

CHAPTER II

THEORY

2.1 Basic Concepts of Liquid Mixing ⁽³⁾

The basic concepts for the representation of the state of mixing and the progress of mixing in agitated liquids will be discussed first.

2.1.1 Defining the state of mixing

When the concentration C at an arbitrary point in a liquid is equal to the average concentration C_m which is calculated from charged ratio, the state of mixing is called homogeneous. In order to represent the degree of mixing in a non-uniform state, a mathematical expression could be desirable to show how far the concentration deviates from the homogeneous state. The concentration distribution of solute in the x-direction is shown in Figure 2.1. The same situation is true for temperature distribution, so the discussion about concentration also includes the temperature. As can be seen in Figure 2.1, the state of mixing of the solute is expressed by the width of concentration blotches, L_s or striation distance, L_D and by the intensity of concentration I_s . The former is called the scale of segregation and the latter the intensity of segregation. The concentration variance σ_c^2 is defined by Eq. (2.1).

$$\delta_c^2 = \int_0^L \{C(X) - C_m\}^2 dx \quad (2.1)$$

where L is the spacial variable occupied by the liquid. δ_c^2 or its dimensionless value δ_c^2/C_m^2 corresponds to I_s . When the solute is in a state of irregular distribution, a correlation function $f(r)$ between two adjacent points separated by r is considered which is defined as shown in Eq.(2.2) where $c(x)=C(x)-C_m$ and $c(x+r)=C(x+r)-C_m$.

$$f(r) = \frac{1}{\delta_c^2} \int_0^L c(x) * c(x+r) dx \quad (2.2)$$

$$L \gg r; \quad \delta_c^2 = \int_0^L c^2(x) dx = \int_0^L c^2(x+r) dx \quad (2.3)$$

Using Eq. (2.2), we can define L_s as follows

$$L_s = \int_0^r f(r) dr \quad (2.4)$$

In a homogeneous state, L_s or L_D is equal to 0 or L . I_s tends to zero. The concentration variation with time is observed at an arbitrary point in a flow system, as shown in Figure 2.2. Similarly with Eqs. (2.2) and (2.4), a correlation $f(r)$ and a time scale L_{st} which is the time for concentration blotches are defined as follows:

$$f(r) = \frac{1}{\sigma_{ct}^2} \int_0^T c(t) c(t+r) dt \quad (2.2')$$

$$L_{st} = \int_0^T f(r) dr \quad (2.4')$$

where T is the duration of the observation and is very large compared with τ . C_{mt} is the time average of concentration.

$$C_{mt} = \frac{1}{T} \int_0^T C(t) dt \quad (2.5)$$

$$\sigma_{ct}^2 = \int_0^T C^2(t) dt = \int_0^T C^2(t+\tau) dt \quad (2.3')$$

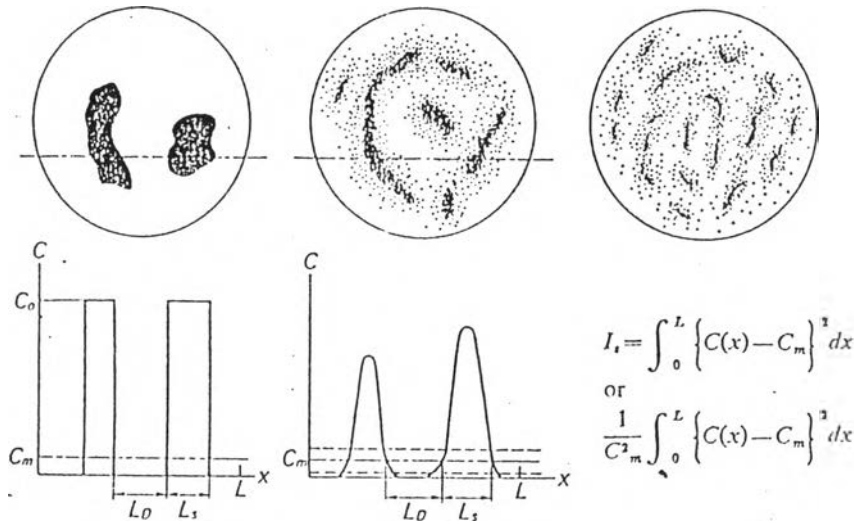


Figure 2.1 Scale of segregation and intensity of segregation.

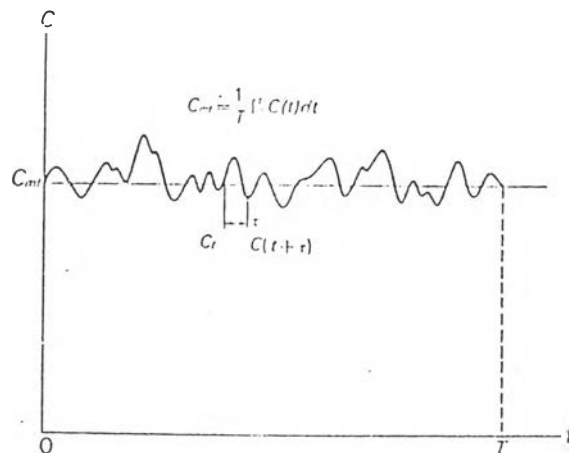


Figure 2.2 Concentration fluctuation with time.

It is difficult to connect strictly the scale of concentration blotches, L_s with the time of concentration variation L_{st} , but the following approximation may be assumed taking the flow velocity at the point of observation as V .

$$L_s = VL_{st} = V \quad (2.6)$$

Actually the observation of a point value is impossible, because the dimensions of the measuring probe and the sample are not infinitesimal. We must be contented with an average concentration corresponding to the resolving ability of the probe. Also the sensitivity of the measuring device has a limit, and the small difference in concentration is difficult to detect. At a satisfactory state of mixing, the intensity of segregation I_s need not be zero, but is allowed to have an error lower than a certain limit ΔI_{sa} .

The time required to lower the value of I_s to ΔI_{sa} is taken as the mixing time θ_m .

