are no economic assumption support and still used in the financial community .The researcher adopts the work of Sercu and Wu (1997). Also apply a contrarian trading strategy by buying (selling) bond that are consider underpriced (overpriced) and weighting each by factor proportional to size of mispricing. The result reveals that parsimonious representations of the term structure perform better than their spline counterparts characterized by a linear functional form. This suggests that the specification of the functional form of the model is important for pricing.

Nonetheless, due to the limitation of Thai bond data I apply a very simple trading strategy in order to measure excess return.

2.2.4 Previous Studies of the Thai Bond Market

The study of term structure of interest rates has become an important topic for many researches in various aspects. Some studies investigate the relationship of term structure of interest rate and economic indicators; others study the development out the approach in order to develop the bond market.

There are two earlier papers which demonstrate that Thai bond market has problem of growth without well-functioning bond market. The first empirical study begins by Herring and Chatusripitak (2000) and subsequently study is Barry and Pipat (2004). These studies are brief described as follow.

Herring and Chatusripitak (2000) study the development of the Thai bond market. They claim that the under development of the Thai bond market can be attributed to several causes. First is lack of the benchmark, market-determined yield curve. Second, the Thai government had constructed a captive market for its securities. Most of these securities were held to maturity. This discouraged secondary market trading and meant that the interest rate did not reflect the true opportunity cost of fund. Third, weak accounting and disclosure standards impeded the evaluation of credit risk and made it difficult for external investors to value risky debt. Also, the problem of tax laws and weak legal infrastructure has power on deterrent the secondary market. The underdevelopment of Thai bond market may cause serious distortion in Thai economy as the financial crisis in 1997. Prior to the crisis, the government bonds solely as a mean of financing deficits rather than a way of maturing the development of a bond market. Hence, if the government begins to reforms designed to stimulate develop of both the primary and secondary bond market, it will be rebuild the stronger financial system.

Barry and Luengnarumitchai (2004) study the development of Asia bond market that included Thailand. They claim that the slow development of Asian bond market is a phenomenon with multi dimensions. So they use the variables from many empirical result that determine the bond market development such as economic size, natural openness, legal system, law, size of banking system, interest rate variability, and exchange rate regime. They test these factors using the multivariate regression analysis of annual data from 1990-2001. The relationship between the bond market characteristics and these factors provide the positive sign in case of economic size, legal system, size of banking system and also the exchange rate. These factors will encourage the development of bond market. However, Asian countries have strong fiscal balance and heavy depend on banks so it is difficult to develop bond market as efficient and well capitalized as the advance industrial countries. Finally, it results in the difference between Asia bond market and the rest of the world.

The studies of bond market development approach show that the absence of active bond market leads to increase instability of financial market.

Besides, there are some studies that investigate the relationship of term structure of interest rate and economic indicators for example expected inflation and GDP. These studies attribute to Fisher (1930) construct the relationship between term structure of interest rate and expected inflation. Also, apply the changing in term structure of interest rate to forecast the business cycle.

Jornjaroen (2000) tests the relationship between term structure of interest rates and economic indicators such as expected inflation and expected GDP by Ordinary Least Square (OLS). The relationship can express as the following equation

$$\pi_t^m = \alpha + \beta i_t^m + \varepsilon_t \tag{14}$$

This relationship based on Fisher Hypothesis that the expectation inflation of m period equal to the difference between the nominal interest rate of m period and real interest rate of m period as the following as

$$E_t \pi_t^m = i_t^m - rr_t^m \tag{15}$$

Since E_t is the expectation at time t

π_t^m is the inflation rate form time t to t + m

i_t^m is the m- period nominal interest rate at time t

rr_t^m is the m- period(ex ante) real interest rate at time t

The real inflation in m period can write in form of the combination of the expected inflation and the forecast error of inflation.

$$\pi_t^m = E_t \pi_t^m + \varepsilon_t^m \qquad (16)$$

which $\varepsilon_t^m = \pi_t^m - E_t \pi_t^m$ represent the forecast error of inflation. If substituted (15) in (16), the result will be

$$\pi_t^m = i_t^m - rr_t^m + \varepsilon_t^m \qquad (17)$$

Since rr_t^m is constant so the equation (17) is same as the equation (15).

