CHAPTER IV

ELECTRON EDM IN THE MSSM

The MSSM Lagrangians constructed in the previous chapter have introduced a number of parameters more than those in the SM Lagrangian. As some of such parameters may be complex, they provide additional sources of CP violation which result in the prediction of fermion EDMs different from those obtained from the Standard Model. In the first section of this chapter, we will see that at GUT scale these parameters are severely constrained in the context of the gravity-mediated supersymmetry breaking model. By doing field redefinitions, the number of independent parameters are finally reduced to only six. Among them, there are two CP-violating phases. In the subsequent section, the one-loop Feynman diagrams contributing to the electric dipole moment of the electron due to chargino and neutralino exchanges will be evaluated.

4.1 CP-violating phases of the MSSM

In the Standard Model, the only source of CP violation is the CKM phase in the quark mixing matrix. Supersymmetric models, however, can have more complex parameters leading to more violation of CP invariance. Of particular interests, the MSSM Lagrangian contains over 40 new complex phases [36] resulting in two sources of CP violation: the SUSY preserving μ parameter and the soft SUSY breaking terms. The additional CP-violating phases are listed as the following:

- φ_μ from the μ parameter in the superpotential.
- φ_b from the soft breaking parameter b involving two Higgs doublets.
- φ_1, φ_2, and φ_3 from the corresponding complex gaugino masses M_1, M_2, and M_3.

- Phases originating in the flavor sector of the Lagrangian, in either the scalar soft mass matrices $\mathbf{m}^2$ or the trilinear matrices $\mathbf{a}$.

Note that the mass squared matrices $\mathbf{m}^2$ are Hermitian so that only off-diagonal terms can be complex, but since the trilinear matrices are general 3×3 matrices then their diagonal elements can also be complex.

In this thesis, for sake of simplicity, the electron EDM will be calculated in the framework of the gravity-mediated supersymmetry breaking model also known as the constrained MSSM (CMSSM), or minimal supergravity (mSUGRA). In this model, the soft SUSY-breaking parameters are "universal" at the grand unification (GUT) scale. This means that the complex parameters are constrained as follows:

- Gaugino mass universality:

$$M_1 = M_2 = M_3 = m_{1/2}. \tag{4.1}$$

- Scalar mass unification (flavor-blind):

$$\mathbf{m}^2_{\tilde{\mathbf{Q}}_\mathbf{L}} = \mathbf{m}^2_{\tilde{\mathbf{u}}_\mathbf{R}} = \mathbf{m}^2_{\tilde{\mathbf{d}}_\mathbf{R}} = \mathbf{m}^2_{\tilde{\mathbf{L}}_\mathbf{L}} = \mathbf{m}^2_{\tilde{\mathbf{e}}_\mathbf{R}} = m_0^2\mathbf{1}$$

$$m^2_{H_u} = m^2_{H_d} = m_0^2. \tag{4.2}$$

- Trilinear scalar coupling unification:

$$\mathbf{a_u} = A_0\mathbf{y_u}; \qquad \mathbf{a_d} = A_0\mathbf{y_d}; \qquad \mathbf{a_e} = A_0\mathbf{y_e}. \tag{4.3}$$

- Proportionality of the soft breaking parameter b to the μ parameter:

$$b = \mu B_0. \tag{4.4}$$

These constraints reduce the number of CP-violating phases, in addition to the CKM phase which is already encoded in the Yukawa couplings, to four: $\varphi_{1/2}$ (phase of $m_{1/2}$), φ_{A_0} (phase of A_0), φ_b, and φ_μ. However, not all of these phases are physical. It is possible to redefine fields and perform the so-called R transformation in such a way that there are only 2 physical phases left [34]. First, consider the bilinear term of the Higgs scalars, bH_uH_d, in the soft supersymmetry breaking Lagrangian. The phase φ_b can be absorbed by redefining both Higgs multiplets with the same phase according to

$$H_u \rightarrow e^{\frac{i}{2}\varphi_b} H_u, \qquad H_d \rightarrow e^{\frac{i}{2}\varphi_b} H_d. \tag{4.5}$$

Then after transfering the phase to the Higgs fields, the b parameter becomes real so that the phases of μ and B_0 are constrained such that

$$\mu B_0 \in \mathbb{R}. \tag{4.6}$$

For the superpotential, and of course the action, to be form-invariant under this phase shift, the μ parameter and all Yukawa couplings must be transformed as

$$\mu \rightarrow \mu' = e^{-i\varphi_b}\mu \tag{4.7}$$

$$\mathbf{y} \rightarrow \mathbf{y}' = e^{-\frac{i}{2}\varphi_b}\mathbf{y}. \tag{4.8}$$

Since we demand that the condition (4.6) always hold, from (4.7), B_0 has to change as

$$B_0 \rightarrow B_0' = e^{i\varphi_b} B_0. \tag{4.9}$$

Now consider the effect of field redefinitions on the trilinear coupling terms in the soft breaking Lagrangian. From Eq. (4.3), since the additional phases of Higgs superfields in Eq. (4.5) and that of Yukawa couplings in Eq. (4.8) cancel each other in the soft supersymmetry breaking Lagrangian, then the trilinear scalar coupling A_0 remains the same, i.e.,

$$A_0 \rightarrow A_0' = A_0. \tag{4.10}$$

All other terms in the Lagrangian are still unaffected. So now the phase φ_b has been completely removed from the Lagrangian.

