INTRODUCTION

The notion of quasi-ideals of semigroups was introduced by O. Steinfeld in [10]. We can see in [2] that the notion of bi-ideals of semigroups was given earlier. Quasi-ideals are a generalization of left ideals and right ideals and bi-ideals are a generalization of quasi-ideals. Let ***BQ*** be the class of semigroups whose sets of bi-ideals and quasi-ideals coincide. S. Lajos has given in [6] that every regular semigroup belongs to ***BQ***. K.M. Kapp has proved in [5] that every left simple semigroup, every right simple semigroup, every left 0-simple semigroup and every right 0-simple semigroup also belongs to this class. Moreover, J. Calais has characterized semigroups in ***BQ*** in [1]. However, this characterization is not practical to use to determine whether a given semigroup belongs to ***BQ***.

Interval semigroups of real numbers under both multiplication and addition seem to be interesting. Therc are exactly 15 types of multiplicative interval semigroups of real numbers which were introduced by K.R. Pearson in [8]. The detail of a proof was given by S. Ritkeao in [9]. By the same idea of this proof, K. Palasri proved in [7] that there are exactly 6 types of additive interval semigroups of real numbers. We characterize such multiplicative interval semigroups and additive interval semigroups belonging to ***BQ*** in Chapter II.

The multiplicative semigroup $\boldsymbol{Z_n}$ of integers modulo a positive integer n is a standard semigroup. It is well-known that for a positive integer n, the multiplicative semigroup $\boldsymbol{Z_n}$ is a group with zero if and only if n is a prime. Therefore if n is a prime, then the multiplicative semigroup $\boldsymbol{Z_n}$ belongs to ***BQ***. G. Ehrlich has proved in [4] that for a positive integer n, the multiplicative semigroup $\boldsymbol{Z_n}$ is regular if and only if n is square-free. Thus if n is square-free, then the multiplicative semigroup $\boldsymbol{Z_n}$ belongs to ***BQ***. In Chapter III, it is show that this converse is not true and we prove that for a positive integer n, the multiplicative semigroup $\boldsymbol{Z_n}$ belongs to ***BQ*** if and only if $n = 4$ or n is square-free.

The four standard transformation semigroups on a set X are the partial transformation semigroup on X, the full transformation semigroup on X, the one-to-one partial transformation semigroup on X (the symmetric inverse semigroup on X) and the symmetric group on X. All of these four transformation semigroups belong to $\boldsymbol{BQ}$ for any set X because they are all regular. The semigroup M_X of all one-to-one transformations of a set X and the semigroup E_X of all onto transformations of X are also standard transformation semigroups but they are not regular if X is infinite. The transformation semigroups M_X and E_X belong to $\boldsymbol{BQ}$ if X is finite because both of them become the symmetric group on X. In Chapter IV, we show that the finiteness of X is also necessary for both M_X and E_X to belong to $\boldsymbol{BQ}$. We also study the other two transformation semigroups in Chapter IV. They are the semigroup C_I of all continuous functions and the semigroup D_I of all differentiable functions of I into itself where I is any interval on the real line with $| I | > 1$ and the topology is the usual topology on I. It is proved that $C_I \notin \boldsymbol{BQ}$ and $D_I \notin \boldsymbol{BQ}$ for any such an interval.

The preliminaries for this research are given in Chapter I.