As a mixing index (M)^(13,14), we can use Eq. (2.2) and (2.7') which consist of a concentration variance of an arbitrary mixing state $\sigma^2 = I$ and that of an initial unmixed state $\sigma^2 = I_0$.

$$M = 1 - \frac{\sigma^2}{\sigma_0^2} = 1 - \frac{I_s}{I_0} \quad (2.7)$$

$$M' = 1 - \frac{\sigma}{\sigma_0} \quad (2.7')$$

M and M' are zero in an unmixed state and are 1 at a completely mixed state.

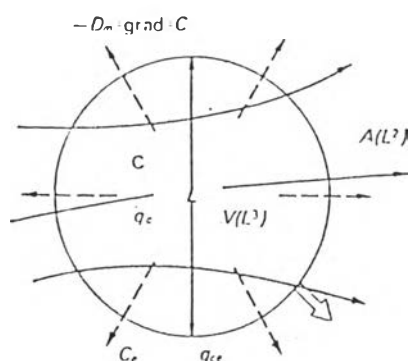
2.1.2 Flow pattern and progress of mixing

There are two mechanisms in promoting mixing for solute. One is the mass transfer caused by a convection flow and the other is that caused by molecular diffusion.

By the convection flow, the blotches of solute are deformed, and are divided into pieces, and the size of segregation L_s or L_p becomes smaller. The contact surface area between zones of high and low concentration of solute increases or is renewed, and the mixing and transport is promoted by the molecular diffusion. The intensity of segregation may not be changed by the convection flow only, but is influenced by the aid of molecular diffusion. As a practical point of view, however, the apparent value of I_s can be reduced to zero or to a certain value smaller than ΔI_{s_0} without molecular diffusion, because the resolving power of the sensor for concentration measurement is finite and the desired mixture is attained at a certain range of an allowable error.

Figure 2.3 shows the process of mass transfer of solute in a lump L_s into surrounding liquid by the action of convection and molecular diffusion. From a mass balance of solute in the lump we have,

$$V(L^3) \frac{dC}{dt} = \iint_{A(L^2)} D_m (\text{grad } C \cdot n)_{\wedge} dA - \iint_{A(L^2)} (C v \cdot n)_{\wedge} dA + \iiint_{V(L^3)} q_c dV \quad (2.8)$$



$V(L^3)$: Volume of a lump

$A(L^2)$: Surface area of a lump

C : Concentration of solute

q_c : Generation of solute

$C_e, q_{c,e}$: C and q_c in surrounding

Figure 2.3 Process of mass transfer of solute in a lump.:

—→ Mass flux by convection,

----→ Mass flux by diffusion

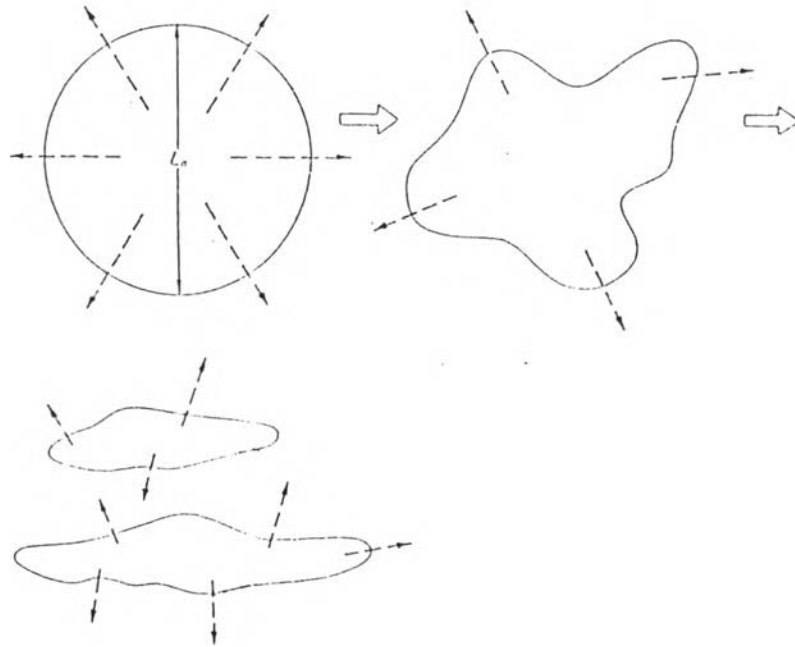


Figure 2.4 Mixing process of a lump of solute with surrounding liquid.

--> Mass transfer by diffusion

Denoting a concentration difference between the lump and the surrounding by $C = C - C_{\infty}$, the solute flux, by $k_c \cdot C$ and the effluent velocity through the interface by v or v_n then we have,

$$V(L^3) \frac{d\Delta C}{dt} = A(L^2)(k_c + v_n) C + V(L^3)(q_c - q_{c_{\infty}}) \quad (2.8 a)$$

$$\frac{d\Delta C}{dt} = - \frac{A(L^2)}{V(L^3)}(k_c + v_n) C + q_c \quad (2.8b)$$

Let us take a lump of solute, flowing in a stream intermingling and transferring solute with the surrounding liquid as shown in Figure 2.4. Assuming that the convection velocity at the interface, v_n is zero, then Eq. (2.8b) is reduced to

$$\frac{d\Delta C}{dt} = - \frac{A(L^2)k_c \Delta C}{V(L^3)} + q_c \quad (2.9)$$

As the ratio of interfacial area to volume A/V becomes larger when L_c gets smaller, the first term of the right hand side of Eq. (2.8b) and Eq. (2.9) as well as the mixing rate become large. Therefore the deformation and subdivision of the lump of solute contribute largely to the mixing in a large space. On the other hand, the spreading of solute by diffusion to the surroundings contributes largely to the mixing in a small space, diminishing I_c and enlarging L_c . Figure 2.5 shows the progress of mixing by the action of convection and diffusion with schematic curves L_c vs. t , L_d vs. t and I_c vs. t .

