Chapter IV

IDENTIFICATION OF A ROCKET SYSTEM

4.1 Introduction

The main problem of the identification of this rocket firing system is to determine the velocity at the end of burning time V_{eb} which occurs at the end of the power flight and the drag coefficient C_D.

4.2 To Calculate V_{eb}

Since the thrust T at the beginning is very high comparing to the gravitational force and the drag, the eqn. (2.2) is simplified as :

$$\frac{dv}{dt} = \frac{T}{m} \tag{4.1}$$

From Appendix C, eqn. (4.1) becomes

$$\frac{dv}{dt} = \frac{\mu u_e}{m} \tag{4.2}$$

where u_e = the effective efflux velocity

$\mu = -\frac{dm}{dt}$

The velocity v can be directly obtained in term of $m(t)$ and $m(o)$ as :

$$v = u_e \, \ell n \frac{m(o)}{m(t)} \tag{4.3}$$

Since the values of the initial mass of the rocket $m(o)$, the burn out mass m_b, are normally known. Therefore, the velocity at the end of burning time V_{eb} is obviously obtained from eqn. (4.3) as :

$$V_{eb} = u_e \ \ell n \frac{m(o)}{m(o) - m_b} \qquad (4.4)$$

A diagram which describes the location of the velocity V_{eb} is shown in Fig. (4.1). Several values of $m(o)$, m_b and u_e have been selected. For example, the typical values used in an experiment are nearly as :

$$m(o) = 2 \text{ kg.}$$

$$m_b = 0.2 \text{ kg.}$$

$$u_e = 1550 \text{ m/sec.}$$

In this case the velocity V_{eb} is :

$$V_{eb} = 1550 \ \ell n \frac{2}{1.8}$$

$$\cong 150 \text{ m/sec.}$$

4.3 To Identify C_D

From eqn. (3.47), we can write :

$$y = x\tan\theta - \frac{ge^{2Cx}}{4C^2(\dot{X}_o)^2} + \frac{gx}{2C(\dot{X}_o)^2} + \frac{g}{4C^2(\dot{X}_o)^2} \qquad (3.47)$$

where $\dot{X}_o$ denotes the rate of change of the horizontal distance at the beginning of the free flight of the rocket. Thus, from Fig. (4.1), we have :

