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APPENDIX A

Standard Useful Constants

Gravitational constant	$G = 6.673 \times 10^{-11}$ N m^2 kg^{-2}
Acceleration due to gravity at sea level	$g_0 = 9.81$ m s^{-2}
Mean radius of the earth	$a = 6.37 \times 10^6$ m
Angular speed of rotation of the earth	$\Omega = 7.292 \times 10^6$ rad s^{-1}
Universal gas constant	$R^* = 8.314 \times 10^3$ J K^{-1} kmol^{-1}
Gas constant for dry air	$R = 287$ J K^{-1} kg^{1}
Specific heat of air at constant pressure	$c_p = 1004$ J K^{-1} kg^{-1}
Specific heat of air at constant volume	$c_V = 717$ J K^{-1} kg^{-1}
Ratio of specific heats	$\gamma = c_p/c_V = 1.4$
Mass of the earth	$M = 5.988 \times 10^{24}$ kg
Standard sea level pressure	$p_0 = 101.325$ kPa
Standard sea level temperature	$T_0 = 288.15$ K
Standard sea level density	$\rho_0 = 1.225$ kg m^{-3}

APPENDIX B

Derivation of the Lorenz Model

Here we present a rather short derivation of the Lorenz model which should provide the reader with a feeling for the approximations involved. For a more rigorous treatment we refer the reader to the original articles by Chandrasekhar, Saltzmann, and Lorenz[Chandrasekhar 1961, Saltzmann 1962, and Lorenz 1963].

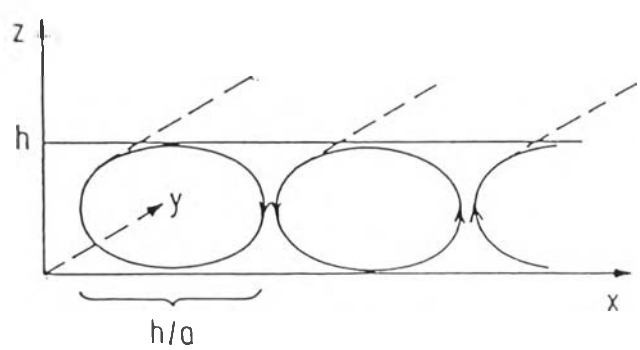

Figure B-1 Convection rolls and geometry in the Rayleigh-Bénard experiment.

Consider the Rayleigh-Bénard experiment as depicted in Fig. (B-1). The fluid is described by a velocity field $\vec{v}(\vec{x}, t)$ and the temperature field $T(\vec{x}, t)$. The basic equations which describe our system are ;

a) the Navier-Stokes equation:

$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} p + \rho \vec{g} + \mu \nabla^2 \vec{v} , \qquad \text{(B-1)}$$

b) the equation for heat conduction:

$$\frac{dT}{dt} = \kappa \nabla^2 T , \qquad \text{(B-2)}$$

c) the continuity equation:

$$\frac{\partial \rho}{\partial t} + div(\rho \vec{v}) = 0 , \qquad \text{(B-3)}$$

d) the equation of state:

$$\rho = \rho_{av}(1 - \alpha \Delta T) , \qquad \text{(B-4)}$$

with the boundary conditions

$$T(x, y, z = 0, t) = T_0 + \Delta T , \qquad \text{(B-5a)}$$

$$T(x, y, z = h, t) = T_0 , \qquad \text{(B-5b)}$$

where ρ is the density of fluid

ρ_{av} is the average of ρ over the entire fluid,

μ is its viscosity,

κ is the thermal conductivity,

α is the coefficient of volume expansion,

$\vec{g}$ is the apparent acceleration of Earth's gravity,

p is the pressure.

To simplify the calculation, it is assumed that;

(i) the system is translationally invariant in the y-direction so that the convection rolls extend to infinity as shown in Fig.(B-1),

(ii) the ΔT - dependence of all coefficients - except in $\rho = \rho_{av} (1 - \alpha \Delta T)$ - can be neglected. The continuity equation thus becomes

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 , \qquad \text{(B-6)}$$

By virtue of Eq.(B-6) we can define a streamfunction ψ (x, z, t) as follows:

$$u = -\frac{\partial\psi}{\partial z}, \qquad and \qquad w = \frac{\partial\psi}{\partial x} \tag{B-7}$$

As a next step we introduce the deviation Θ (x, z, t) from the linear temperature profile via

$$T(x, z, t) = T_0 + \Delta T - \frac{\Delta T}{h} z + \Theta(x, z, t) \ , \tag{B-8}$$