Next, consider the gaugino mass terms in the $\mathcal{L}_{\text{soft}}^{\text{MSSM}}$. The phase $\varphi_{1/2}$ of the gaugino mass parameter can be absorbed by redefining all gauginos with the same phase, i.e.,

$$\tilde{g} \longrightarrow e^{\frac{i}{2}\varphi_{1/2}}\tilde{g}, \quad \widetilde{W} \longrightarrow e^{\frac{i}{2}\varphi_{1/2}}\widetilde{W}, \quad \widetilde{B} \longrightarrow e^{\frac{i}{2}\varphi_{1/2}}\widetilde{B}. \tag{4.11}$$

Such phase shifts clearly affect the interaction terms in $\mathcal{L}_{\text{int}}$ which couple gauginos to chiral superfields. For $\mathcal{L}_{\text{int}}$ to be invariant under the field redefinitions (4.11), we now make a transformation on the component fields of the chiral supermultiplets,

$$\phi \longrightarrow \phi' = \phi \tag{4.12}$$

$$\psi \longrightarrow \psi' = e^{-\frac{i}{2}\varphi_{1/2}}\psi. \tag{4.13}$$

The transformations in Eqs. (4.11)-(4.13) constitute the so-called R-transformation which is conveniently described in the superspace formulation of supersymmetry as follows. In the superspace formalism, we enlarge the spacetime by including the fermionic coordinates θ_α transforming as a Weyl spinor and their hermitian conjugates $\bar{\theta}^{\dot{\alpha}}$ to form a superspace.[1] Then the fields in a chiral or vector supermultiplet become the coefficients of some power of θ and $\bar{\theta}$ in a "superfield." For a chiral supermultiplet, a "chiral superfield" takes the form $\Phi = \phi + \theta\psi + \ldots$, while for a vector supermultiplet the associated "vector superfield" takes the form $V = \theta\sigma^\mu\bar{\theta}A_\mu + \bar{\theta}^2\theta\lambda + \ldots$ (valid up to some normalization of the component fields). The Lagrangian $\mathcal{L}_W$ which involves a superpotential $W(\phi)$ comes from an integration over the anticommuting coordinates of the superpotential with ϕ being replaced by Φ, i.e., $\mathcal{L}_W = \int d^2\theta W(\Phi)$. The R-transformation relevant to this thesis is defined by $\phi \longrightarrow \phi$, $\psi \longrightarrow e^{i\alpha}\psi$, $A_\mu \longrightarrow A_\mu$, $\lambda \longrightarrow e^{-i\alpha}\lambda$, and $\theta \longrightarrow \theta' = e^{-i\alpha}\theta$ (so

[1] In this formalism, it is customary to use a bar instead of a dagger for a left-handed spinor.

that $d^2\theta = e^{-2i\alpha}d^2\theta'$) for some α, so as to make both chiral and vector superfields invariant. Amazingly enough, such an R-transformation results in the same form of the Lagrangian except that the superpotential changes by a phase rotation, $W \rightarrow e^{-2i\alpha}W$.

In our case, the R-transformation changes the superpotential by a phase shift,

$$W \rightarrow e^{i\varphi_{1/2}}W. \tag{4.14}$$

This extra phase of the superpotential must be absorbed into:

1. the Higgs bilinear term - the μ parameter gets an extra phase $\mu \rightarrow e^{i\varphi_{1/2}}\mu$ which results in the transformation $B_0 \rightarrow e^{-i\varphi_{1/2}}B_0$ to preserve the condition that μB_0 be real; this leads us to identify a physical phase

$$\text{Physical phase} = \text{Arg}(B_0 m^*_{1/2}) = \text{Arg}(\mu^* b m^*_{1/2}) \tag{4.15}$$

which cannot be removed by any phase transformation of the fields;

2. the Yukawa coupling terms - the Yukawa couplings also get an extra phase $\mathbf{y} \rightarrow e^{i\varphi_{1/2}}\mathbf{y}$ which causes the trilinear coupling A_0 to change by $A_0 \rightarrow e^{-i\varphi_{1/2}}A_0$ upon using the condition (4.3) and demanding that the trilinear scalar couplings $\mathbf{a}_i$ be invariant; thus we obtain another physical phase

$$\text{Physical phase} = \text{Arg}(A_0 m^*_{1/2}). \tag{4.16}$$

From the above discussion, we see that there are only two physical phases which, if non-zero, cannot be removed by any phase transformation of the fields. In practice, however, we usually treat φ_μ and φ_{A_0} as physical phases due to their appearance in the calculations, and all other parameters as real numbers.

At the GUT scale, therefore, the minimal supergravity model with CP violation is completely parameterized by six real parameters: $m_{1/2}$, m_0, $|A_0|$, b and two physical phases above. The values of these parameters at the electroweak scale are determined by using renormalization group equations.

4.2 One-loop contributions to the electron EDM

As mentioned in Chapter 2, in renormalizable theories the effective electric dipole interaction (2.62) is induced at the loop level. For a theory of a fermion ψ_f interacting with other massive fermions ψ_i and massive scalars ϕ_j, the general form of interactions that contains CP violation is given by

$$\mathcal{L}_{\text{int}} = -\sum_{ij} \left[\bar{\psi}_f \left(\Gamma_{Lij} P_L + \Gamma_{Rij} P_R\right) \psi_i \phi_j\right] + \text{h.c.}, \tag{4.17}$$

where $P_{L,R} = (1 \mp \gamma^5)/2$, the usual left- and right-handed projection operators. $\Gamma_{Lij}, \Gamma_{Rij}$ are complex couplings. Here, all fields are assumed to be mass eigenstates at the tree level. For an electron, the EDM will receive one-loop contributions from the neutralino and chargino exchange diagrams.