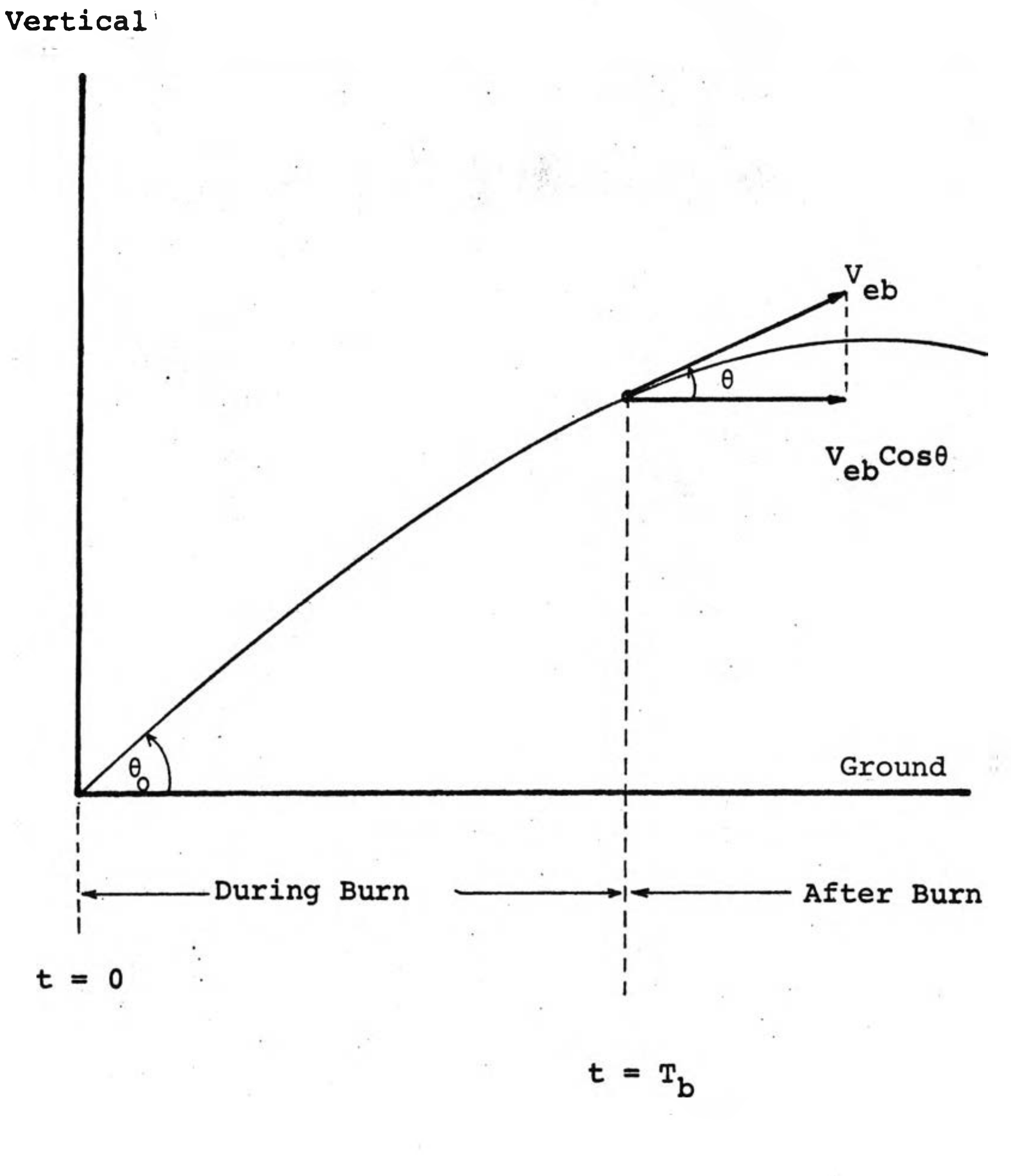

Figure 4.1

A Diagram Showing the Velocity at the End of Power Flight.

$$\dot{X}_o = V_{eb} \text{Cos}\theta \tag{4.5}$$

Substituting eqn. (4.5) into eqn. (3.47), we obtain :

$$y = x\tan\theta - \frac{ge^{2Cx}}{4C^2(V_{eb}\text{Cos}\theta)^2} + \frac{gx}{2C(V_{eb}\text{Cos}\theta)^2} + \frac{g}{4C^2(V_{eb}\text{Cos}\theta)^2}$$

Rearranging, we have :

$$y = x\tan\theta - \frac{gx^2}{2(V_{eb}\text{Cos}\theta)^2}\left[\frac{e^{2Cx} - 2Cx - 1}{\frac{1}{2}(2Cx)^2}\right]$$

Now, let $z = 2Cx$, the above equation becomes :

$$y = x\tan\theta - \frac{gx^2}{2(V_{eb}\text{Cos}\theta)^2}\left[\frac{e^{z} - z - 1}{\frac{1}{2}z^2}\right] \tag{4.6}$$

Consider the trajectory of the rocket after burning out as shown in Fig. (4.2), we have :

$$h = x\tan\theta \tag{4.7}$$

$$y = x\tan\theta - d \tag{4.8}$$

From eqn. (4.6) and eqn. (4.8), we obtain :

$$d = \frac{gx^2}{2(V_{eb}\text{Cos}\theta)^2}\left[\frac{e^{z} - z - 1}{\frac{1}{2}z^2}\right] \tag{4.9}$$

Since the values of θ, x and d may be measured from the experiment. Therefore the value of z can be calculated from eqn. (4.9).