Using (B-7) and (B-8) the basic equations can be written as

$$\frac{\partial}{\partial t} \nabla^2 \psi = -\frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, z)} + \vartheta \nabla^4 \psi + g\alpha \frac{\partial \Theta}{\partial x} \ , \tag{B-9}$$

$$\frac{\partial \Theta}{\partial t} = -\frac{\partial(\psi, \Theta)}{\partial(x, z)} + \frac{\Delta T}{h}\frac{\partial \psi}{\partial x} + \kappa \nabla^2 \Theta \ , \tag{B-10}$$

where $\vartheta = \frac{\mu}{\rho_{av}} \equiv$ the kinematic viscosity,

$$\frac{\partial(F, G)}{\partial(x, z)} = \frac{\partial F}{\partial x}\frac{\partial G}{\partial z} - \frac{\partial F}{\partial z}\frac{\partial G}{\partial x} \ ,$$

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial z^4} + 2\frac{\partial^4}{\partial x^2 \partial z^2} \ ,$$

$\nabla^2 \psi = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \equiv$ the vorticity of the motion in the xz-plane.

In order to simplify (B-9) and (B-10), Lorenz used free boundary conditions:

$$\Theta(0, 0, t) = \Theta(0, h, t) = \psi(0, 0, t) = \psi(0, h, t) = 0 \ , \tag{B-11a}$$

$$\nabla^2 \psi(0, 0, t) = \nabla^2 \psi(0, h, t) = 0 \ , \tag{B-11b}$$

and retained only the lowest order terms in the Fourier expansions of ψ and Θ. Accordingly, we shall let

$$\frac{a}{\kappa(1 + a^2)} \psi(x, z, t) = \sqrt{2}\, X \sin\left(\frac{\pi a}{h} x\right) \sin\left(\frac{\pi}{h} z\right) \ , \tag{B-12}$$

$$\frac{\pi R}{R_c \Delta T} \Theta(x, z, t) = \sqrt{2}\, Y \cos\left(\frac{\pi a}{h} x\right) \sin\left(\frac{\pi}{h} z\right) - Z \sin\left(\frac{2\pi}{h} z\right) \ , \tag{B-13}$$

where X, Y and Z are functions of time alone. In fact,

X is proportional to the intensity of convective motion,

Y is proportional to the temperature difference between ascending and descending currents,

Z is proportional to the distortion of the vertical temperature profile.

When expressions (B-12) and (B-13) are substituted into Eqs.(B-9) and (B-10), and trigonometric terms others than those occuring in (B-12) and (B-13) are omitted, we obtain the equations

$$\begin{aligned} \frac{dX}{d\tau} &= \sigma (Y - X), \\ \frac{dY}{d\tau} &= rX - Y - XZ, \\ \frac{dZ}{d\tau} &= XY - bZ, \end{aligned} \qquad \text{(B-14)}$$

Here $\tau = \pi^2 h^{-2} (1 + a^2) \kappa t$

σ is the so-called the ***Prandtl number*** ($\sigma = \upsilon/\kappa$),

r is the external control parameter ($r = R_a/R_c \; \alpha \; \Delta T$);

R_a is the ***Rayleigh number*** ($R_a = g\alpha H^3 \Delta T \upsilon^{-1} \kappa^{-1}$),

R_c is a ***critical number*** ($R_c = \pi^4 a^{-2}(1 + a^2)^3 = 27\pi^4/4$; when $a^2 = 1/2$)

$b = 4 (1 + a^2)^{-1}$.

APPENDIX C

Linear Stability Analysis

For conservative systems (in one dimension), a global phase portrait is particular easy to construct. The *equilibrium points* (or the *fixed points*) play an important role and clearly have a characteristic *local behaviour*, that is, sets of closed curves around the stable points and hyperbolic-looking regions in the neighborhood of the unstable points. For nonconservative systems, a global phase portrait is difficult to construct unless an explicit solution is known to the equations of motion. However, it is always possible to build up an approximate local phase portrait by identifying the equilibrium points and drawing the phase curves in their neighborhood, the nature of which will depend on their stability. Equilibrium points can be thought of as the *"organizing centers"* of a system's phase-space dynamics. Thus by identifying them and their stability, we can build up a fairly global picture of a system's behaviour.

Consider a system whose state may be described by 2 variables (x, y). Let the system be governed by the set of equations

$$\frac{dx}{dt} = F(x, y) \ , \qquad \text{(C-1a)}$$

$$\frac{dy}{dt} = G(x, y) \ , \qquad \text{(C-2b)}$$

where time t is the single independent variable, the functions F and G possess continuous first partial derivatives. Such a system may be studied by means of *phase space.* Each point in phase space represents a possible instantaneous state of the system. A state which is varying in accordance with Eqs.(C-1) is represented by a moving particle in phase space, traveling along a trajectory in phase space. For completeness, the position of a stationary particle, representing a equilibrium state, is included as a trajectory.