Before calculating the EDM, it should be reminded that the EDM operator must flip the chirality of the external electron. In the MSSM Lagrangian, the gauginos couple an external electron to a slepton of the same chirality via the gauge interactions, while the Higgsinos couple an electron to a slepton with the opposite chirality via the Yukawa interactions. Therefore, in one-loop diagrams there are two possible ways that the chirality flip can happen:

1. Both two vertices that couple to the external electron are due to either gauge interactions or Yukawa interactions, and the exchanged slepton changes its chirality via the left-right mixing terms of the slepton mass squared matrices.

2. One vertex is due to Yukawa interactions and the other is due to gauge interactions. In this case, the gauginos and the Higgsinos are mixed while the chirality of the exchanged slepton is preserved.

Since the neutralinos are mixtures of neutral Higgsinos and both $U(1)$ and $SU(2)$ gauginos, the neutralino exchange diagrams therefore include both types of chirality flip processes. On the other hand, the chargino contribution to the electron

EDM contains only the second process because the $SU(2)$ gaugino components of the charginos only couple to the left-handed fields.

4.2.1 One-loop calculations

In this section the one-loop calculations contributed to the EDM of the electron will be performed. There are two types of diagrams [37]. The first one is shown in Fig. 4.1 where the photon line is attached to the slepton. The second one is shown in Fig. 4.2 where the photon line is attached to the exchanged fermion. By convention, the mass of the slepton (selectron or sneutrino) is denoted by $m_{\tilde{l}}$ while the mass of the exchanged fermion (gauginos or higgsinos) is m_χ. Similarly, charges of the electron, slepton, and exchanged fermion in the diagrams are denoted in the unit of the positron electric charge by Q_e, $Q_{\tilde{l}}$, and Q_χ respectively. With the conservation of charge at each vertex, the relation $Q_e = Q_{\tilde{l}} + Q_\chi$ is satisfied. Now let us start the calculations.

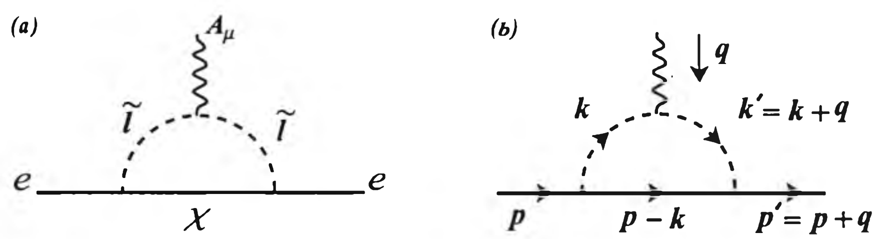

Figure 4.1: Loop diagrams with a photon line attached to the slepton: (a) the diagram with particle labels; and (b) the same diagram with momentum labels.

- The contribution from the loop diagram in Fig. 4.1:

$$\bar{u}(p')\delta\Gamma^\mu u(p) = \int \frac{d^4k}{(2\pi)^4}\, \bar{u}(p')\Gamma_R^*\frac{(1-\gamma^5)}{2}\frac{i}{k'^2 - m_{\tilde{l}}^2 + i\epsilon}[-Q_{\tilde{l}}e(2k+q)^\mu]$$
$$\frac{i}{k^2 - m_{\tilde{l}}^2 + i\epsilon}\cdot\frac{i(\gamma^\nu(p_\nu - k_\nu) + m_\chi)}{(p-k)^2 - m_\chi^2 + i\epsilon}\Gamma_L\frac{(1-\gamma^5)}{2}u(p).$$

Since $\{\gamma^\mu, \gamma^5\} = 0$ and $(1-\gamma^5)(1+\gamma^5) = 0$, then the nonzero terms must consist of even number of γ^μ. Moreover, from the definition of the electric dipole moment, (2.60) and (2.61), we are interested only in terms containing γ^5. Therefore

$$\bar{u}(p')\delta\Gamma^\mu u(p) \propto \frac{-i}{2} Q_{\tilde{l}} e(\Gamma_R^* \Gamma_L) \int \frac{d^4k}{(2\pi)^4} \bar{u}(p') m_\chi (2k+q)^\mu \gamma^5 u(p) \cdot \frac{1}{\left[k'^2 - m_{\tilde{l}}^2 + i\epsilon\right]\left[k^2 - m_{\tilde{l}}^2 + i\epsilon\right]\left[(p-k)^2 - m_\chi^2 + i\epsilon\right]}. \tag{4.18}$$

In order to evaluate the integral, we have to combine the three denominator factors above into the form (complete square in k)^3 so that we can perform integration in the spherical coordinate system without difficulty. Applying the identity

$$\frac{1}{A_1 A_2 \cdots A_n} = \int_0^1 dx_1 dx_2 \cdots dx_n \delta(\Sigma x_i - 1) \frac{(n-1)!}{[x_1 A_1 + x_2 A_2 + \cdots x_n A_n]^n} \tag{4.19}$$

known as the method of Feynman parameters to the denominator of (4.18), we get

$$\text{Denominator} = \int_0^1 dxdydz\ \delta(x+y+z-1) \frac{2}{D^3} \tag{4.20}$$

where

$$D = x(k'^2 - m_{\tilde{l}}^2) + y(k^2 - m_{\tilde{l}}^2) + z[(p-k)^2 - m_\chi^2] + i\epsilon.$$

Substitute the momentum conservation condition $k' = k + q$ into the above equation:

$$D = k^2 + 2k \cdot (xq - zp) + xq^2 + zp^2 - (1-z)m_{\tilde{l}}^2 - zm_\chi^2 + i\epsilon.$$