The range of rapid decrease in size of a concentration blotch L_c corresponds to the period of controlling convection and the range where L_c converts to a rapid increase or the intensity of segregation I_c goes down abruptly, corresponds to the period of diffusion controlling.

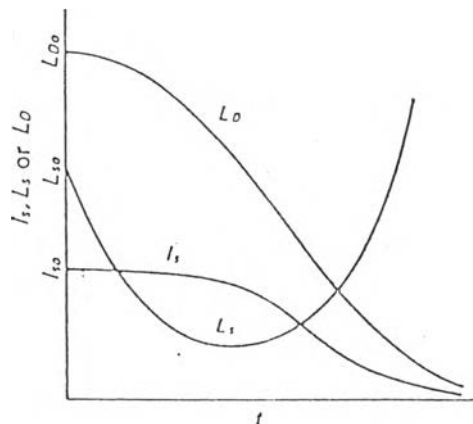


Figure 2.5 Change of L_o and I_o with lapse of mixing time.

2.1.3 Mixing in a laminar flow range

In high viscosity liquids, mixing is carried out in a laminar state and the molecular diffusion is very small. The mass balance equation is obtained by taking the diffusion in convection flow into consideration, i.e., by taking the limit of L tending to zero in Eq. (2.8), and the well-known differential equation is derived;

$$\frac{\partial C}{\partial t} + v_x \frac{\partial C}{\partial x} + v_y \frac{\partial C}{\partial y} + v_z \frac{\partial C}{\partial z} = D_m \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) + q_c \quad (2.10)$$

Also the equation of continuity must be considered

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \quad (2.11)$$

Let us convert Eq. (2.10) into a dimensionless form.

$$\begin{aligned}x^* &= x/L_0, \quad y^* = y/L_0, \quad z^* = z/L_0 \\V_x^* &= V_x/V_0, \quad V_y^* = V_y/V_0, \quad V_z^* = V_z/V_0 \\C^* &= C/c_0, \quad t^* = tV_0/L_0\end{aligned}\quad (2.12)$$

where C_0 , L_0 , and V_0 are values taken at a standard condition. By using dimensionless notations, Eq. (2.10) is written:

$$\frac{\partial C^*}{\partial t^*} = -(V_x^* \frac{\partial C^*}{\partial x^*} + V_y^* \frac{\partial C^*}{\partial y^*} + V_z^* \frac{\partial C^*}{\partial z^*}) + \frac{D_m}{D_0 L_0} (\frac{\partial^2 C^*}{\partial x^{*2}} + \frac{\partial^2 C^*}{\partial y^{*2}} + \frac{\partial^2 C^*}{\partial z^{*2}}) \quad (2.10')$$

or by a vector notation

$$\frac{\partial C^*}{\partial t^*} = - (U^* \nabla^* C^*) + \frac{1}{Pe} (\Delta^* C^*) \quad (2.13)$$

where $Pe = Re \cdot Sc$, $Re = V_0 L_0 / \nu$ and $Sc = \nu / D_m$.

When the mixing process by convection flow only is considered, the diffusion term in Eq. (2.13) disappears,

$$\frac{\partial C^*}{\partial t^*} = - (U^* \nabla^* C^*) \quad (2.13')$$

Therefore, in a geometrically similar system, the progress of mixing by a convection flow becomes identical. As the flow pattern of agitated liquid is controlled by Reynolds number, the state of similarly is also determined by the Reynolds number. When the diffusional term is also to be considered, we must consider the equality of Pe number, which corresponds to the equality of Re and Sc numbers.

2.1.4 Turbulent mixing

Although the mechanism of turbulent mixing is not essentially different from that of laminar, a statistical treatment as a mass of fluid elements composing irregular motion is necessary in the former case. The so-called turbulent diffusion is assumed for the mixing phenomena in a large space compared with the average size of turbulent eddies. The velocity and concentration of convection flow are represented by the sum of their time average value denoted with bars, and their fluctuation components shown by small letters as follows;

$$V_x = \bar{V}_x + v_x, \quad V_y = \bar{V}_y + v_y, \quad V_z = \bar{V}_z + v_z \quad (2.14)$$

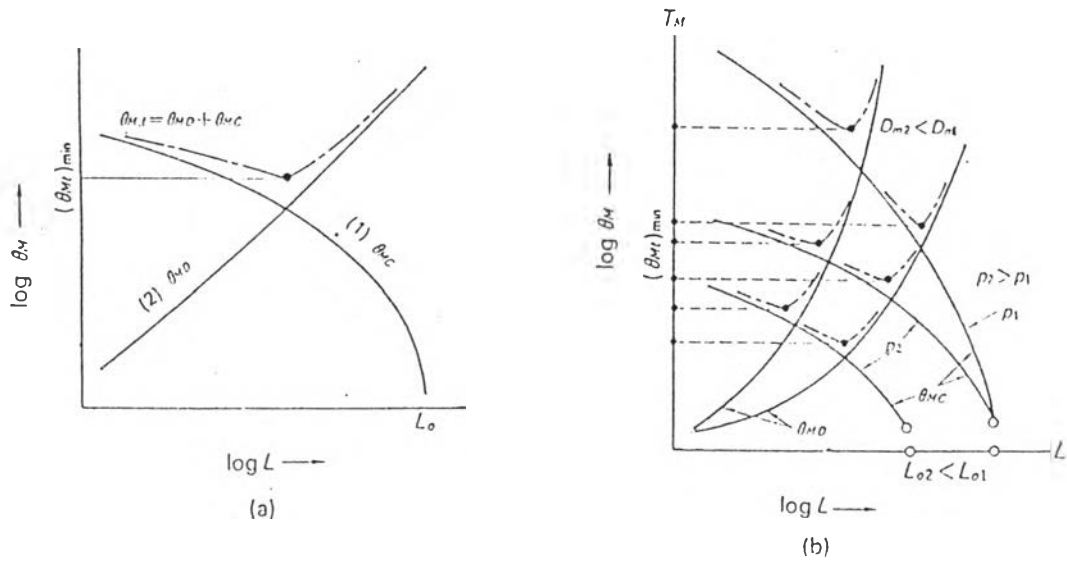


Figure 2.6 Relationships among mixing time θ_M and dividing action of convection (p), molecular diffusion (D_m) and space variables (L_o).

