Chapter 5

Geostatistical Estimation

Several computer-oriented methods have been used in the context of statistical estimation. The simplest statistical estimate involves calculating the arithmetic mean of a series of numerical data (or value), which give the equally average value within the geologically defined area. This means under one estimation, the arithmetic mean is considered to represent the block of interest. Therefore, when using this non-spatial statistical method, it is essential that samples are independent of one another. As a matter of fact, weighting of each sample data is reasonably important in geological sense. So the closer the value is, the more important it would be (Evan, 1951).

5.1 Kind of Estimation

It is notably important to briefly discuss, first of all, methods conventionally applied to the estimation. These include inverse distance, polygonal, and triangulation methods. Then the widely used geostatistical method - regionalized variables, is subsequently explained.

5.1.1 Inverse distance method

These assign weights to the samples which are inversely proportional to their distances from the panel (data point) being estimated. Hence, samples nearer to the center of the data point are given more weight than the more distance ones. Thus no direction of mutually related sample data is concerned . The spatial relationships are put on a very rough and empirical basis, with a lack of consideration towards the statistical characteristics of sample (see Fig. 5.1).

5.1.2 Polygonal method

Polygonal method, in which an area of influence is defined for each drill-hole and a weighted average grade is calculated on the assumption that the thickness or accumulation of the bore-hole in each polygon extends uniformly over the whole area of the polygon; the weighting factor is the area of the polygon. A pronounced disadvantage of the polygonal method is that it always promotes randomness, individual bore-holes are evaluated independently and without searching for mutual relations and interdependence.

5.1.3 Triangulation method

This method is most commonly used in contour map. The first computer programs for contouring were direct implementations of methods used by surveyors for the hand mapping of topography. Control points, assumed to be located without any particular regularity, are first connected by straight lines. This forms a mesh of triangles that covers the map (Fig. 5.3). By interpolating down the sides of the triangles, locations can be encountered where the ground elevation is a constant, specified value. Connecting these points of equal elevation produces a contour line. In effect, the surface is modeled as a series of flat, triangular plates, each held at its conners by a control point.

With these conventional methods of modeling, it is impossible to calculate the error of estimations. The under-estimation of unimportance (e.g., low-grade) blocks and the over-estimation of more important (e.g. high-grade) blocks are other undesirable features which may arise from the use of the conventional method. Calculation of the variances (discussed in previous chapters), therefore, gives a measure of the error of estimation of the mean (Davis, 1986).

The geostatistical method of interpolation of variable values (such as ore-grade) is called "kriging". The estimation is also known as the Best Line Unbiased Estimator (BLUE), in which the values of the same pleasure multiplied by weighting factors which are determined from a set of linear equations. It is generally accepted that variables in a coal deposit (grade, thickness, quality, etc.) are a function of the geological environment. Therefor, a change in geological and structural conditions result in variation (Scott&Whateley, 1995). The advantage of geostatistical method over conventional contouring methods is their ability to quantity the potential size of errors. The size of error depends on factors such as size of block being estimated (Whitchurch, et.al, 1987).

5.2 Overview of Kriging

5.2.1 Backgrounds & Conditions

This is sometimes called kriging variance (see Puvichit & Thongpenyai, 1996, Pairat, 1987, Scott & Whateley, 1995). As stated above, in the estimation of ore-grade value, the weight average of the given samples posses higher level of confidence when they become nearer, so the estimation can be solved using the following formula

$$\boxed{T^* = a_1Z_1 + a_2Z_2 + a_3Z_3 + \ldots + a_nZ_n} \qquad 5.1$$

when T^* = estimation value

a_i (i = 1,n) = weight given to each sample value

Z_i (i = 1,n) = value of sample

n = total numbers used in calculation

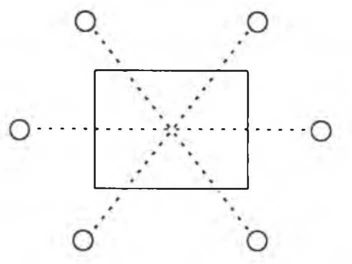


Fig. 5.1 Inverse distance method for geostatistical estimation (Royle and others, 1980).

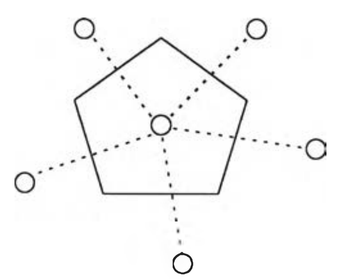
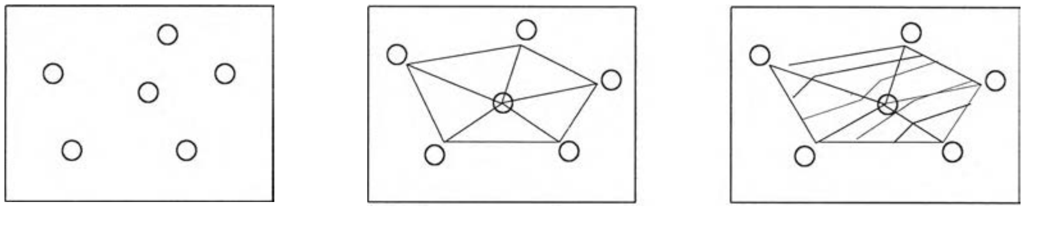


Fig. 5.2 Polygonal method for geostatistical estimation (Royle and others, 1980).



Triangulation method of estimating positions of contour lines

a) Irregularly spaced data points

b) Triangles formed across map area with data points as vertices

c) Contour lines drawn through sides of triangles where points of specified elevation have been found by linear

Fig. 5.3. Triangulation method for geostatistical estimation.

Theoretically, kriging can be calculated under two cryptic conditions:

1. The condition involved must be non-biased, that is

$$\sum_{i=1}^{n} a_i \quad = \quad 0 \qquad \text{whereby } a = \text{weight of sample}$$

2. The condition under consideration must have minimum variance of estimation so variance of estimation can be defined as

$$\text{variance of estimation (or } \sigma^2 E) \quad = \quad E\{(Z_o - Z_o^*)^2 \}$$

whereby Z_o = estimator, and

Z_o^* = value to be estimated.

In general, and as a rule, variance of estimation is the expectation of squared difference between ' true ' (Z_o) and 'estimated' values (Z_o^*)

5.2.2 Weight Determination

In order to quantify the of estimation condition until $\Sigma a_i = 1$ (i.e., non-bias condition)*, the Lagrange method is introduced, that is

$$[E(Z_o - Z_o^*)^2 - 2\alpha (\Sigma a_i - 1)] \qquad 5.2$$

whereby α = Lagrange multiplier, a_i = weight given, Z_i = estimator, Z_o^* = value to be estimated. This method gives rise to the minimum of variance or $\{E(Z_o - Z_o^*)^2 \}$ on the constrain that $\Sigma a_i = 1$ (non-bias condition). Another word, the values a_1, a_2, a_3, ... , a_n and α which reduce the value from the equation 5.2 being minimal. That is substituted values to Z_o^* and takes derivative with respect to a_i and α , equal zero (0), then the equations will be taken as follows :

$$
\left.\begin{array}{lcl}
a_1C(Z_1, Z_1) + a_2C(Z_1, Z_2) + \ldots + a_nC(Z_1, Z_n) - \alpha & = & C(Z_1, Z_0) \\
a_1C(Z_2, Z_1) + a_2C(Z_2, Z_2) + \ldots + a_nC(Z_2, Z_n) - \alpha & = & C(Z_2, Z_0) \\
. & & . \\
. & & . \\
. & & . \\
a_1C(Z_m, Z_1) + a_2C(Z_n, Z_2) + \ldots + a_nC(Z_n, Z_n) - \alpha & = & C(Z_n\ Z_0) \\
a_1 + a_2 + a_3 + \ldots + a_n + 0 & = & 1
\end{array}\right] \qquad 5.3
$$

when $C(Z_i\ Z_j)$ > Covariance of Z between I and j and the equation in the form of matrix can be drawn

$$
\boxed{Ka = D} \qquad 5.4
$$

whereby

$$
K = \begin{bmatrix}
C(Z_1, Z_1) & C(Z_1, Z_2) & \ldots & C(Z_1, Z_n) & 1 \\
C(Z_2, Z_1) & C(Z_2, Z_2) & \ldots & C(Z_2, Z_n) & 1 \\
. & & & & . \\
. & & & & . \\
. & & & & . \\
C(Z_m, Z_1) & C(Z_n, Z_2) & \ldots & C(Z_n, Z_n) & 1 \\
1 & 1 & 1 & & 0
\end{bmatrix}
$$

$$
a = \begin{bmatrix} a1 \\ a2 \\ . \\ . \\ . \\ an \\ -\alpha \end{bmatrix}
\qquad
D = \begin{bmatrix} C(Z_1, Z_0) \\ C(Z_2, Z_0) \\ . \\ . \\ . \\ C(Z_n, Z_0) \\ 1 \end{bmatrix}
$$