To complete the square, we change the variable k to $l \equiv k + xq - zp$,

$$\begin{aligned} D &= l^2 - (xq - zp)^2 + xq^2 + zp^2 - (1-z)m_{\tilde{l}}^2 - zm_\chi^2 + i\epsilon \\ &= l^2 + x(1-x)q^2 + z(1-z)p^2 + 2xzpq - (1-z)m_{\tilde{l}}^2 - zm_\chi^2 + i\epsilon. \end{aligned}$$

The momentum relation $p'^2 = (p+q)^2 = p^2 + 2p\cdot q + q^2$ together with the on-shell constraints of both external electron lines $p'^2 = p^2 = m_e^2$ imply $2p \cdot q = -q^2$. Substituting this into D, then

$$\begin{aligned} D &= l^2 + x(1-z-x)q^2 + z(1-z)m_e^2 - (1-z)m_{\tilde{l}}^2 - zm_\chi^2 + i\epsilon \\ &= l^2 - \Delta + i\epsilon \end{aligned} \tag{4.21}$$

where $\Delta = -xyq^2 - z(1-z)m_e^2 + (1-z)m_{\tilde{l}}^2 + zm_\chi^2$ and $x+y+z=1$.

Now consider the numerator of (4.18). It can be rewritten in terms of the variable l as

$$\text{Numerator} \;=\; \bar{u}(p')m_\chi(2l^\mu + (1-2x)q^\mu + 2zp^\mu)\gamma^5 u(p).$$

Notice that D depends only on the magnitude of l. This implies that

$$\int \frac{d^4l}{(2\pi)^4}\frac{l^\mu}{D^3} = 0. \tag{4.22}$$

The most general form of the vertex function Γ^μ is a linear combination of all Lorentz vectors involved in the Feynman diagram: γ^μ, p^μ, and p'^μ. For convenience, it is written as

$$\gamma^\mu \cdot A + (p'+p)^\mu \cdot B + q^\mu \cdot C. \tag{4.23}$$

Rearranging the numerator into this form:

$$\begin{aligned}\text{Numerator} &= \bar{u}(p')m_\chi(z(p'+p)^\mu + (1-2x-z)q^\mu)\gamma^5 u(p).\\ &= \bar{u}(p')m_\chi(z(p'+p)^\mu + (y-x)q^\mu)\gamma^5 u(p).\end{aligned}$$

It can be seen that the denominator is symmetric under the interchange $x \leftrightarrow y$, while the coefficient of q^μ is antisymmetric, hence vanishes after integrating over x and y. Now consider

$$\begin{aligned}\bar{u}(p')\gamma^\mu\gamma^5 u(p) &= \bar{u}(p')\gamma^\mu\gamma^5\frac{p_\nu\gamma^\nu}{m_e}u(p) = -\frac{1}{m_e}\bar{u}(p')\gamma^\mu\gamma^\nu\gamma^5 u(p)p_\nu\\ &= -\frac{1}{m_e}\bar{u}(p')\left(\frac{1}{2}\{\gamma^\mu,\gamma^\nu\} + \frac{1}{2}[\gamma^\mu,\gamma^\nu]\right)\gamma^5 u(p)p_\nu\\ &= -\frac{1}{m_e}\bar{u}(p')\left(g^{\mu\nu} + \frac{1}{2}[\gamma^\mu,\gamma^\nu]\right)\gamma^5 u(p)p_\nu\\ &= -\frac{1}{m_e}\bar{u}(p')(p^\mu - i\sigma^{\mu\nu}p_\nu)\gamma^5 u(p),\end{aligned} \tag{4.24}$$

and similarly

$$\begin{aligned}\bar{u}(p')\gamma^\mu\gamma^5 u(p) &= \frac{1}{m_e}\bar{u}(p')\gamma^\nu\gamma^\mu\gamma^5 u(p)p'_\nu\\ &= \frac{1}{m_e}\bar{u}(p')(p'^\mu + i\sigma^{\mu\nu}p'_\nu)\gamma^5 u(p).\end{aligned} \tag{4.25}$$

Subtracting (4.25) from (4.24) gives an identity

$$\bar{u}(p')(p'+p)^\mu\gamma^5 u(p) = \bar{u}(p')(-i\sigma^{\mu\nu}q_\nu)\gamma^5 u(p). \tag{4.26}$$

By using this identity, the numerator becomes

$$\text{Numerator} \quad = \quad \bar{u}(p')2m_e m_\chi z\left(-\frac{i\sigma^{\mu\nu}q_\nu}{2m_e}\right)\gamma^5 u(p).$$

Therefore $\delta\Gamma^\mu$ gets a contribution

$$\begin{aligned}\delta\Gamma'^\mu \propto -\frac{1}{2}Q_{\tilde{l}}e(\Gamma^*_R\Gamma_L)\frac{\sigma^{\mu\nu}\gamma^5 q_\nu}{2m_e}\int_0^1 dxdydz\;\delta(x+y+z-1)2 \\ \cdot\int\frac{d^4l}{(2\pi)^4}\frac{2m_e m_\chi z}{(l^2-\Delta+i\epsilon)^3}.\end{aligned} \tag{4.27}$$

Only remaining work is to perform the momentum integration in (4.27) and extract the EDM term. To do so, we use a mathematical trick known as a Wick rotation by changing the integration variables l to the Euclidean 4-momentum variables l_E defined by

$$l^0 \equiv il^0_E, \qquad \vec{l} = \vec{l}_E.$$

The second integral in (4.27) becomes

$$(-i)\int\frac{d^4l_E}{(2\pi)^4}\frac{2m_e m_\chi z}{(l_E^2+\Delta)^3}. \tag{4.28}$$

In Section 3.1, we have used the momentum cutoff technique in evaluating the momentum integral. In this chapter, however, we will use a more standard technique called the dimensional regularization method,[2] originated by 't Hooft and Veltman [38]. This method provides the integration formula

$$\int\frac{d^dl_E}{(2\pi)^d}\frac{1}{(l_E^2+\Delta)^n} \quad = \quad \frac{1}{(4\pi)^{d/2}}\frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}}. \tag{4.29}$$

[2] Actually, the technique we use is known as "dimensional reduction," in which the algebras of gamma matrices are first performed in four dimensions, and then the momentum integrals are evaluated in d dimensions.