$$C = C + c \tag{2.14'}$$

Substitute these notations; and introduce the turbulent solute transfer into Eqs. (2.10) and (2.11), then the equation expressing the distribution of the time average of solute concentration is obtained.

$$\frac{\partial C}{\partial t} + v_x \frac{\partial C}{\partial x} + v_y \frac{\partial C}{\partial y} + v_z \frac{\partial C}{\partial z} = (D_t + D_m) \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) + q_c \tag{2.15}$$

In a fully developed turbulent flow field, D_t is extremely large compared with D_m and the contribution of turbulent diffusivity to the mixing process is predominant and important compared with the

case of laminar flow. The dimensionless variables U^* and C^* are also introduced for the time average values of flow velocities and concentration. When molecular diffusivity is neglected compared with turbulent diffusivity, Eq. (2.15) is reduced to,

$$\frac{\partial C^*}{\partial t^*} = -(U^* \nabla^* C^*) + \frac{1}{Pe_{\epsilon}} (\Delta^* C^*) \quad (2.15')$$

where $Pe_{\epsilon} = V_0 L_0 / D_{\epsilon}$ is a Peclet number in a turbulent field. Eq. (2.15') corresponds to Eq. (2.13) for the laminar field. In a fully developed turbulent flow, the state of flow is independent of $Re = V_0 L_0 / \nu$, and $Sc_{\epsilon} = \nu_{\epsilon} / D_{\epsilon}$. $Re_{\epsilon} = V_0 L_0 / \nu_{\epsilon}$ (or Pe_{ϵ}) takes a constant value. The flow pattern is considered to be similar in geometrically similar vessels, so that the dimensionless mixing velocity $\partial C^* / \partial t^*$ takes an identical value irrespective of the volume of mixing space and the law of similarity becomes applicable. For example, in turbulent flow in a mixing vessel, the relation between $C^* = C / C_0$ and $t^* = t V_0 / L_0$ (or t_n) becomes identical, irrespective of vessel volume and a similar state of mixing is attained at an equal dimensionless mixing time.

$$\Theta_M \cdot n = N_M = \text{const.}$$

$$\Theta_M \approx 1/n \quad (2.16)$$

Actually the turbulence in mixing vessels is found to be composed of an average convection flow with a turbulent fluctuation velocity superimposed. Therefore, in a similar way as in laminar flow, the state of mixing proceeds as follows; (1) mixing by convection flow (diminution of L_D), (2) mixing by turbulent diffusion (diminution of I_s in macroscopic observation), (3) mixing by a local convection flow which causes the disintegration of turbulent eddies (diminution of L_D), (4) mixing by molecular diffusion (real decrease in I_s).

Mixing in a large space volume is accomplished by (1) convection due to a mean flow and (2) turbulent diffusion. Mixing by the disintegration of turbulent eddies and by the molecular diffusion is important for micro mixing, but is generally negligible for macro mixing in a large space.

2.2 Method of Measuring Mixer Performance ⁽³⁾

After injection of a small amount of tracer in an agitated liquid, concentration change and mixing time are measured in an appropriate location of the vessel. The following methods were commonly used.

a) Using a small amount of electrolyte as a tracer, the concentration change was measured by a small probe. ^(5.8.7.8)

b) After introducing warm liquid, the temperature change is measured with thermometers located in various positions. ⁽¹²⁾.

c) After injection of a dyed solution, the spreading patterns are followed by eye, by taking photographs or by using a detector comprised of photoelectric converter.⁽¹¹⁾

d) After injection of a decolorizing agent as a tracer into a dyed liquid, the state of decolorization is traced.⁽³⁾

f) Schlieren method⁽⁹⁾ which is a technique to trace the uniformity of solute concentration by measuring the difference in refractive index.

Figure 2.7 shows a measuring circuit for electric conductivity presented by Kramers et al.⁽³⁾ and Figure 2.13 shows Schlieren technique⁽¹⁵⁾ which detects the concentration difference throughout the vessel.

As has already been stated in connection with Figure 2.1 and 2.2, it is essential to set the allowable degree of unevenness of solute concentration. Figure 2.9 shows the variation in observed mixing time θ_m with the difference of criteria for sufficient mixing. From this diagram we can compare the sensitivity of various methods in measuring mixing time.

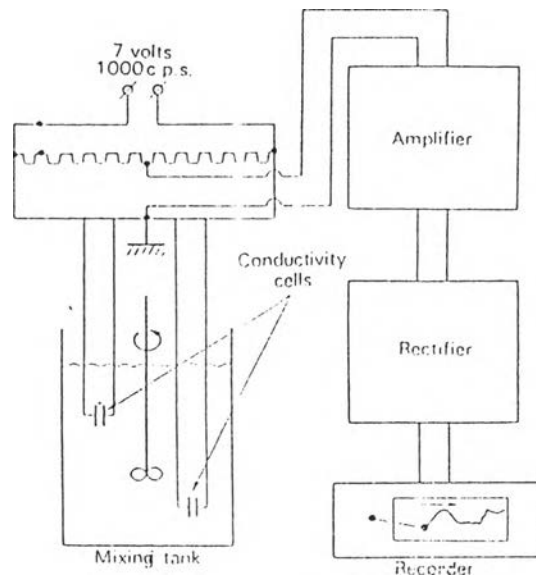


Figure 2.7 Method of measuring fluctuations by electrical conductivity.

