

การพัฒนาแบบจำลองพลวัตชนิดไม่เป็นเชิงเส้นของลอเรนซ์

นาย สมพร ชันเงิน

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DEVELOPMENT OF LORENZ NONLINEAR DYNAMIC MODEL

Mr. Somporn Khan-ngern

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By Mr. Somporn Khan-ngern
Department Physics
Thesis Advisor Professor Virulh Sa-yakanit, F.D.

Accepted by the Graduate School, Chulalongkorn University in
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Santi Thoongsuwan

..... Dean of Graduate School

(Associate Professor Santi Thoongsuwan, Ph.D.)

Thesis Committee

Ahpisit Ungkitchanukit

..... Chairman

(Ahpisit Ungkitchanukit, Ph.D.)

Virulh Sa-yakanit

..... Thesis Advisor

(Professor Virulh Sa-yakanit, F.D.)

Dusadee Sukawat

..... Member

(Dusadee Sukawat, Ph.D.)

David Ruffolo

..... Member

(David Ruffolo, Ph.D.)

พิมพ์ต้นฉบับบทคัดย่อวิทยานิพนธ์ภายในกรอบสี่เหลี่ยมนี้เพียงแผ่นเดียว

สมพร ชันเงิน : การพัฒนาแบบจำลองพลวัตชนิดไม่เป็นเชิงเส้นของลอเรนซ์ (DEVELOPMENT OF LORENZ NONLINEAR DYNAMIC MODEL) อ. ที่ปรึกษา : ศ.ดร. วิรุฬห์ สายคณิต, 91 หน้า. ISBN 974-631-137-9

วิทยานิพนธ์นี้มีจุดมุ่งหมาย เพื่อศึกษาการประยุกต์แบบจำลองของลอเรนซ์กับระบบพลวัตชนิดไม่เป็นเชิงเส้น ในการศึกษานี้ได้พัฒนาแบบจำลองดังกล่าวมาใช้อธิบายการเคลื่อนที่ของมวลอากาศตามแนวราบบริเวณละติจูดกลางของซีกโลกเหนือ โดยได้กำหนดฟังก์ชันกระแสของระบบให้อยู่ในรูป :

$$\psi(x, y; t) = \sqrt{2} X(t) \cos y + 2 Y(t) \cos 2x \sin y + 2 Z(t) \sin 2x \sin y$$

ผลจากการวิเคราะห์เสถียรภาพของสถานะคงตัวของระบบในกรณีต่างๆ แสดงให้เห็นถึงอิทธิพลของสภาพภูมิประเทศ, แรงขับเนื่องจากความร้อนต่อพฤติกรรมของระบบ นอกจากนี้เมื่อทำการทดลองเชิงตัวเลขเพื่อหาค่าฟังก์ชันกระแสที่เปลี่ยนแปลงตามเวลาของระบบ พบว่า ระบบดังกล่าวมีพฤติกรรมแบบเคออส (*chaotic behaviour*)

ภาควิชา..... ฟิลิกส์
สาขาวิชา..... ฟิลิกส์
ปีการศึกษา..... 25.37

ลายมือชื่อนิสิต.....
ลายมือชื่ออาจารย์ที่ปรึกษา.....
ลายมือชื่ออาจารย์ที่ปรึกษารวม.....

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The purpose of this thesis is to study the applications of Lorenz model for nonlinear dynamic systems. This shows a development of the model for a planetary-scale motion of the atmosphere in the domain area of middle latitude of the Northern Hemisphere. Streamfunctions of the system are assumed to be in the form :

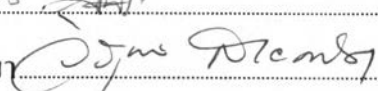
$$\psi(x, y; t) = \sqrt{2} X(t) \cos y + 2 Y(t) \cos 2x \sin y + 2 Z(t) \sin 2x \sin y.$$

Steady-state solutions and their stabilities have been calculated. The result shows that perturbations are induced by topography and by thermal driving force. Moreover, when we do the numerical experiments to obtain streamfuntions of the system, the result shows that there is chaotic behaviour occurring.

ภาควิชา..... ฟิสิกส์.....

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 ลายมือชื่ออาจารย์ที่ปรึกษา..... .....
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