As the covariance equation and estimation variogram are liked (see chapter 4), the relationship can be made using this equation

$$\boxed{\gamma(h) = C(o) - C(h)}$$

when	$\gamma(h)$	=	variogram of Z at the distance h ;
	$C(o)$	=	covariance of Z at the same point;
	$C(h)$	=	covariance of Z at the distance h ;
therefore	$C(h)$	=	$C(o) - \gamma(h)$

substituting the values in equation 5.3, then

$$\left.\begin{array}{lcl} a_1\gamma(Z_1, Z_1) + a_2\gamma(Z_1, Z_2) + \ldots + a_n\gamma(Z_1, Z_n) - \alpha & = & \gamma(Z_1, Z_0) \\ a_1\gamma(Z_2, Z_1) + a_2\gamma(Z_2, Z_2) + \ldots + a_n\gamma(Z_2, Z_n) - \alpha & = & \gamma(Z_2, Z_0) \\ . & & . \\ . & & . \\ . & & . \\ a_1\gamma(Z_m, Z_1) + a_2\gamma(Z_n, Z_2) + \ldots + a_n\gamma(Z_n, Z_n) - \alpha & = & \gamma(Z_n Z_0) \\ a_1 + a_2 + a_3 + \ldots + a_n + 0 = 1 & & \end{array}\right] \qquad 5.5$$

It should be pointed out that covariance of n in the first equation (5.1) is deteriorated and then $a_1 + a_2 + \ldots + a_n = 1$. From equation 5.5, matrix can be drawn as

$$\boxed{Ga' = D'} \qquad 5.6$$

whereby

$$
G = \begin{bmatrix}
\gamma(Z_1, Z_1) & \gamma(Z_1, Z_2) & \cdots & \gamma(Z_1, Z_n) & 1 \\
\gamma(Z_2, Z_1) & \gamma(Z_2, Z_2) & \cdots & \gamma(Z_2, Z_n) & 1 \\
. & & & & . \\
. & & & & . \\
. & & & & . \\
\gamma(Z_m, Z_1) & \gamma(Z_n, Z_2) & \cdots & \gamma(Z_n, Z_n) & 1 \\
1 & 1 & & 1 & 0
\end{bmatrix}
$$

$$
a' = \begin{bmatrix} a1 \\ a2 \\ . \\ . \\ . \\ an \\ -\alpha \end{bmatrix}
\qquad
D' = \begin{bmatrix} \gamma(Z_1, Z_0) \\ \gamma(Z_2, Z_0) \\ . \\ . \\ . \\ \gamma(Z_n, Z_0) \\ 1 \end{bmatrix}
$$

Therefore, from the relationship between covariance and distance or between variogram and distance, weight and lag range multiplier can be solved statistically.

In this study, kriging method is used for the geostatistical estimation of the coal quality for both Sin Pun and Saba Yoi areas. Kriging method can determine the error of estimation statistically using the SURFER program. This method is deferent from the others in that the estimated parameters taken from variogram model being selected are involved in the calculation. Results of estimation are presented in the light of coal quality model of kriging estimation and error estimation.

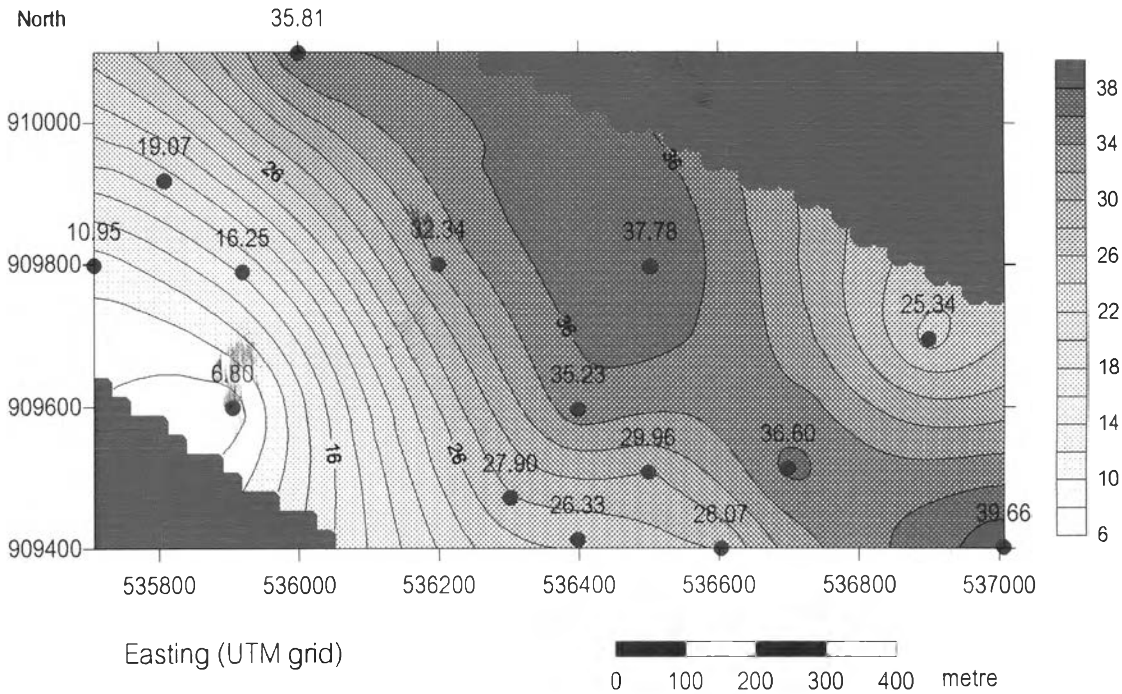
5.3 Coal quality model of kriging estimation

In this section, several quality models of individual variables are simulated. Since the same method is applied to each variable, only some selected examples are shown herein by areas and by coal seams below.

5.3.1. Ash Content (%)

From the summary of variogram analysis of Nong Wa P1 seam (Sin Pun area), as shown in Table 4.14, variogram model appropriate for kriging estimation is spherical model with nugget value of 0, sill of 900, and range of 900 m. Results of kriging estimate by SURFER program illustrated in Fig.5.4. It is figured out that the relationship among samples is in the east-west direction, corresponding with the variogram model (see Table 4.15). Results from kriging (Fig.5.5) reveal that there is a gradual decrease in ash values to the west. Therefore, it is concluded at this point that the good quality of coals in term of ash content is more concentrated to the west rather than the east. In addition, results from this estimate can give rise to the values of error estimation. As shown in Fig 5.6, area with low value of estimation error is posted as symbol " L "

Similarly, in the case of the Saba Yoi S2 seam, analysis of variogram parameter indicates that variogram model applied for kriging estimation is the spherical one (see Table 4.15), which has nugget effect of 3, sill of 5, range of 250 m and R-minor of 130 m. Result of kriging estimated by SURFER program is illustrated in Fig. 5.6. It is figured out that the relationship among samples is in the north-south direction, corresponding with the variogram model (see Table 4.16). Results from kringing (Fig. 5.6) reveal that there is a gradual decrease in ash value to the west. Therefore, it is concluded that the good quality of coals in term of ash content is more concentrated to the west rather than the east. Results from this estimate can give rise to the values of error estimation. As shown in Fig. 5.7, area with low values of estimation error is posted as symbol "L".