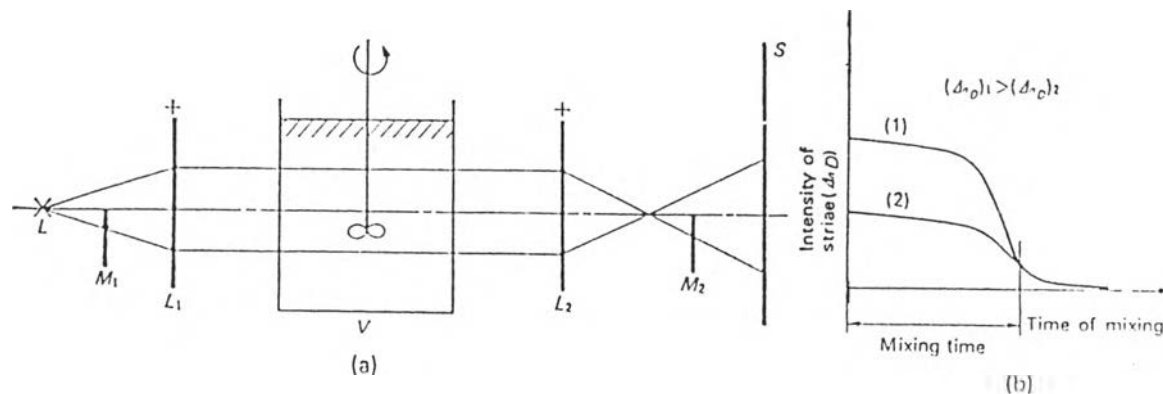


Figure 2.8 Experimental assembly by Schlieren method.

(a) Block diagram of instrument.

L = Light source, L_1 = First "Schlieren" lens,

L_2 = Second "Schlieren" lens, V = Mixing vessel
with stirrer,

S = Screen in plane of image of V ,

M_1 = First knife-edge intercepting half of the beam,

M_2 = Second knife-edge (in plane of image of M_1),
intercepting the other half of the beam

(b) Measurement of mixing time

$$(n_D)_1 > (n_D)_2$$

n_D = Refractive index difference.

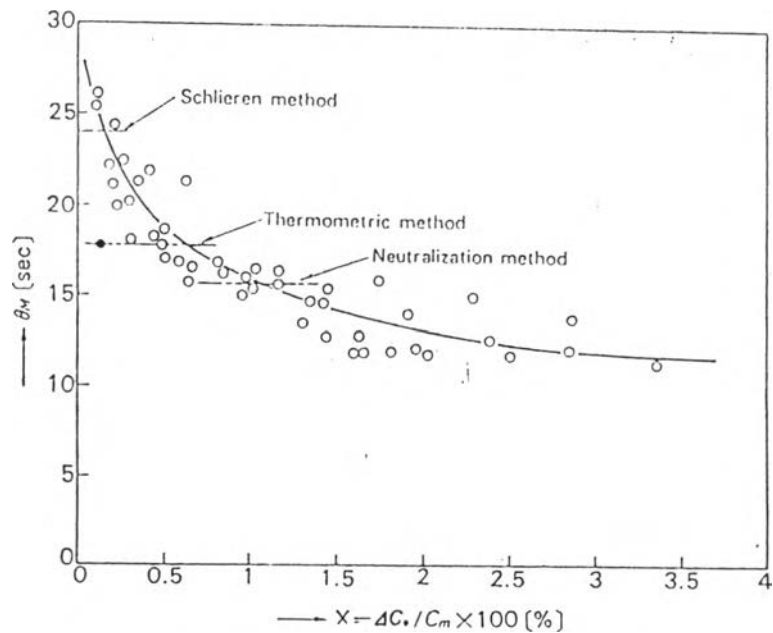


Figure 2.9 Variation of θ_M with change of criteria for sufficient mixing.

Conditions of agitation; standard type agitator

$D = H = 30$ cm, 8 baffles;

$B_w/D = 0.075$, 8-flat-blade-paddle; $d/D = 1/2$, $b/D = 1/10$,

$C/H = 1/2$, $N = 60$ r.p.m., $Re = 2.2 \times 10^4$ Agitated liquid;

tap water (17 l)

O : Conductometric method, injected liquid: 0.6 cc of
2N KCl aq. soln./1 of tap water

* Schlieren method; 6 cc of 56 % saccharose soln./1 of
tap water

* Thermometric method; 15 cc of hot water (95 °C)/1 of
cold water (30 °C): sensitivity; $1/30$ °C

($x = 0.1$ %; 0)

* Neutralization method; 2 cc of 3N KOH aq. soln./1 of
 6×10^{-3} N. HCl aq. soln., indicator; phenolphthalein

In the decolorizing method, a reactant in the mother liquid and a reactant in a tracer undergo an instantaneous chemical reaction at their interface, so the observation of mixing process is replaced by the shift of the interface, and is different somewhat from other techniques.

The decolorizing method is conveniently used for detecting dead space, but is influenced by the ratio in quantity of two reactants. In the case of neutralization reaction where alkaline solution with phenolphthalein is neutralized by acid, a tracer containing 20-30 % excess acid compared with that equivalent to the base is adequate to give a steady mixing time.

The measuring method stated above is used for macro mixing. A direct method of measuring the micro mixing state is difficult to find. Merely by an improvement of electric probes, there is a limit of resolving power in 10-100 (scale), especially in high viscosity liquids where the resolving scale becomes even coarser because a stagnant layer may arise around the probes by adherence. Thus, for the measurement of micro mixing state, an optical method is recommended. As another approach, it is well-known that the rate of second order chemical reactions is influenced by the state of mixing. Using an inverse relation, we may be able to evaluate the degree of micro mixing from the rate of chemical reaction.

2.3 Mixing Time Correlations ⁽³⁾

For the mixing in an agitated vessel, the generalized equation for convection mixing may be applied.

$$\frac{\partial C^*}{\partial t^*} = - (U^* \nabla^* C^*) + \frac{1}{P_e} (\Delta^* C^*) \quad (\text{in laminar flow}) \quad (2.13)$$

$$\frac{\partial C^*}{\partial t^*} = - (U^* \nabla^* C^*) + \frac{1}{P_e} (\Delta^* C^*) \quad (\text{in turbulent flow}) \quad (2.15)$$

Therefore, the dimensionless mixing time $N_M = \theta_M \cdot n$ is influenced by the geometry of the mixing vessel, by the injection methods for the ingredients, and by Re or Pe which controls the convection flow or diffusion.

