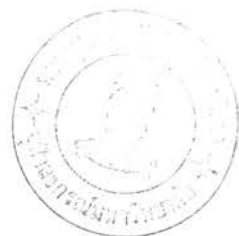


## INTRODUCTION



Every commutative congruence-free semigroup with a multiplicative identity  $1$  is isomorphic to the integers modulo  $p$  where  $p$  is  $1$  or  $p$  is a prime number or is a group with zero of order two. Every congruence-free commutative ring with a multiplicative identity  $1$  is a field. Thus in both these cases every non-zero element has a multiplicative inverse. The purpose of this thesis is to characterize congruence-free commutative semirings with  $1$ . The research was driven by this natural question: "Does every non-zero element have a multiplicative inverse?" The answer to this question as is proved in Chapter V, is no. However in many important cases the answer is yes.

Chapter I introduces notation and provides a few necessary results from the literature. Chapter II contains some basic theorems and enables us to classify congruence-free commutative semirings with  $1$ . Chapter III proves that every such semiring with an additive identity is a field. Chapter IV considers the case of congruence-free commutative semirings with  $1$  which have an additive zero and gives a sharp characterization of these semirings as semifields. Chapter V is concerned with congruence-free commutative semirings with  $1$  without an additive identity or an additive zero. Basic theorems on the additive structure of these semirings are given as well as some interesting and important examples.