Explanation

(shaded box)	degree of ash content, very high in dark blue and decrease to white
(dot)	location of true value for ash content
X-coordinate	easting (UTM grid)
Y-coordinate	northing (UTM grid)
contour interval	2% (ash), contour line begin at 6-38 % ash

Fig. 5.4. Krige estimate as shown by colour-shaded relief and contour lines for ash content (%), P1 seam, Nong Wa deposit, Sin Pun area.

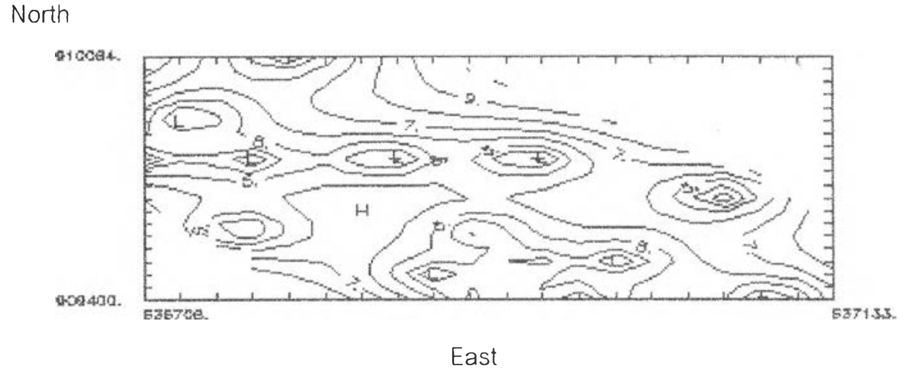
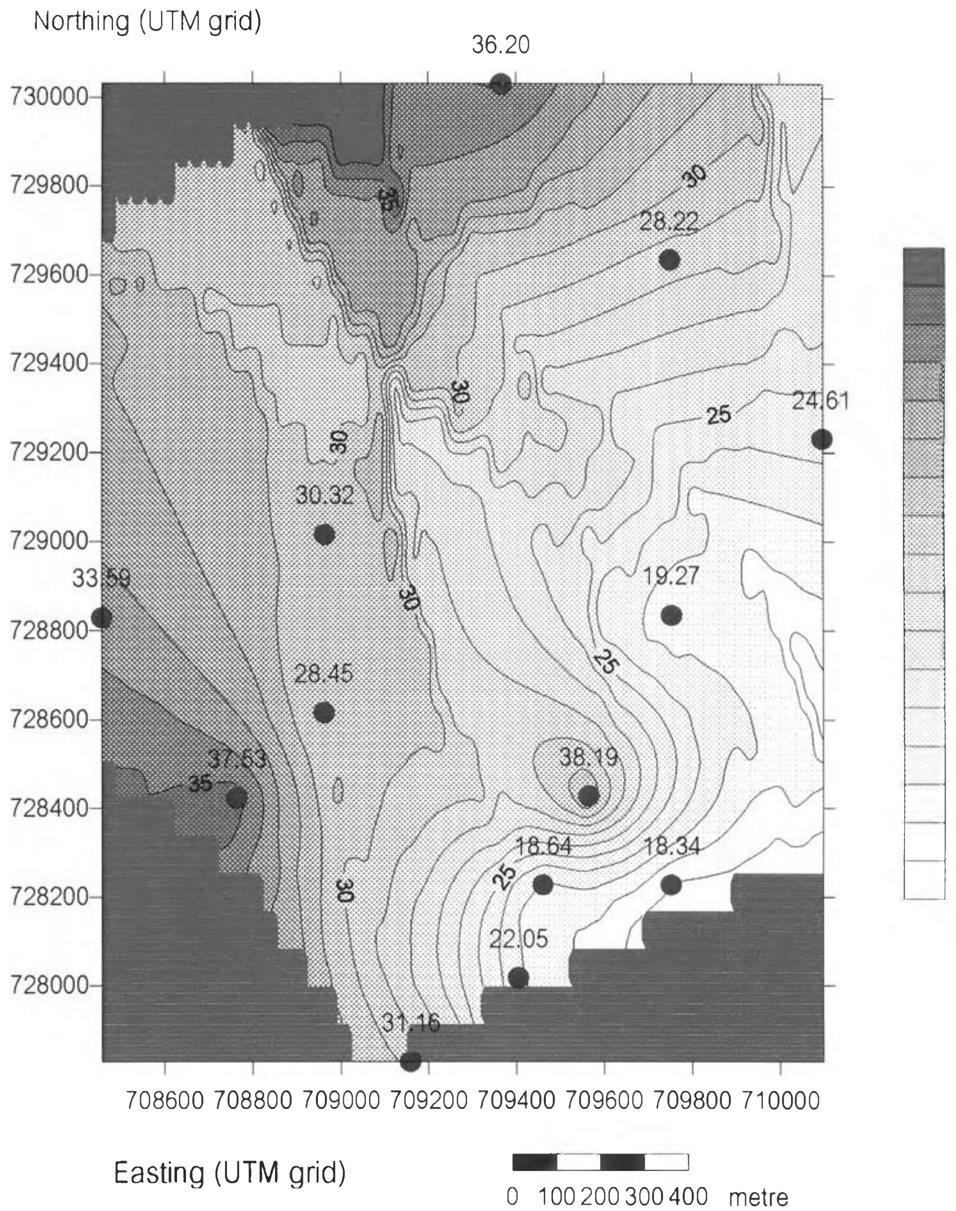


Fig. 5.5. Standard error of estimation for ash content (%), P1 seam, Nong Wa deposit, Sin Pun area.



Explanation

	degree of ash content, very high in dark blue and decrease to white
●	location of true value for ash content
X-coordinate	easting (UTM grid)
Y-coordinate	northing (UTM grid)
contour interval	2% (ash), contour line begin at 6-38 % ash

Fig. 5.6. Krige estimate as shown by colour-shaded relief and contour lines for ash content (%), S3 seam, Ban Sao deposit, Saba Yoi area.