By direct measurement, only the macro mixing can be observed. Therefore let us neglect the activity of molecular diffusion in the two equations stated above; then we have;

$$\text{For laminar mixing} \quad \frac{\partial C^*}{\partial t^*} = - (U^* \nabla^* C^*) \quad (2.13')$$

$$\text{For turbulent mixing} \quad \frac{\partial C^*}{\partial t^*} = - (U^* \nabla^* C^*) + \frac{1}{P_e} (\Delta^* C^*) \quad (2.15')$$

In geometrically similar vessels, the state of liquid flow is classified roughly by Reynolds number, $Re = d^3 n / \nu$ as follows;

1) a low Reynolds range where the impeller speed is not large and the secondary circulation flow due to the centrifugal effect is negligible.

2) a medium Reynolds range where the secondary circulation flow is appreciable (laminar range).

3) a high Reynolds range (a complete turbulent state).

In each individual range, the liquid flow pattern is similar. In a completely turbulent flow range, the turbulent Peclet number (Pe_t) gives a certain constant value, so that the dimensionless mixing time $N_m = \theta_m \cdot n$ shows a constant value and θ_m is inversely proportional to n .

2.4 Radioactive Tracer Applications ⁽¹⁶⁾

Tracer methods using dyes and chemical salts have been in use for some considerable time. With the introduction of radioactive materials as tracers, there was an initial tendency for more emphasis to be placed on chemical labelling in biochemistry and biological/medical research and this tended to overshadow the development of physical tracers. Although chemical labelling, and carbon-14 studies in particular, continues to dominate the field, there has been considerable development in industrial tracer techniques over the last 20 years. Most industrial process investigation work involves physical or phase tracing application.

The basic requirements of a tracer are as follows

1. It should behave in the same way as the material under investigation (this is not usually a problem unless there is a change of phase in the system).
2. It should be easily detectable at low concentrations; detection should be unambiguous;
3. Injection, detection and/or sampling should be performed without disturbing the system.
4. The residual tracer concentration in the product from the system should be minimal.

All these criteria can be met by the use of radioisotope tracers and by careful selection of the most appropriate tracer for a particular application. Frequently more than one radioisotope can be chosen, and the factors which are important in the selection of the tracer are half-life, specific activity, type of radiation, energy of radiation and physical and chemical forms. In most instances the freedom of choice is restricted and specification of two or three of these factors will reduce the choice of the others. The relevant properties of some radioactive tracers are shown in Table 2.1.

- Half-life

The choice of half-life is, like many of the others, a compromise. The half-life must be long enough to allow time to transfer the tracer from the nuclear reactor, to the work site, prepare the tracer for use and complete the measurement. In order to reduce the level of residual tracer in the exit streams, however, a short half-life tracer is desirable.

- Specific activity

For any tracer investigation it is clearly imperative that the total amount of activity which is added to the system is such that, allowing for dilution and possible splitting of the process stream, sufficient remains for detection or measurement downstream. The amount of activity added and the specific activity will also be dependent upon whether the activity is to be detected in situ or analysed later in the laboratory. In the case of analysis, the samples or the sample stream should contain sufficient activity for the analyses to be performed with an acceptable error.

- Type of radiation

The advantages of using gamma-emitters for industrial process investigation work are well known. There are two specific properties of gamma-radiation which are significant. The first of these is the ability of electromagnetic radiation to penetrate dense materials such as steel pipe or vessel walls.

The second important property of gamma-radiation is that the radiation from a specific radioisotope is characterized by a unique energy spectrum, thus making identification and measurement of the radioisotope relatively simple. Consequently, dual-tracer experiments have greater scope and are much more effective when gamma-emitters are used.

- Energy of radiation

The choice of the energy of the radiation of the tracer will depend upon the measurement system which can be used and also upon shielding considerations. For gamma-emitter the actual energy

is not critical, but clearly for an in-situ measurement the energy must be high enough to penetrate the vessel walls without having to use large quantities of material.

Although a high-energy gamma-emitting radioisotope may be desirable for the reasons given above, it is likely to be less so when considering shielding requirements for transportation and dispensing.

- Physical and chemical behaviour

Most industrial radioisotope tracer applications are based on phase tracing rather than chemical labelling; consequently, except in a few special circumstances, the physical form of the tracer is of greater importance when selecting tracer material. In assessing the behaviour of the tracer in a system, however, and particularly with regard to its ultimate fate, the chemical form should also be selected with care.

The final choice of a radioisotope tracer for a particular investigation will be made after consideration of all of the factors discussed, many of which may be mutually exclusive. Once the decision has been made, the next stage in the procedure is the planning and execution of the measurement.

Table 2.1 Some radioiactive tracer in aqueous solutions ⁽¹⁷⁾

Isotope	half-life	radiation of interest(MeV)	chemical form
²⁴ Na	15 h	1.37, 2.75	Na ₂ CO ₃ , NaHCO ₃
⁵⁶ Mn	2.6 h	0.85, 1.8, 2.1	MnSO ₄
⁸² Br	36 h	0.55, 1.32	NH ₄ Br
¹⁴⁰ La	40 h	0.33-2.54	LaCH ₃ COO, La-EDTA
¹⁹⁸ Au	2.7 d	0.41	HAuCl ₄
¹³¹ I	8 d	0.36, 0.64	NaI, KI
³⁸ Cl	37 min	1.6, 2.15	HCl
⁴⁸ Sc	84 d	0.89, 1.48	ScCl, Sc-EDTA
⁵¹ Cr	27.8 d	0.325	Cr-EDTA
^{99m} Tc	6 h	0.14	NaTcO ₄

2.5 Radiation Measurement-Statistical Considerations ⁽¹⁸⁾

Radioisotopes decay in a random manner and the laws of probability apply to the behaviour of any given radioactive nucleus. Consequently, radiation measurement is always subject to some degree of statistical fluctuation. These inherent fluctuations represent an

unavoidable source of uncertainty in all measurements and are often the predominant sources of error.