Hence (4.28) reads

$$(-i)\int \frac{d^4l_E}{(2\pi)^4}\frac{2m_em_\chi z}{(l_E^2+\Delta)^3} = \frac{-i}{(4\pi)^2}\frac{\Gamma(1)}{\Gamma(3)}\left(\frac{2m_em_\chi z}{\Delta}\right)$$
$$= \frac{-i}{(4\pi)^2}\frac{m_em_\chi z}{\Delta}. \tag{4.30}$$

From (4.28) and (4.30), Eq. (4.27) becomes

$$\delta\Gamma^\mu = i\frac{Q_{\tilde{l}}e}{16\pi^2}(\Gamma_R^*\Gamma_L)\frac{\sigma^{\mu\nu}\gamma^5 q_\nu}{2m_e}\int_0^1 dxdydz\ \delta(x+y+z-1)$$
$$\cdot\frac{m_em_\chi z}{-xyq^2-z(1-z)m_e^2+(1-z)m_{\tilde{l}}^2+zm_\chi^2}.$$

By comparing the one-loop correction to the vertex function above with Eq. (2.60), we obtain the coefficient $F_3(q^2)$ which is related to the electric dipole moment of the electron by $d_e = -F_3(q^2=0)/2m_e$, see Eq. (2.61). Thus the first loop diagram contributes to the electron EDM as

$$\frac{d_e^{(1)}}{e} = \frac{Q_{\tilde{l}}}{16\pi^2}\text{Im}(\Gamma_R^*\Gamma_L)\frac{1}{2m_e}\int_0^1 dz\int_0^{1-z} dy\frac{m_em_\chi z}{-z(1-z)m_e^2+(1-z)m_{\tilde{l}}^2+zm_\chi^2}$$
$$= \frac{1}{2}\frac{Q_{\tilde{l}}}{16\pi^2m_{\tilde{l}}^2}\text{Im}(\Gamma_R^*\Gamma_L)m_\chi\int_0^1 dz\frac{z(1-z)}{1-z+rz-sz(1-z)}$$
$$= \frac{1}{2}\frac{1}{16\pi^2m_{\tilde{l}}^2}m_\chi\text{Im}(\Gamma_R^*\Gamma_L)Q_{\tilde{l}}I(r,s), \tag{4.31}$$

where $r = m_\chi^2/m_{\tilde{l}}^2$, $s = m_e^2/m_{\tilde{l}}^2$, and

$$I(r,s) = \int_0^1 dz\frac{z(1-z)}{1-z+rz-sz(1-z)}. \tag{4.32}$$

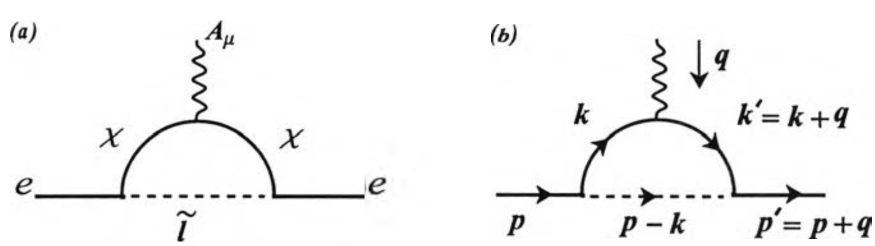

Figure 4.2: Loop diagrams with a photon line attached to the chargino: (a) the diagram with particle labels; and (b) the same diagram with momentum labels.

• The contribution from the loop diagram in Fig. 4.2:

$$\bar{u}(p')\delta\Gamma^{\mu}u(p) = \int \frac{d^4k}{(2\pi)^4}\,\bar{u}(p')\Gamma^{*}_{R}\frac{(1-\gamma^5)}{2}\frac{i(\gamma^{\nu}k'_{\nu}+m_{\chi})}{k'^2-m_{\chi}^2+i\epsilon}[-Q_{\chi}e\gamma^{\mu}]\frac{i(\gamma^{\rho}k_{\rho}+m_{\chi})}{k^2-m_{\chi}^2+i\epsilon}$$
$$\cdot\frac{i}{(p-k)^2-m_{\tilde{l}}^2+i\epsilon}\Gamma_{L}\frac{(1-\gamma^5)}{2}u(p). \quad (4.33)$$

Again we are interested in terms containing γ^5 and even number of γ^{μ}, i.e.,

$$\bar{u}(p')\delta\Gamma^{\mu}u(p) \propto \frac{-i}{2}Q_{\chi}e(\Gamma^{*}_{R}\Gamma_{L})\int\frac{d^4k}{(2\pi)^4}\bar{u}(p')m_{\chi}(\gamma^{\nu}\gamma^{\mu}k'_{\nu}+\gamma^{\mu}\gamma^{\nu}k_{\nu})\gamma^5u(p)$$
$$\cdot\frac{1}{\left[k'^2-m_{\chi}^2+i\epsilon\right]\left[k^2-m_{\chi}^2+i\epsilon\right]\left[(p-k)^2-m_{\tilde{l}}^2+i\epsilon\right]}. \quad (4.34)$$