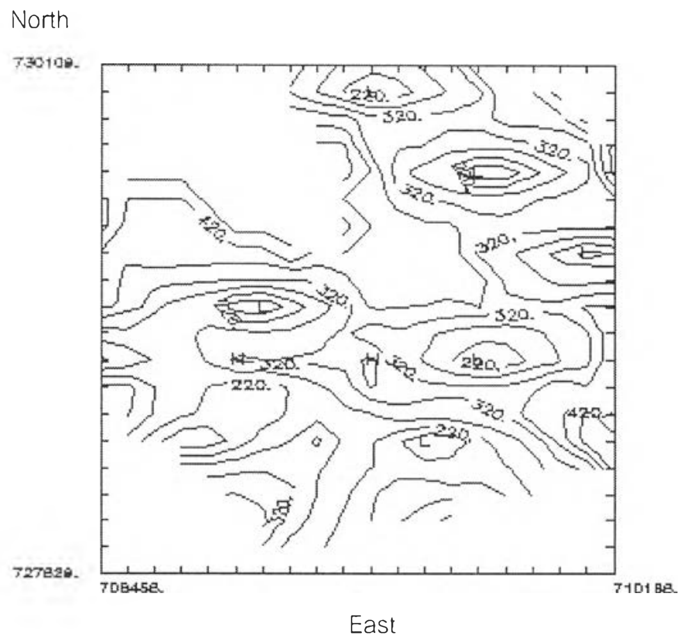


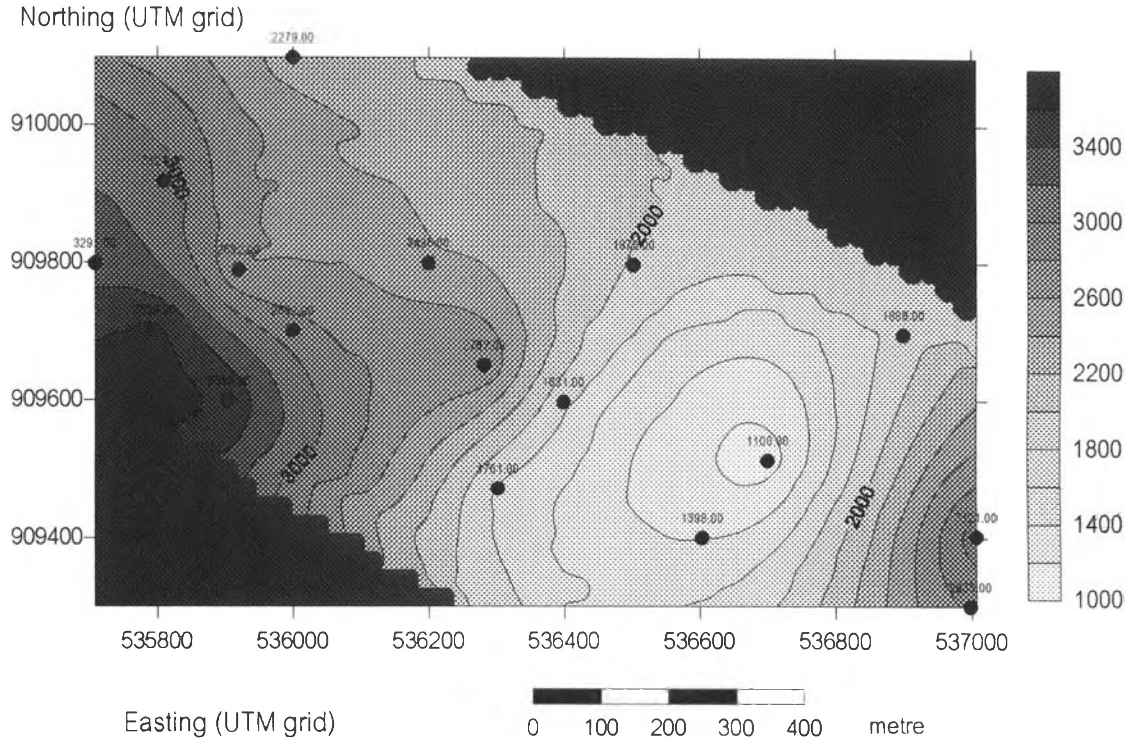
Fig. 5.7. Standard error of estimation for ash content (%), S3 seam, Ban Sao deposit, Saba Yoi area.

5.3.2. Calorific Value (kcal/kg)

Similar procedure is also applied for kriging estimation on calorific value of Sin Pun and Saba Yoi coal seams. Fig. 5.8 predicts the example of kriging result illustrated by contour-shade relief and overlaid contour maps of Nong Wa deposit, P2 seam, Sin Pun area. It is observed that with the use of nugget effect at 0, sill at 675000, range at 600 m and R-minor at 250 m, the result indicates that the major concentration of heat contents seem to be higher to the west than the east, suggesting the higher-quality coal in the western part of Nong Wa area.

The error estimation shown in Fig. 5.9 demonstrates the low values (L) in the central zone of Nong Wa sub-basin.

An example of Saba Yoi coal seam is taken from the S2 seam. It is visualized from Fig. 5.10 that ordinary kriging estimate is applicable with the application of spherical model at the nugget effect at 4000, sill 62000, range 400 m, and R-minor at 180 m. Results from kriging reveal that there is a gradual increase in calorific value to the west. Therefore, it is concluded at this point that the good quality of coal in term of CV is more concentrated to the west rather than the central and the east area.



Explanation

	degree of calorific value (CV), very high in dark red and decrease to white
	location of true value for CV
X-coordinate	easting (UTM grid)
Y-coordinate	northing (UTM grid)
contour interval	200 kcal/kg (CV), contour line begin at 1000-3600 kcal/kg.

Fig. 5.8. Krige estimate as shown by colour-shaded relief and contour lines for calorific value (kcal/kg), P2 seam, Nong Wa deposit, Sin Pun area.

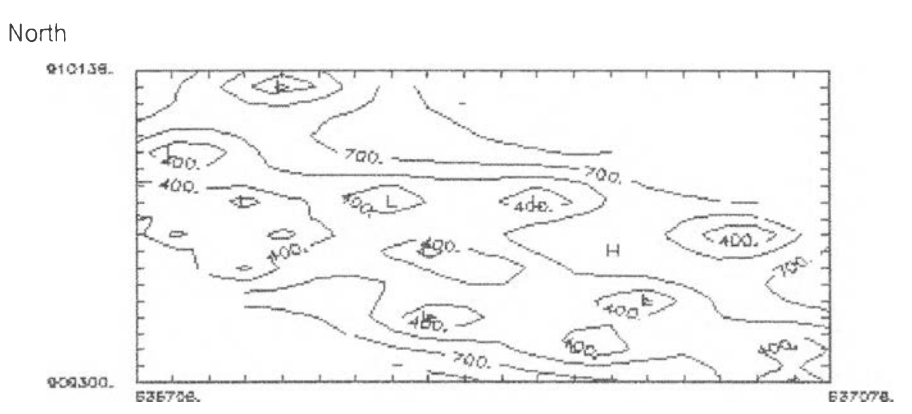
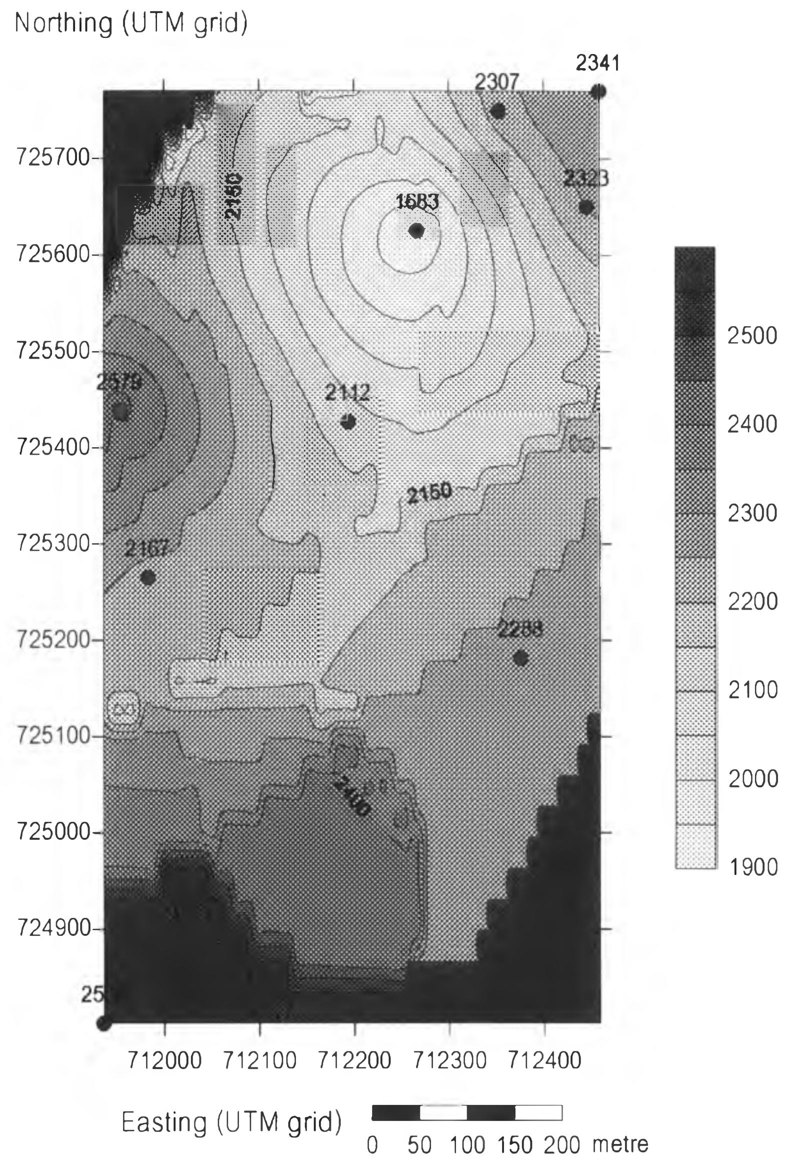


Fig. 5.9. The standard error of estimation computed for a kriged estimation of calorific value, P2 seam, Nong Wa deposit, Sin Pun area.