2.5.1 Counting statistics

Suppose we collect a set of counts from a radiation counter, each count reading being of the same duration and having been taken under identical conditions. The count readings are:

$$x_1, x_2, x_3, \dots, x_i, \dots, x_n.$$

Two elementary properties of this set of data are

(i) "sum": $\sum x_i$

(ii) "experimental mean": $\bar{x} = \sum x_i / N$

It is usual to characterise the width of the distribution by defining its standard deviation :

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N-1}} \quad (2.17)$$

Various mathematical models exist which can predict the distribution function that will describe the results of a given radiation measurement. In descending order of complexity these models are :

a: Binomial distribution - most general

b: Poisson distribution - simplification of binomial model valid if observation time is small compare with half-life of the source

c: Gaussian or normal distribution - simplification of Poisson model, valid provided that the total number of measurements (N) is large or x is large, widely applicable to many problem in counting statistics.

These models will not be further discussed here, but it should be noted that

(i) Both the Poisson and the Gaussian/Normal distributions adequately describe the vast majority of situations encountered in the industrial application of radioisotopes.

(ii) Both the Poisson and the Gaussian/Normal models make the fundamentally important prediction that the standard deviation of the distribution is the square root of the mean value which characterizes that same distribution:

$$\sigma = \sqrt{x} \quad (2.18)$$

2.6 Product Data Calculations ^(A)

2.6.1 Simplified Analysis Batch Operation.

Batch mixers are normally evaluated on the basis of the mixing time required to obtain a specified mixture.

Properties of interest are examined as a function of the mixing time, and the required mixing time is the shortest time at which there is satisfactory assurance that the values of the properties are within selected limits. Decisions may be based on visual examination of a plot of time of sampling (abscissa and independent variable) versus measured values of the property (ordinate and dependent variable) for many operations. Evaluations may also be made at different operating conditions (for example, mixer speeds) and the effect of important operating variables determined by examining the required mixing time as a function of the operating variables.

AICHe Equipment Testing Procedure ⁽⁸⁾ recommended the selection of the mixing time of the curing-rate additive as the time required to achieve concentration of mixture within acceptable limit as shown in Figure 2.10 or to reach composition equilibrium by statistical analysing as shown in Figure 2.11

Kramers et al. ⁽⁵⁾ selected required mixing time as the time for concentration variations to become less than 0.1% of the average KCl concentration as shown in Figure 2.13.

Noi et al. ⁽⁷⁾ selected required mixing time as the time to achieve variation of concentration of radioactive material (count rate) at both detectors less than 1 time of standard deviation as shown in Figure 2.13 (a) and Figure 2.13 (b).