Observe that the denominator of this loop integral is similar to that of the first diagram except the interchange $m_{\tilde{l}} \leftrightarrow m_{\chi}$. Hence it can be arranged into the form (4.20) where

$$D = l^2-\Delta+i\epsilon; \quad \Delta = -xyq^2-z(1-z)m_e^2+(1-z)m_{\chi}^2+zm_{\tilde{l}}^2. \quad (4.35)$$

Consider the numerator written in terms of the variable l,

$$\text{Numerator} = \bar{u}(p')m_{\chi}[\gamma^{\nu}\gamma^{\mu}(l_{\nu}+(1-x)q_{\nu}+zp_{\nu})+\gamma^{\mu}\gamma^{\nu}(l_{\nu}-xq_{\nu}+zp_{\nu})]\gamma^5u(p).$$

We have to rearrange this numerator into the form (4.23). To do so, substitute $q = p'-p$ into the equation above and use the Dirac equations $\bar{u}(p')\gamma^{\nu}p'^{\nu} = m_e\bar{u}(p')$ and $\gamma^{\nu}p_{\nu}u(p) = m_eu(p)$, together with Eq. (4.22), we get

$$\begin{aligned}\text{Numerator} &= \bar{u}(p')m_{\chi}\left[(1-x)m_e\gamma^{\mu}+(-1+x+z)2p^{\mu}\right.\\ &\qquad \left.-x(2p'^{\mu}-m_e\gamma^{\mu})-m_e\gamma^{\mu}\right]\gamma^5u(p)\\ &= \bar{u}(p')m_{\chi}[-2yp^{\mu}-2xp'^{\mu}]\gamma^5u(p)\\ &= \bar{u}(p')m_{\chi}[-(x+y)(p'+p)^{\mu}+(x-y)q^{\mu}]\gamma^5u(p).\end{aligned}$$

With the same reason as before, the coefficient of q^{μ} vanishes after integrating over variables x and y. Now using the identity (4.26),

$$\text{Numerator} = -\bar{u}(p')2m_em_{\chi}(1-z)\left(-\frac{i\sigma^{\mu\nu}q_{\nu}}{2m_e}\right)u(p)\gamma^5.$$

This contributes to $\delta\Gamma^\mu$ as

$$\begin{aligned}\delta\Gamma^\mu &\propto +\frac{1}{2}Q_\chi e(\Gamma_R^*\Gamma_L)\frac{\sigma^{\mu\nu}\gamma^5 q_\nu}{2m_e}\int_0^1 dxdydz\ \delta(x+y+z-1)2\cdot\int\frac{d^4l}{(2\pi)^4}\frac{2m_e m_\chi(1-z)}{(l^2-\Delta+i\epsilon)^3}\\ &= -i\frac{Q_\chi e}{16\pi^2}(\Gamma_R^*\Gamma_L)\frac{\sigma^{\mu\nu}\gamma^5 q_\nu}{2m_e}\int_0^1 dxdydz\ \delta(x+y+z-1)\\ &\quad\cdot\frac{m_e m_\chi(1-z)}{-xyq^2-z(1-z)m_e^2+(1-z)m_\chi^2+zm_{\tilde{l}}^2}.\end{aligned}$$

In the second step, we performed a Wick rotation and used the dimensional regularization method. Therefore the second loop diagram contributes to the EDM of the electron as

$$\begin{aligned}\frac{d_e^{(2)}}{e} &= -\frac{Q_\chi}{16\pi^2}\mathrm{Im}(\Gamma_R^*\Gamma_L)\frac{1}{2m_e}\int_0^1 dz\int_0^{1-z} dy\frac{m_e m_\chi(1-z)}{-z(1-z)m_e^2+(1-z)m_\chi^2+zm_{\tilde{l}}^2}\\ &= -\frac{1}{2}\frac{(Q_e-Q_{\tilde{l}})}{16\pi^2}\mathrm{Im}(\Gamma_R^*\Gamma_L)m_\chi\int_0^1 dz\frac{(1-z)^2}{-z(1-z)m_e^2+(1-z)m_\chi^2+zm_{\tilde{l}}^2}.\end{aligned}$$

Here we have used the standard convention in replacing Q_χ with $(Q_e - Q_{\tilde{l}})$. Changing the integration variable from z to $1-z$, we obtain

$$\begin{aligned}\frac{d_e^{(2)}}{e} &= -\frac{1}{2}\frac{(Q_e-Q_{\tilde{l}})}{16\pi^2}\mathrm{Im}(\Gamma_R^*\Gamma_L)m_\chi\int_0^1 dz\frac{z^2}{-z(1-z)m_e^2+zm_\chi^2+(1-z)m_{\tilde{l}}^2}\\ &= -\frac{1}{2}\frac{(Q_e-Q_{\tilde{l}})}{16\pi^2 m_{\tilde{l}}^2}\mathrm{Im}(\Gamma_R^*\Gamma_L)m_\chi\int_0^1 dz\frac{z^2}{1-z+rz-sz(1-z)}\\ &= -\frac{1}{2}\frac{1}{16\pi^2 m_{\tilde{l}}^2}m_\chi\mathrm{Im}(\Gamma_R^*\Gamma_L)(Q_e-Q_{\tilde{l}})J(r,s),\end{aligned} \tag{4.36}$$

where $J(r,s)$ is defined by

$$J(r,s)=\int_0^1 dz\frac{z^2}{1-z+rz-sz(1-z)} \tag{4.37}$$

with the same definitions of r and s.

