Chapter 1

Introduction

Anderson localization (Anderson, 1958) has been studied during the past decades (Lee and Ramakrishnan,1985; Kramer and MacKinnon, 1993). In ordered systems, such as in the lattice, electrons can move freely and their wave functions are the Bloch waves. When the disorder appears in the system, the periodic structure of the lattice is distorted and the spatial extent of the wave function is reduced. Then the wave function may become localized.

In this thesis we will adopt the idea of Anderson localization to study the influence of disorder on the quantum transport of vortices in two-dimensional Josephson junction arrays. Oudenaarden *et al.* have studied this problem experimentally, in one-dimensional arrays of Josephson junctions (Oudenaarden *et al.*, 1996). A number of experiments have been performed on Josephson-junction arrays (Oudenaarden and Mooij, 1996), in particular to disordered junction arrays.

For superconductors, Cooper pair electrons act as a boson and can move through the superconductors. The total number N of Cooper pairs in each grain size and its phase ϕ associated with each grain size, are found to be conjugate variables (Anderson, 1964), that is

$$[N, \phi] = i. \tag{1.1}$$

Our system is a two dimensional granular superconductor with weak couplings among the grains throughout the sample. The disorder appeared in our system is due to the randomness of the grain size (or the capacitance of the grain) which is inversely proportional to the charging energy.

When the dimension of the grain size and capacitance involved are small, the charging energy E_c is large, and quantum fluctuations of the phase associated with the grain are important (Fazio and Schön, 1991). In addition, for each grain size, the charge and phase operators, $\hat{Q}$ and $\hat{\phi}$, are conjugate variables (Eckern and Schmid, 1989),

$$[\hat{Q}, \hat{\phi}] = -i(2e). \tag{1.2}$$

By taking both the charging energy and the Josephson coupling energy into account, the quantum mechanics of the system is described by the following Hamiltonian:

$$H = \sum_i U_i n_i^2 - \sum_{\langle ij \rangle} (E_J)_{ij} \cos(\phi_i - \phi_j) \tag{1.3}$$

where $(E_J)_{ij}$ is the Josephson coupling energy between site i and site j, $\sum_{\langle ij \rangle}$ denotes the sum over nearest neighbors ij, and U_i is the charging energy which equals $\frac{e^2}{2c_i}$ (where c_i is the capacitance at site i).

In the first part of our work, we are interested in a zero-temperature phase transition between the superconducting phase and the insulating phase. The transition can be tuned by changing parameters, such as the strength of the charging energy in Josephson junction arrays, the strength of disorder, or the magnitude of the external magnetic field (Cha and Girvin, 1994). In the superconducting state, delocalized bosons condense whereas vortices are localized. On the other hand, in the insulating phase, bosons are localized and vortices condense. As for second part of our study, the quantum transport of vortices in the insulating phase at low temperature is considered.

In this thesis, we will perform the Monte Carlo Simulation of Josephson junction arrays, by mapping our system to the spin$-1/2$ XY model. We have used the idea of computer simulation from the work of Gawiece and Grempel (Gawiece and Grempel, 1991), and their results converge to our result in the limit where the charging energy effect is completely ignored.

To address completely the quantum transport of quantum vortex, we perform the Monte Carlo simulation by assuming that our system is in the insulating phase, and therefore, the transport property of quantum vortices is due to variable-range hopping mechanism which was first introduced by Mott (Mott, 1968) sometimes ago.

The organization of this thesis is as follows. In Chapter 2 we review some theoretical backgrounds concerning the idea of Anderson localization, Josephson tunneling effect, and the quantum vortex. In Chapter 3 we present the analytic result for the one-dimensional ordered Josephson junction arrays via a mean field approach. Our work is then presented in Chapters 4 and 5. Chapter 4 is devoted to the study of the impurity influence on the phase transition by using the method of Monte Carlo simulation. Chapter 5, we study the transport of vortices in the insulating phase using the same method. Finally, the conclusion and discussion are made in Chapter 6.