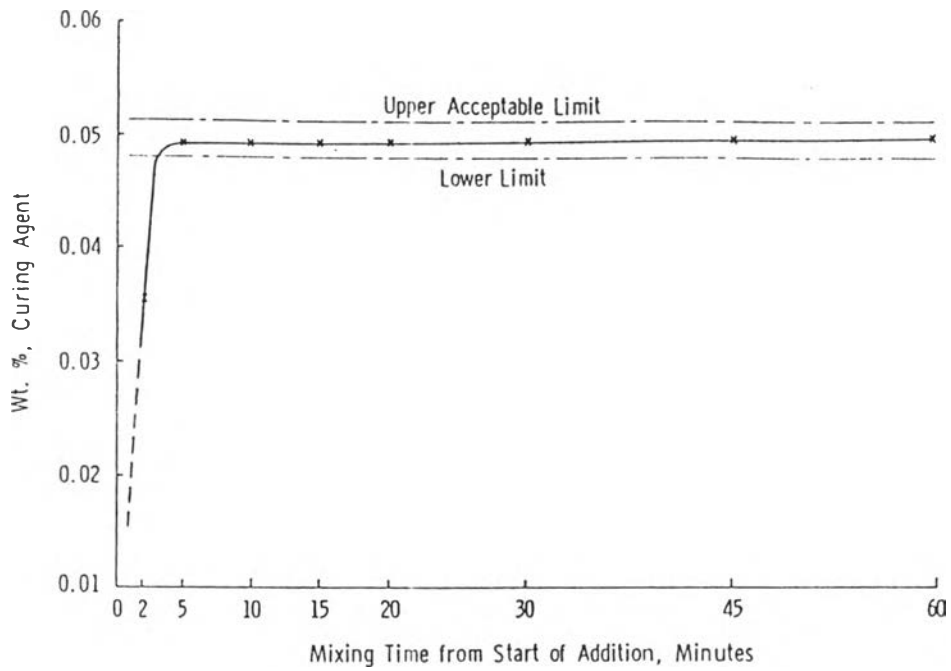


Figure 2.10 Composition vs. mixing time in 150 gal. dough mixer

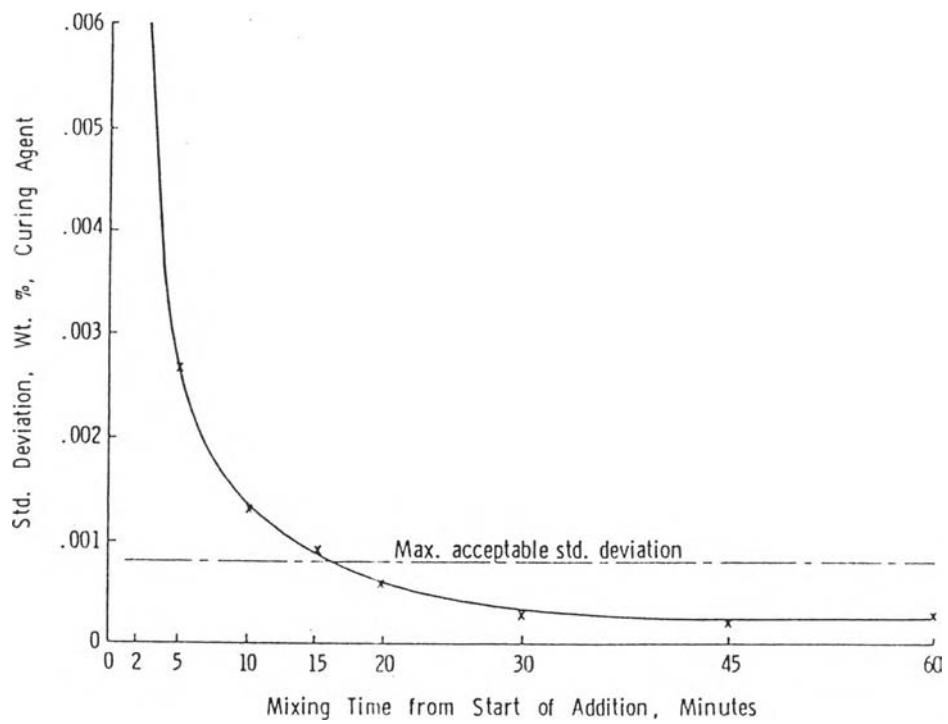


Figure 2.11 Standard variation vs. mixing time in 150 gal. dough mixer

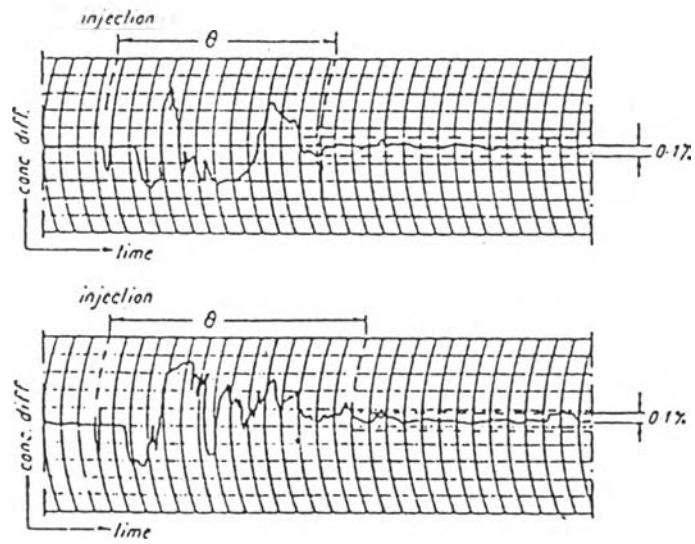


Figure 2.12 Method of determining mixing time[from Kramers et al.]

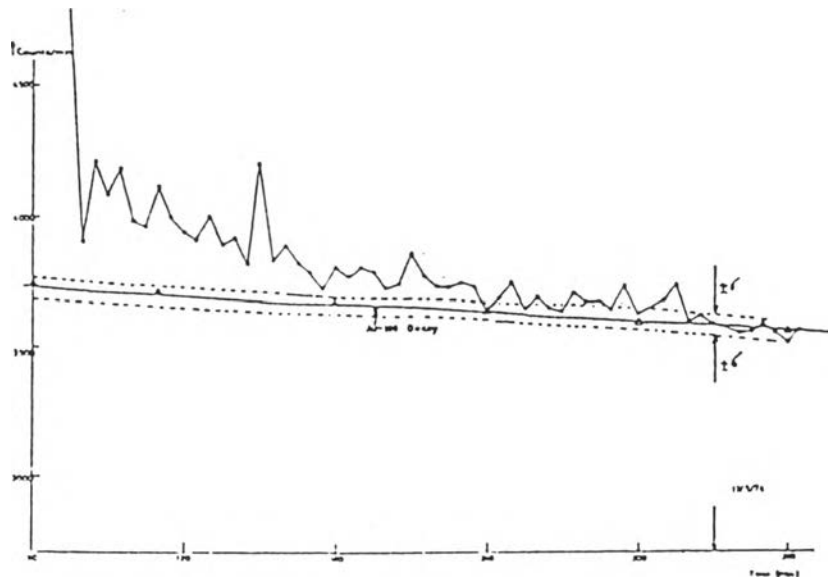


Figure 2.13 (a) Scalar counts from detector No. 1

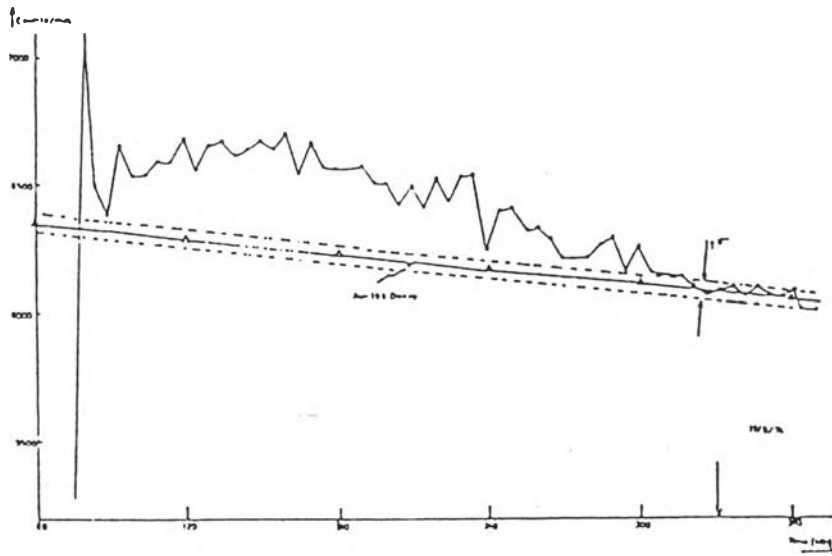


Figure 2.13 (b) Scalar counts from detector No.2