Having found the contributions from the one-loop Feynman diagrams, we can now write down the total contributions to the electron EDM due to a single fermion χ exchange,

$$d_e^\chi = 2\cdot\left(d_e^{(1)}+d_e^{(2)}\right) \tag{4.38}$$

where the factor 2 came from the complex conjugates of both loop diagrams. Finally, as the total electron EDM is obtained from one-loop contributions involving chargino and neutralino exchanges, then it can be written as

$$d_e = d_e^{\tilde{C}} + d_e^{\tilde{N}}. \tag{4.39}$$

In the next two sections, we will identify the coefficient $\Gamma_R^* \Gamma_L$ in cases of the exchanged fermion being charginos and neutralinos respectively.

4.2.2 The chargino contribution

The chargino contribution comes from the diagram in Fig. 4.2 together with its complex conjugate. In order to specify the vertex coupling Γ_L and Γ_R, firstly we have to write down explicitly the relevant interactions of the form (4.17) in terms of gauge eigenstates. For the chargino contribution to the electron EDM, ψ_f is the external electron field, ψ_i is a charged gaugino or higgsino, and ϕ is a sneutrino. After transforming all fields into mass eigenstates, we will then obtain the wanted complex couplings.

The interactions of type (4.17) comes from two sources:

1. From the slepton-lepton-charged wino couplings in the interaction Lagrangian,

$$\begin{aligned}
\mathcal{L}_{\text{int}} &\supset -\sqrt{2} g^{(a)} \left[(\phi^* T^a \psi) \lambda^a + \text{h.c.} \right] \\
&\supset -\sqrt{2} g \left[\left(\tilde{L}^* \tau^1 L \right) \widetilde{W}^1 + \left(\tilde{L}^* \tau^2 L \right) \widetilde{W}^2 \right] + \text{h.c.} \\
&\supset -\sqrt{2} g \left[\left(\frac{1}{2} \tilde{\nu}_e^* e_L \right) \frac{1}{\sqrt{2}} (\widetilde{W}^+ + \widetilde{W}^-) + \left(\frac{-i}{2} \tilde{\nu}_e^* e_L \right) \frac{i}{\sqrt{2}} (\widetilde{W}^+ - \widetilde{W}^-) \right] + \text{h.c.} \\
&= -\sqrt{2} g \left[\frac{1}{\sqrt{2}} \, \tilde{\nu}_e^* e_L \widetilde{W}^+ \right] + \text{h.c.} .
\end{aligned}$$

In terms of mass eigenstates, it can be rewritten as

$$\mathcal{L}_{\text{int}} \supset -g \left[\, e_L \mathrm{V}_{i1}^* \widetilde{C}_i^+ \tilde{\nu}_e^* \right] + \text{h.c.} . \tag{4.40}$$

2. From the superpotential

$$W_{\text{MSSM}} \supset -\bar{\tilde{e}}\mathbf{y}_e\widetilde{L}H_d \supset -y_e\bar{\tilde{e}}\tilde{\nu}_e\widetilde{H}_d^-$$

which leads to the Lagrangian

$$\mathcal{L}_W \supset y_e\bar{e}\tilde{\nu}_e\widetilde{H}_d^- + \text{h.c.} = y_e e_R^\dagger\tilde{\nu}_e\widetilde{H}_d^- + \text{h.c.}.$$

In terms of mass eigenstates:

$$\mathcal{L}_W \supset y_e e_R^\dagger \mathbf{U}_{i2}^* \widetilde{C}_i^- \tilde{\nu}_e + \text{h.c.}. \tag{4.41}$$

Combining the results from (4.40) and (4.41), we get

$$\Gamma_R^*\Gamma_L = -g y_e \mathbf{U}_{i2}^*\mathbf{V}_{i1}^* = -g^2\kappa_e\mathbf{U}_{i2}^*\mathbf{V}_{i1}^* \tag{4.42}$$

where

$$\kappa_e = \frac{y_e}{g} = \frac{m_e}{\sqrt{2}m_W\cos\beta}. \tag{4.43}$$

Moreover, the factor g^2 is conventionally expressed as

$$g^2 = \frac{e^2}{\sin^2\theta_W} = \frac{4\pi\alpha_{\text{EM}}}{\sin^2\theta_W} \tag{4.44}$$

where $\alpha_{\text{EM}} = e^2/(4\pi) \approx 1/137$ is the fine structure constant. The total chargino contribution to the electric dipole moment of the electron is therefore given by

$$\frac{d_e^{\tilde{C}}}{e} = \frac{\alpha_{\text{EM}}}{4\pi\sin^2\theta_W}\frac{\kappa_e Q_e}{m_{\tilde{\nu}_e}^2}\sum_{i=1}^{2} m_{\tilde{C}_i}\text{Im}\left(\mathbf{U}_{i2}^*\mathbf{V}_{i1}^*\right) J\left(\frac{m_{\tilde{C}_i}^2}{m_{\tilde{\nu}_e}^2}, \frac{m_e^2}{m_{\tilde{\nu}_e}^2}\right). \tag{4.45}$$

4.2.3 The neutralino contribution

Now we turn to consider the contribution to the electron EDM due to neutralino exchange. The contributing Feynman diagrams are that in Fig. 4.1 and its complex conjugate. In this case, ψ_f and ϕ in Eq. (4.17) are again the external electron and a selectron respectively, while ψ_i denotes a neutral gaugino or higgsino.