Explanation

	degree of calorific value (CV), very high in dark red and decrease to white
	location of true value for CV
X-coordinate	easting (UTM grid)
Y-coordinate	northing (UTM grid)
contour interval	200 kcal/kg (CV), contour line begin at 1700-2600 kcal/kg.

Fig. 5.10. Krige estimate as shown by colour-shaded relief and contour lines for calorific value (kcal/kg), S2 seam, Ban Khok Ok deposit, Saba Yoi area.

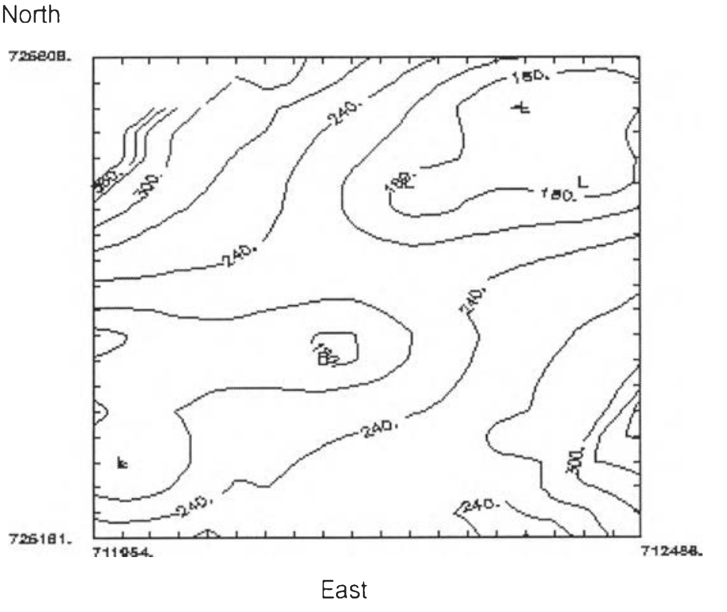


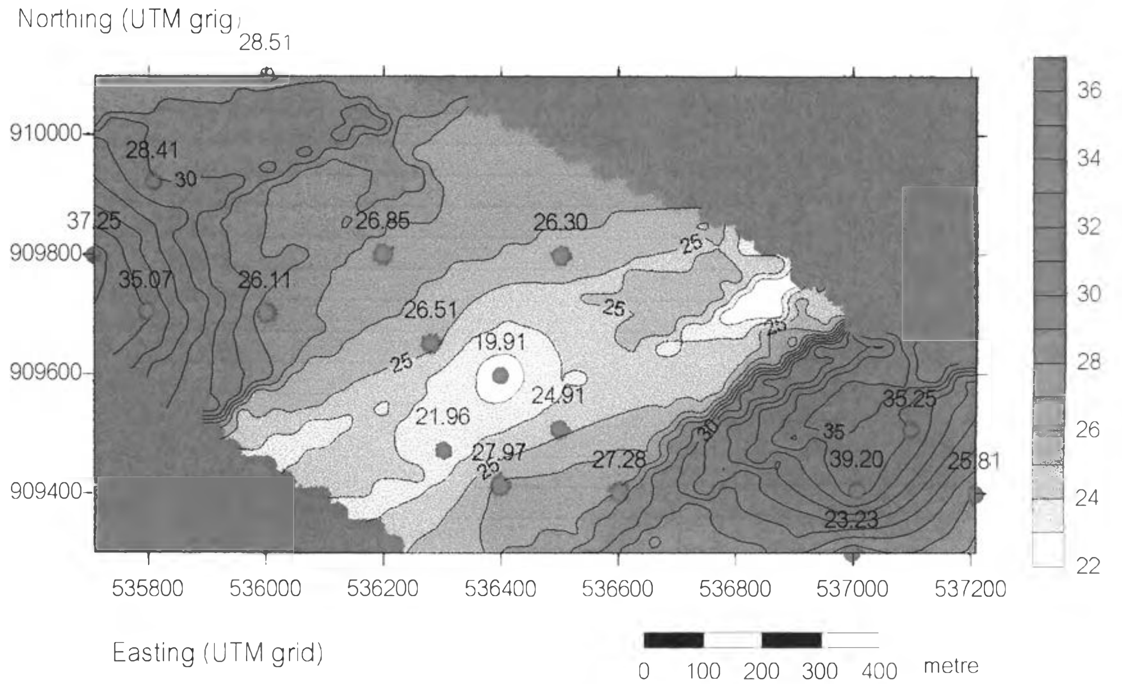
Fig. 5.11. The standard error of estimation computed for a kriged estimation of calorific value, S2 seam, Ban Khok Ok deposit, Saba Yoi area.

5.3.3. Moisture Content (%)

Similar procedure is also applied for kriging estimation on moisture content of Sin Pun and Saba Yoi coal seam. Fig. 5.12 predicts the example of kriging result illustrated by contour-shaded relief and overlaid contour maps of Nong Wa deposit, P4 seam, sin Pun area. It is observed that with the use of nugget effect at 5, sill at 22.5, range at 550 m and R-minor at 150 m. The result indicates that the major concentration of moisture content is located in the central part of the Nong Wa area and higher to the east and west. However, as noted by Davis (1986), variations in moisture contents can be caused by several involving factors-the timing on sampling and the duration of samples kept before analysis.

The error estimation shown in Fig. 5.13 demonstrates the low values (L) in the southeastern of Nong Wa sub-basin.

An example of Saba Yoi coal seam is taken from S3 seam. It is visualize from Fig. 5.14 that ordinary kriging estimate with the application of spherical model is determined by using nugget effect at 0, sill at 26, range 1400 m, and R-minor at 630 m. The results of moisture from kriging estimate shown high values in the middle zone of the study area. The overall estimated errors are located 2-4% model content.



Explanation

	degree of moisture content. very high in dark green and decrease to white
	location of true value for moisture content
X-coordinate	easting (UTM grid)
Y-coordinate	northing (UTM grid)
contour interval	2% (moisture). contour line begin at 6-38 % moisture content

Fig. 5.12. Krige estimate as shown by colour-shaded relief and contour lines for moisture content (%), P4 seam, Nong Wa deposit, Sin Pun area.