From eqn. (3.30) and eqn. (3.38), where $z = 2Cx$, we obtain :

$$C_D = \frac{mz}{\rho\, d_i^2\, x} \tag{4.10}$$

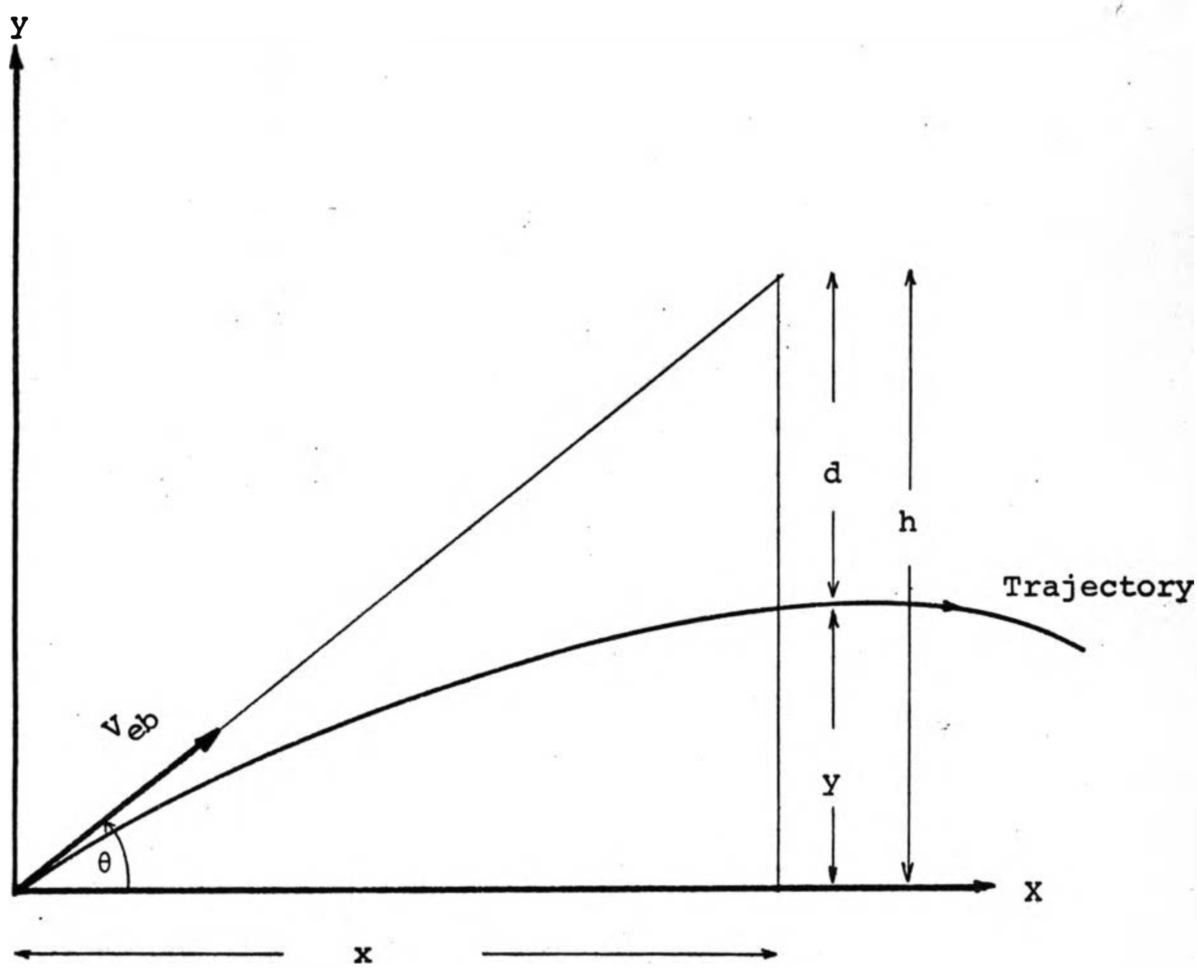

Figure 4.2

A Diagram Showing the Path of Rocket after Burn Out.

Then, the drag coefficient C_D can be calculated where ρ is the density of air and d_i is the diameter of the rocket body, and also the lift coefficient C_L can be calculated by the aerodynamics relation

$$C_L = \sqrt{\P (AR) C_D} \qquad (4.11)$$

where (AR) is the aspect ratio of the rocket (see Appendix D).