Jornjaroen (2000) finds that the spread of term structure of interest rates without relationship with expected inflation that differs from the foreign researches. The difference result may due to high volatility of real return from investment in the study period. For the GDP, in case of upward sloping of term structure of interest rate present the future economic growth. Besides, the researcher adds future consumption and investment in order to confirm the relationship between these factors and term structure. Overall result concludes that term structure of interest rates has relationship with changing of some economic indicators with high significant level. However, he forecast only the inflation in the next period by using the spread of interest rate. Also, the data does not consider the property of stationary. All of these reasons might affect the result of study.

Thammapukkul (2000) examines whether or not the interest rate spreads have the predictive power on economic growth in Thailand during 1997-2000. The interest rate spreads are categorized into three groups as the interest rate spreads with the difference in risk, the interest rates with the difference in maturity and the loan-deposit spread. The study employed OLS as the econometric and the economic growth by testing with the percentage change of GDP, private investment and private consumption for the whole period of the time during 1997-2000. The study also tests the relationship before and after the economic crisis which the study had considered the second quarter of year 2000 as the breakpoint reflecting the economic crisis. The statistical result showed that the interest rate spreads of government bond with difference in maturity could explain a change in economic growth.

Thanakamonant (2000) studies the relationship between maturity of bond and term structure of interest rates by multiple regression. The spread between long term and short term government bonds used to analyze term structure of interest rate. Also, use duration as the proxy of changing in structure maturity. The result shows that structure of maturity has relationship with the changing of term structure of interest rates. For the duration variable, if the duration increases the number of long term bond would greater than the short term bond by relatively. Moreover, spreads between long term and short term bonds will increase and result in the term structure of interest rate changes.

Phannikorn (2001) analyses the relationship between yield and inflation under Fisher hypothesis and to predict inflation in the period of 1995-2001. This study extends from Jornjaroen (2000) by considering both in case of level and spread analysis. Also, test the ADF and Engle Granger cointegration. There are two groups of yields that employed in this study: TFB Implied Risk Free Yield Curve and Repo & ThaiBDC Government Bond Yield Curve. The result represents that Repo & ThaiBDC Government Bond Yield Curve relate to general inflation and core inflation more than TFB Implied Risk Free Yield Curve both in case of level analysis and spread analysis. On the other hand, the result of forecasting shows the poor forecasting ability of yield curve. It may due to the small size and illiquidity of Thai bond market which represent in efficient bond market.

To sum up these previous literatures in Thai market showed that bond market has relationship with economic indicators such as GDP and expected inflation. Furthermore, I find that two types of the yield of bond could not be uses. First is the yield of the bond that has time to maturity greater than 1 year in case of Repo & ThaiBDC Government Bond Yield Curve. Second is the yield of bond which greater than 2 years in case of TFB Implied Risk Free Yield Curve. They were published in recent years, so it is the reason why these curves could not be use in. Therefore, they have not enough information of the lag of inflation to find relationship between long term yields. Moreover, the Repo& ThaiBDC Government Bond Yield Curve, which is the combination of repurchase rate of the Bank of Thailand and the ThaiBDC Government Bond Yield Curve of ThaiBDC, were terminated since March1, 2001 and changed into T-Bill Government Bond Yield Curve.

Overall, the important points of the previous studies were the liquidity of bond market, including number of bond issued, bond trading and maturities, that not enough to make market depth and standard on bid-ask price. Besides, there are no discussions about the construction detail of yield curve in their papers that will affect their result.

This study differs from earlier studies in Thailand because it develops a new way of constructing the yield curve. Prior studies used either the Repo or the ThaiBDC as the benchmarks while this study introduces a new revolutionary way of constructing the benchmark by using the equilibrium models. The introduction of these models is strongly beneficial to the Thai bond market as whole. For example, policy makers and financial intermediaries could use these to monitor the stability of the economy and to forecast into future. Investor can also weight exposed and expected return of their investment in the Thai bond market as well.

In the next section, this study discusses with the data description and then presents the estimation method.