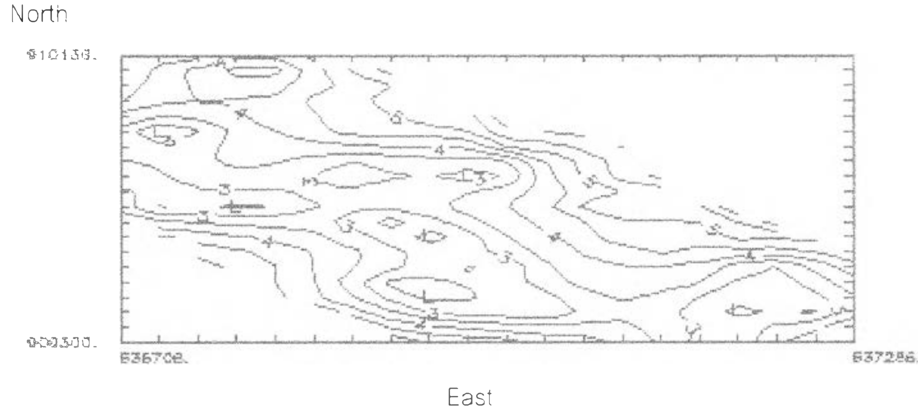
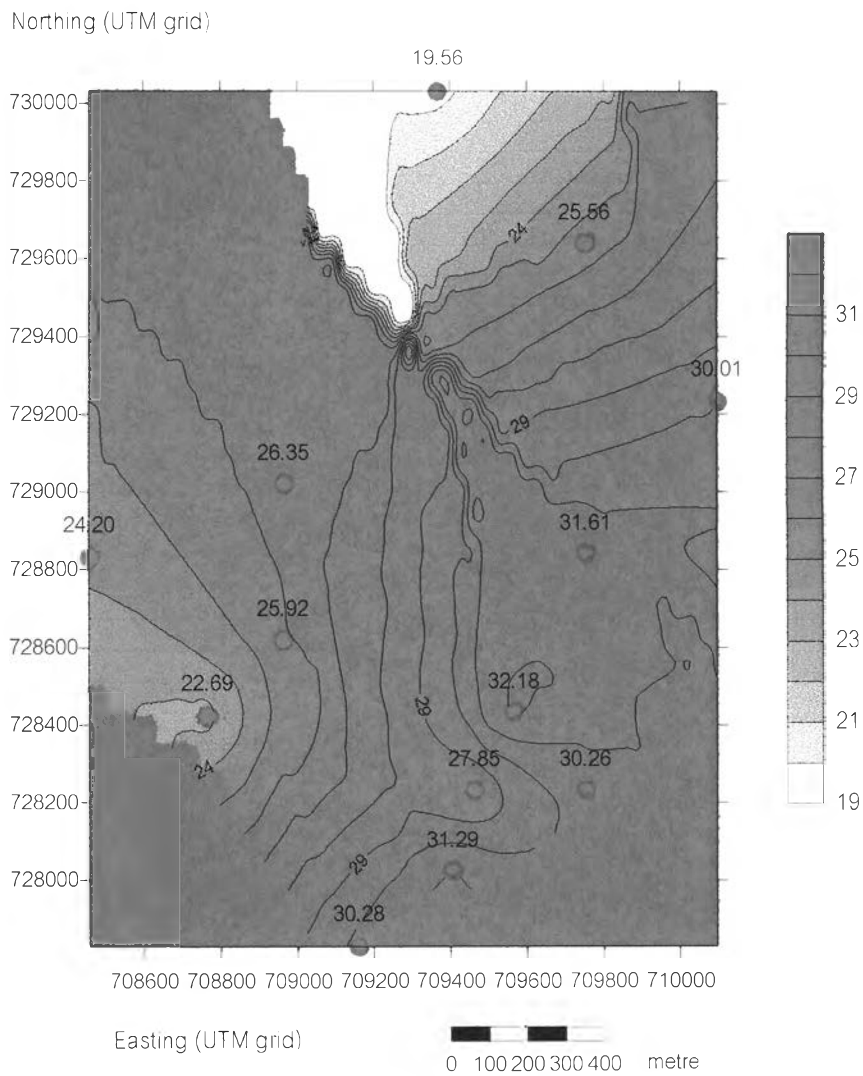


Fig. 5.13. Standard error of estimation for moisture content (%), P4 seam. Nong Wa deposit, Sin Pun area.



Explanation

	degree of moisture content. very high in dark green and decrease to white
	location of true value for moisture content
X-coordinate	easting (UTM grid)
Y-coordinate	northing (UTM grid)
contour interval	2% (moisture). contour line begin at 6-38 % moisture content

Fig. 5.14. Krige estimate as shown by colour-shaded relief and contour lines for moisture content (%),S3 seam, Ban Sao deposit, Saba Yoi area.

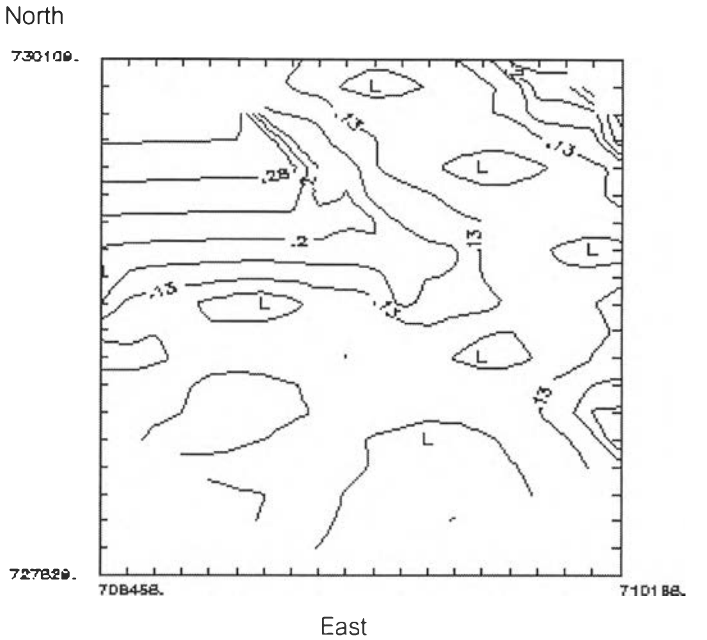
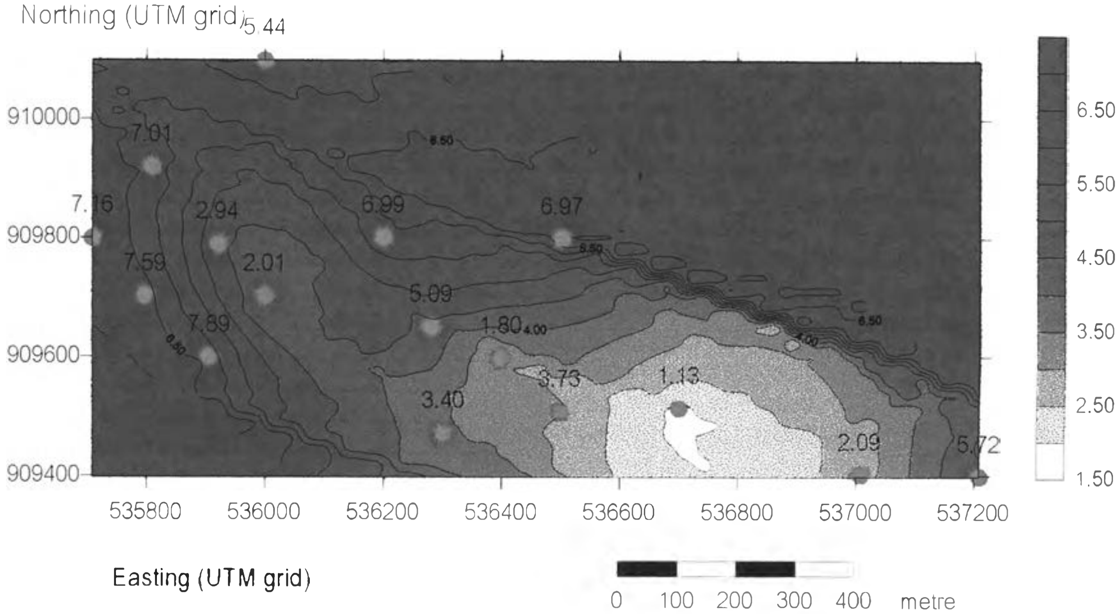


Fig. 5.15. Standard error of estimation for moisture content (%), S3 seam, Ban Sao deposit, Saba Yoi area.

5.3.4. Sulphur Content (%)

Similar procedure is also applied for kriging estimation on sulphur content of Sin Pun and Saba Yoi coal seam. Fig. 5.16 predicts the example of kriging result illustrated by contour-shaded relief and overlaid contour maps of Nong Wa deposit, P3 seam, sin Pun area. It is observed that with the use of nugget effect at 2.3, sill at 5, range at 650 m and R-minor at 200 m. The result indicates that the major concentration of sulphur content is located in the central part of the Nong Wa area. The error estimation shown in Fig. 5.17 demonstrates the low values (L) in the southeastern of Nong Wa sub-basin.

An example of Saba Yoi coal seam is taken from S3 seam. It is visualize from Fig. 5.18 that ordinary kriging estimate with the application of linear model, nugget effect at 0, sill at 0.03, range 1200 m, and R-minor at 330 m, can give rise to the trend of sulphur content in the approximate NW-SE direction with more values to the north. It is noted that the sulphur content is relatively constant throughout the Ban Sao deposit. However, the estimation error shown in Fig. 5.19, reveals that the low estimated error is located mostly in the east.