The equilibrium points of the motion are those values of x and y, denoted as x_0 and y_0, for which the phase flow is stationary, that is

$$F(x_0, y_0) = G(x_0, y_0) = 0 , \qquad \text{(C-2)}$$

There can be any number of points (x_0, y_0) satisfying these conditions, depending on the precise functional form of F and G. Having identified these points, their stability can be determined by examining the evolution of some small displacement (δx, δy) about (x_0, y_0). Expanding F and G in powers of δx, δy, we obtain

$$\frac{dx}{dt} = F_x(x_0, y_0)\,\delta x + F_y(x_0, y_0)\,\delta y + F_{xy}(x_0, y_0)\,\delta x\delta y + \cdots , \qquad \text{(C-3a)}$$

$$\frac{dy}{dt} = G_x(x_0, y_0)\,\delta x + G_y(x_0, y_0)\,\delta y + G_{xy}(x_0, y_0)\,\delta x\delta y + \cdots , \qquad \text{(C-3b)}$$

where

$$F_x \equiv \frac{\partial F}{\partial x}; \qquad F_y \equiv \frac{\partial F}{\partial y}; \qquad \text{and} \qquad F_{xy} \equiv \frac{\partial^2 F}{\partial x \partial y},$$

with similar expressions for G.

To first order, Eqs.(C-3) can be written as the linear system (C-4), and we refer to them as the ***linearized equations,*** that is

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} F_x(x_0, y_0) & F_y(x_0, y_0) \\ G_x(x_0, y_0) & G_y(x_0, y_0) \end{pmatrix}\begin{pmatrix} \delta x \\ \delta y \end{pmatrix}, \qquad \text{(C-4)}$$

where the 2 x 2 matrix (to be denoted as M) is often refer to as the *stability matrix*. The system of first order linear-equations (which is easily generalized for an system of the form $\dot{X}_i = F_i(X_i, \cdots, X_N)$, $i = 1, \cdots, N$) is easily solved with the eigenvalues of M determining the stability of the associated equilibrium point. The eigenvalues are just the roots of the equation

$$det(M - \lambda I) = 0 , \tag{C-5}$$

where I is the unit matrix. For the problem of two differential equations, there are two eigenvalues which can possibly have both real and imaginary parts. These eigenvalues take the general form

$$\lambda_1 = a_1 + b_1 i \qquad \text{and} \qquad \lambda_2 = a_2 + b_2 i$$

where $i = \sqrt{-1}$. The values of real parts (a_1, a_2) and imaginary parts (b_1, b_2) determine the nature of the stability (or instability) in the neighborhood of the equilibrium points. These possibilities are summarized in Table (C-1).

Table C-1 Classification of equilibrium (or fixed) points.

a_1, a_2	b_1, b_2	Stability Analysis
Negative	Zero	Stable point
Positive	Zero	Unstable point
One Positive, One Negative	Zero	Metastable; (hyperbolic point or saddle point)
Negative	Nonzero	Stable spiral point
Positive	Nonzero	Unstable spiral point
Zero	Nonzero	Neutrally stable (Elliptic point)

APPENDIX D

Numerical Integrations

Consider a system represented by the equations

$$\frac{dX_i}{dt} \quad = \quad F_i(X_1, \cdots, X_N); \qquad i = 1, \cdots, N , \tag{D-1}$$

where time t is the single independent variable, the functions F_i possess continuous first partial derivatives. Such a system may be studied by means of *phase space* - an N-dimensional Euclidean space Γ whose coordinates are $X_1, \cdots, X_N$.

To solve Eqs.(D-1) numerically we may choose an initial time t_0 and a time increment Δt, and let

$$X_{i,\,n} \quad = \quad X_i(t_0 + n\,\Delta t) \ . \tag{D-2}$$

We then introduce the approximations

$$X^*_{i\,,(n+1)} \quad = \quad X_{i,\,n} \ + \ F_i(P_n)\,\Delta t \ , \tag{D-3a}$$

$$X_{i,(n+1)} \quad = \quad X_{i,\,n} \ + \ F_i(P_{n+1})\,\Delta t \ , \tag{D-3b}$$

where P_n and P_{n+1} are the points whose coordinates are

$$(X_{1,\,n}, \cdots, X_{N,\,n}) \qquad \text{and} \qquad \left(X^*_{1,(n+1)}, \cdots, X^*_{N\,,(n+1)}\right)$$

respectively.

This numerical procedure is so-called the *Euler-backward difference procedure.*

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