Similar to the chargino case, the interactions of the form (4.17) come from two sources:

1. From the slepton-lepton-neutral wino and the slepton-lepton-bino couplings in (3.44),

$$\begin{aligned} \mathcal{L}_{\rm int} &\supset -\sqrt{2}g^{(a)}\left[(\phi^* T^a \psi)\lambda^a + \text{h.c.}\right] \\ &\supset -\sqrt{2}g\left(\widetilde{L}^* \tau^3 L\right)\widetilde{W}^0 - \sqrt{2}g'\left(\widetilde{L}^* Y L + \tilde{e}_R^* Y e_R\right)\widetilde{B} + \text{h.c.}. \end{aligned}$$

Since the wanted terms are those involving electron, selectron and neutral gauginos, then

$$\begin{aligned} \mathcal{L}_{\rm int} &\supset -\sqrt{2}\,\tilde{e}_L^* e_L\left(g' Y\widetilde{B} + g\tau^3\widetilde{W^0}\right) - \sqrt{2}g'\tilde{e}_R^* e_R Y\widetilde{B} + \text{h.c.} \\ &= -\sqrt{2}g\,\tilde{e}_L^* e_L\left(\tan\theta_W(Q_e - \tau^3)\widetilde{B} + \tau^3\widetilde{W^0}\right) - \sqrt{2}g\tan\theta_W Q_e \tilde{e}_R^* e_R\widetilde{B} + \text{h.c.}. \end{aligned}$$

In terms of mass eigenstates,

$$\begin{aligned} \mathcal{L}_{\rm int} \supset\; & -\sqrt{2}g\;e_L\left(\tan\theta_W(Q_e - \tau^3)\mathrm{N}_{1i} + \tau^3\mathrm{N}_{2i}\right)\mathrm{S}_{e1j}^*\widetilde{N}_i\tilde{e}_j^* \\ & -g\;e_R\left(\sqrt{2}\tan\theta_W Q_e \mathrm{N}_{1i}\mathrm{S}_{e2j}^*\right)\widetilde{N}_i\tilde{e}_j^* + \text{h.c.}. \end{aligned} \tag{4.46}$$

2. From the superpotential

$$W_{\rm MSSM} \supset -\bar{\tilde{e}}\mathbf{y_e}\widetilde{L}H_d = y_e\bar{\tilde{e}}\tilde{e}H_d^0$$

which gives rise to the Lagrangian

$$\begin{aligned} \mathcal{L}_W &\supset -y_e\left[\bar{\tilde{e}}e\widetilde{H}_d^0 + \bar{e}\tilde{e}\widetilde{H}_d^0\right] + \text{h.c.} \\ &= -y_e\left[e_L\tilde{e}_R^*\widetilde{H}_d^0 + e_R^\dagger\tilde{e}_L\widetilde{H}_d^0\right] + \text{h.c.}, \end{aligned}$$

or, written in terms of mass eigenstates,

$$\mathcal{L}_{\rm W} \supset -y_e\left[e_L\left(\mathrm{N}_{3i}\mathrm{S}_{e2j}^*\right)\widetilde{N}_i\tilde{e}_j^* + e_R^\dagger\left(\mathrm{N}_{3i}\mathrm{S}_{e1j}\right)\widetilde{N}_i\tilde{e}_j\right] + \text{h.c.}. \tag{4.47}$$

The complex coupling obtained from Eqs. (4.46) and (4.47) is

$$\begin{aligned} (\Gamma_R^*\Gamma_L)_{ij} &= g^2\left[-\sqrt{2}\left\{\tan\theta_W(Q_e - \tau^3)\mathrm{N}_{1i} + \tau^3\mathrm{N}_{2i}\right\}\mathrm{S}_{e1j}^* - \kappa_e\mathrm{N}_{3i}\mathrm{S}_{e2j}^*\right] \\ &\quad\cdot\left[\sqrt{2}\tan\theta_W Q_e\mathrm{N}_{1i}\mathrm{S}_{e2j} - \kappa_e\mathrm{N}_{3i}\mathrm{S}_{e1j}\right]. \end{aligned} \tag{4.48}$$

Therefore, the total neutralino contribution to the electron EDM is given by

$$\frac{d_e^{\tilde{N}}}{e} = \frac{\alpha_{\rm EM}}{4\pi \sin^2\theta_W} \sum_{j=1}^{2}\sum_{i=1}^{4} \frac{m_{\tilde{N}_i} Q_e}{m_{\tilde{e}_j}^2} {\rm Im}\left(\frac{1}{g^2}\left(\Gamma_R^* \Gamma_L\right)_{ij}\right) I\left(\frac{m_{\tilde{N}_i}^2}{m_{\tilde{e}_j}^2}, \frac{m_e^2}{m_{\tilde{e}_j}^2}\right). \quad (4.49)$$

It should be emphasized that $d_e^{\tilde{C}}$ receives contributions from the loop diagram of Fig. 4.2-type only while $d_e^{\tilde{N}}$, in contrast, receives Fig. 4.1-type contributions only. This is due to the fact that sneutrino and neutralinos have zero electric charges. Moreover, sources of CP non-conservation, the phases φ_μ and φ_{A_0}, are all encoded in the transformation matrices $\mathbf{U}$, $\mathbf{V}$, $\mathbf{N}$, and $\mathbf{S}_e$.

We also note that since the experimental lower bounds on the slepton masses are much larger than the electron's mass, then $s = m_e^2/m_{\tilde{l}}^2$ is always assumed to be zero. The kinematic functions $I(r,s)$ and $J(r,s)$ evaluated at $s \simeq 0$ are given by

$$I(r,0) = \int_0^1 dx \frac{x(1-x)}{1-x+rx} = \frac{1}{2(1-r)^2}\left[1 + r + \frac{2r}{1-r}\ln r\right] \quad (4.50)$$

$$J(r,0) = \int_0^1 dx \frac{x^2}{1-x+rx} = \frac{-1}{2(1-r)^2}\left[3 - r + \frac{2}{1-r}\ln r\right]. \quad (4.51)$$