EExplanation

	degree of sulphur content, very high in dark purple and decrease to white
	location of true value for sulphur content
X-coordinate	easting (UTM grid)
Y-coordinate	northing (UTM grid)
contour interval	0.5% (sulphur), contour line begin at 1.5-7 % sulphur

Fig. 5.16. Krige estimate as shown by colour-shaded relief and contour lines for sulphur content (%), P3 seam, Nong Wa deposit, Sin Pun area

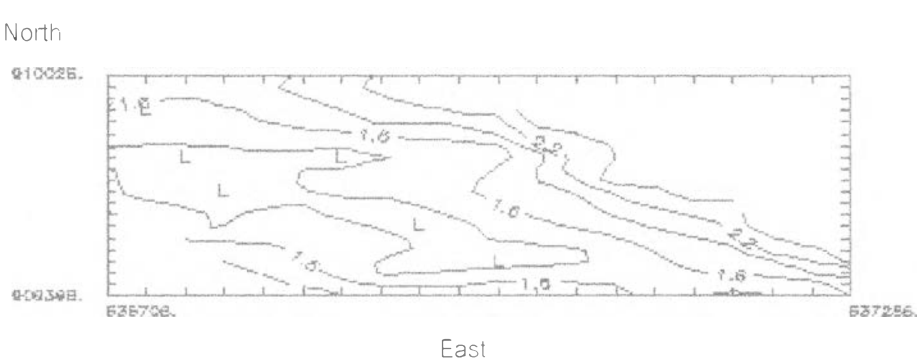
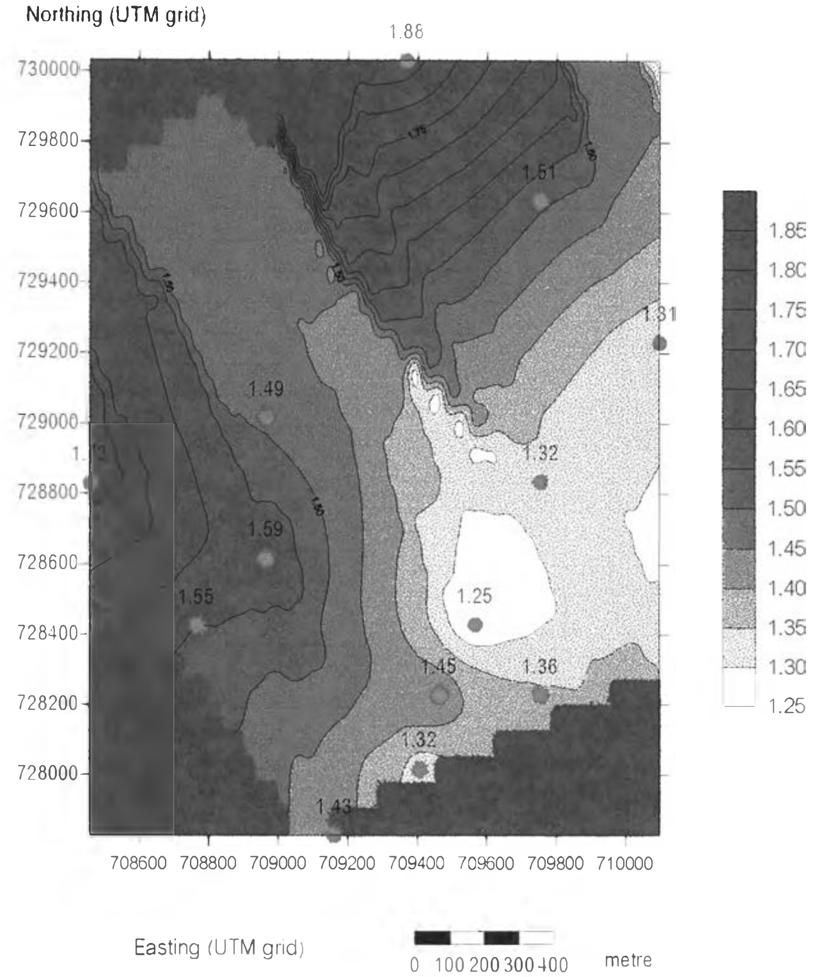


Fig. 5.17. Standard error of estimation for sulphur content (%), P3 seam, Nong Wa deposit, Sin Pun area.



Explanation

	degree of sulphur content, very high in dark purple and decrease to white
	location of true value for sulphur content
X-coordinate	easting (UTM grid)
Y-coordinate	northing (UTM grid)
contour interval	2% (sulphur), contour line begin at 1.25-1.85 % sulphur

Fig. 5.18. Krige estimate as shown by colour-shaded relief and contour lines for sulphur content (%),S3 seam, Ban Sao deposit, Saba Yoi area.

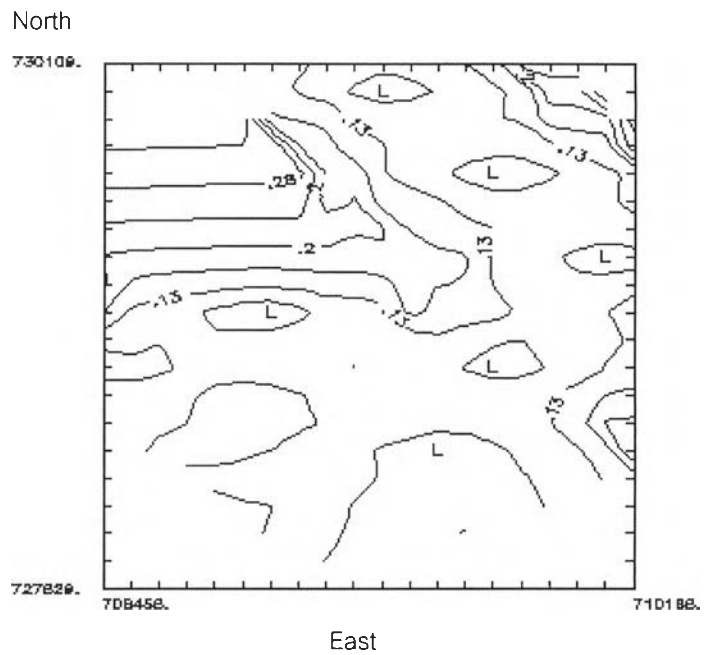


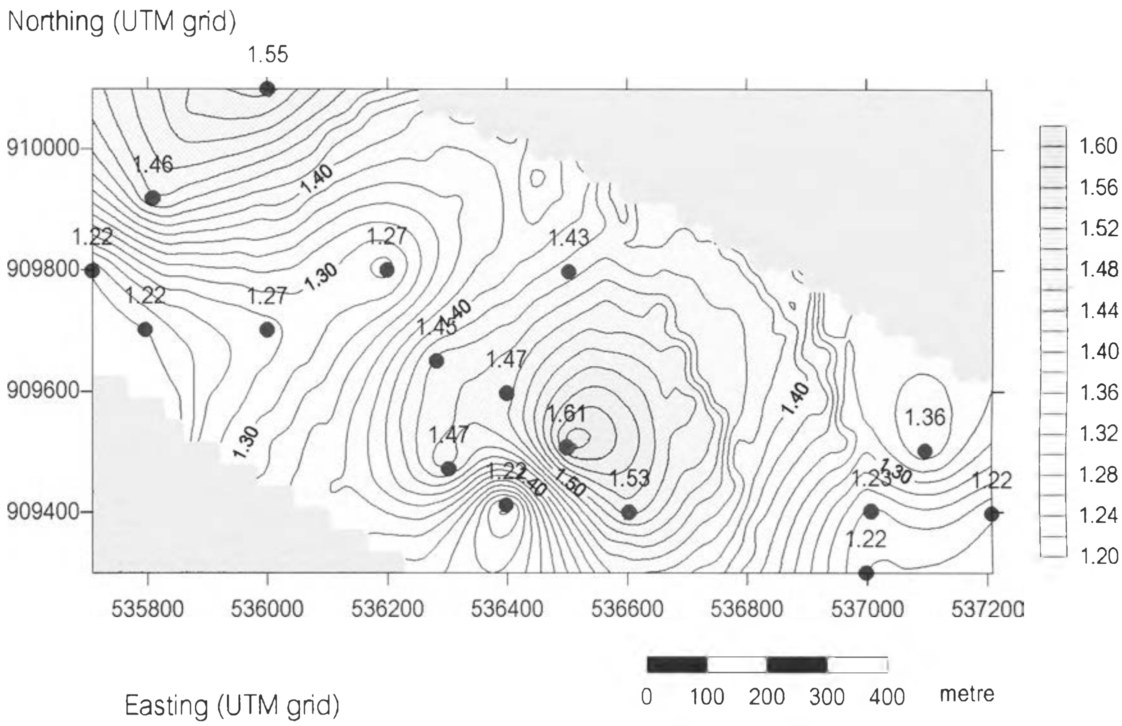
Fig. 5.19. Standard error of estimation for sulphur content (%), S3 seam, Ban Sao deposit, Saba Yoi area.

5.3.5. Density (g/cc)

Similar procedure using the previously described parameters is also applied for kriging estimation on density of Sin Pun and Saba Yoi coal seam. Fig. 5.20 predicts the example of kriging result illustrated by contour-shaded relief and overlaid contour maps of M seam Nong Wa deposit, Sin Pun area. It is observed that with the use of nugget effect at 0, sill at 0.014, range at 800 m and R-minor at 160 m. The result indicates the smooth contours, and the density varies from 1.2 to 1.5 (see Fig. 5.20).

The error estimation shown in Fig. 5.21 demonstrates the low values (L) in the western zone of Nong Wa sub-basin. The high value of estimated error in the northeast is interpreted as a result of small amount of drilled-hole data.

An example of Saba Yoi coal seam is taken from S4 seam. It is visualize from Fig. 5.22 that ordinary kriging estimate with the application of linear model, nugget effect at 0.0001, sill at 0.0018, range 1100 m, and R-minor at 480 m can indicate low density value in the northern zone of Ban Sao sub-basin.



Explanation

	degree of density, very high in dark yellow and decrease to white
●	location of true value for density
X-coordinate	easting (UTM grid)
Y-coordinate	northing (UTM grid)
contour interval	0.04% (density), contour line begin at 1.22-1.5 % density

Fig. 5.20. Krige estimate as shown by colour-shaded relief and contour lines for ash content (%), M seam, Nong Wa deposit, Sin Pun area.

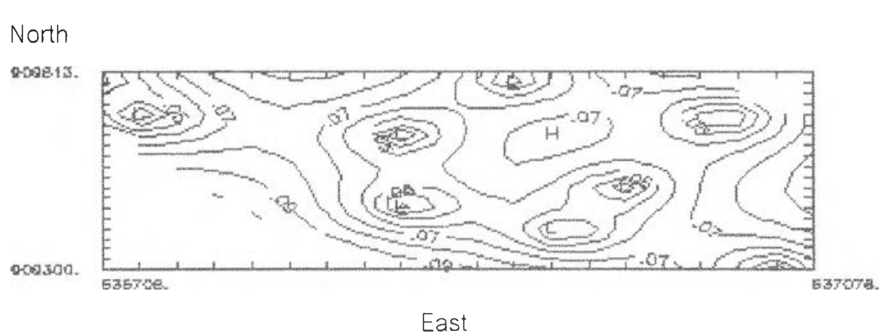
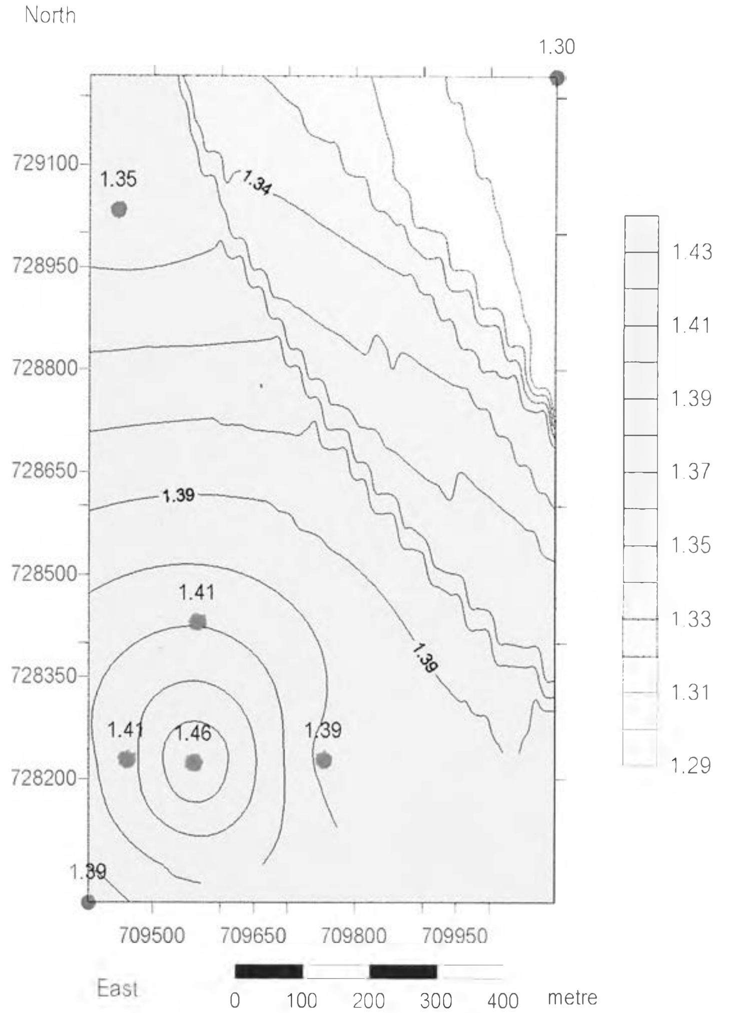


Fig. 5.21. Standard error of estimation for density (g/cc), M seam, Nong Wa deposit, Sin Pun area.



Explanation

▭	degree of density. very high in dark yellow and decrease to white
●	location of true value for density
X-coordinate	easting (UTM grid)
Y-coordinate	northing (UTM grid)
contour interval	0.02% (density). contour line begin at 1.29-1.43 % density

Fig. 5.22. Kriging estimation as shown by colour-shaded relief and contour lines for density (g/cc), S4 seam, Ban Sao deposit. Saba Yoi area.

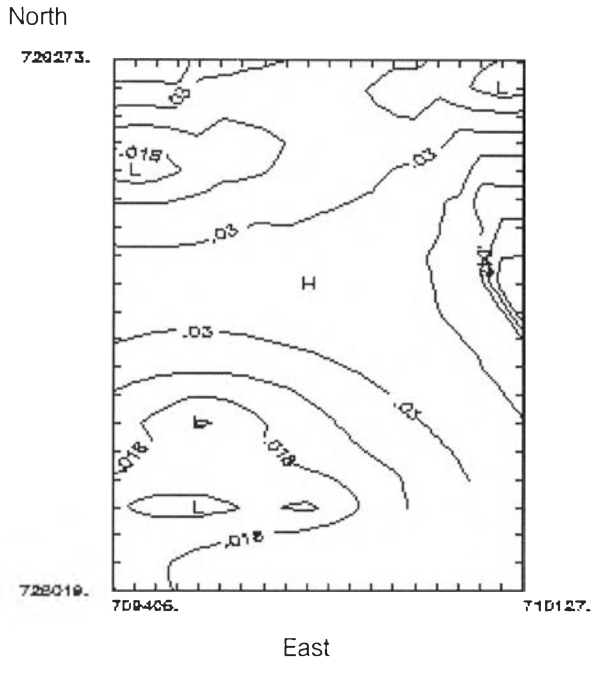


Fig. 5.23. Standard error of estimation for density (g/cc), S4 seam, Ban Sao deposit, Saba Yoi area.