CHAPTER III

PATH INTEGRAL APPROACH TO THE DENSITY OF STATES OF A TWO-DIMENSIONAL ELECTRON GAS

Introduction

In this chapter we present a straightforward evaluation of the density of states of a two-dimensional electron system in the presence of a transverse magnetic field and a disorder potential by using the Feynman path integral method. In our model we consider an electron in a system of very dense, random and weak scatterers. In this manner, the scattering potential can be represented by a gaussian random potential with finite correlation length L and then we can treat the correlation L as one adjustable parameter. The comparison of our numerical results with the direct measurement of Kukushkin and Timofeev will be seen in the the last section, the conclusion and discussion will be contained in chapter IV.

The Density of States

It is convenient to consider the density of states in the form defined by

$$n(E) = (1/V) < \sum_{i=1}^{\infty} \delta(E - E_i) >, \qquad (3.1)$$

where E_i is the energy of the ith eigenstates of a Hamiltonian, V is a container of N scatterers in d dimensions, and where the angular bracket <.....> indicates an average over an ensemble of the scatterer positions. Similar to that of (2.81), one can verify that (3.1) can be rewritten in the form

$$n(E) = (V/2\pi\hbar) \int_{-\infty}^{\infty} \bar{K}(0,0;T)\, e^{(i/\hbar)ET}\, dT, \qquad (3.2)$$

where $\bar{K}(0,0;T)$ is the average propagator of an electron.

Path Integrals of a Two-Dimensional Random Model

To evaluate the propagator $\bar{K}$ mentioned in the preceding section, we first consider a simple model which can be conveniently expressed in terms of Feynman-like path integrals. The model is that of an electron in a system of very dense, random, weak scatterers. If ρ is the density of the scatterers, and $v(\vec{r})$ is the scattering potential, then the model is obtained by taking the limit $\rho \to \infty$, $v(\vec{r}) \to 0$ and $\rho v^2(\vec{r}) \to$ finite. To begin the calculation, we first consider a propagator $K(\vec{r}'', \vec{r}'; T, \{\vec{R}i\})$ for an electron in the presence of N scatterers at the fixed positions $\{\vec{R}_i \mid i=1, 2, ...\}$. It is obvious that this propagator must depend explicitly on the positions of the scatterers. Since the scatterers are randomly distributed, such properties of the system as the density of states are obtained from the average propagator $\bar{K}(\vec{r}'', \vec{r}'; T)$, the average of K over the random scatterer positions.

For the model of two-dimensional N scatterers, in the presence of a transverse magnetic field, the Hamiltonian of the electron is

$$H(\{\vec{R}_i\}) = H_o + \sum_{i=1}^{N} v(\vec{r} - \vec{R}_i), \qquad (3.3)$$

where $$H_o = (1/2m^*)(\vec{p} + (e/c)\vec{A})^2, \qquad (3.4)$$

$\vec{A}$ is the vector potential with $\vec{\nabla}x\vec{A} = \vec{B}$, m* is the effective mass of the electron and $v(\vec{r}-\vec{R}_i)$ is the scattering potential at position $\vec{r}$. The position $\vec{R}_i$ of the scatterers are taken to be random. The probability for the scattering centers to be $\vec{R}_i$ is, therefore

$$P(\{\vec{R}_i\}) \quad = \quad 1/S^N, \qquad (3.5)$$

where S is the area of the system. The propagator of such a system satisfies

$$[\, i\hbar\frac{\partial}{\partial T} - H(\{\vec{R}_i\})\,]\, K(\vec{r}'',\vec{r}';T,\{\vec{R}_i\}) \quad = \quad i\hbar\delta(\vec{r}''-\vec{r}')\delta(T). \qquad (3.6)$$

In path integral representation, the propagator is expressed as

$$K(\vec{r}'',\vec{r}';T,\{\vec{R}_i\}) \quad = \int D[\vec{r}(t)]\exp[(i/\hbar)\int_0^T dt\{(m/2)(\dot{x}^2+\dot{y}^2+\Omega(x\dot{y}+y\dot{x})) - \sum_i v(\vec{r}(t)-\vec{R}_i)\}]. \qquad (3.7)$$

The average propagator is thus

$$\bar{K}(\vec{r}'',\vec{r}';T) \quad = \quad <K(\vec{r}'',\vec{r}';t;\{\vec{R}_i\})> \quad = \quad (1/S)^N \int\int ...\int \prod_{i=1}^{N} d\vec{R}_i\, K(\vec{r}'',\vec{r}'T;\{\vec{R}_i\}). \qquad (3.8)$$

Since (3.8) implies that the average propagator $\bar{K}$ is the usual Green' s function, $\bar{K}$ can be thought to describe the propagation of a particle, even though it does not correspond to a physical electron in a specific configuration. The function $\bar{K}$ can also be considered as a propagator describing the motion of a ficitious average electron in the average system.

As pointed out first by Edwards and Gulyaev (32), the average of (3.8) can be explicitly obtained because the set of $\{\vec{R}_i\}$ is random. Therefore, since the scattering potentials $v(\vec{r}(t) - \vec{R}_i)$ are independent random variables, the average propagator becomes

$$\bar{K}(\vec{r}'',\vec{r}';T) = \int D[\vec{r}(t)] \exp\left[(im/2\hbar) \int_0^T (\dot{x}^2 + \dot{y}^2 + \Omega(x\dot{y}+y\dot{x}))dt \right.$$
$$\left. + \rho \int d\vec{R}\{\exp[(-i/\hbar) \int_0^T dt\, v(\vec{r}(t)-\vec{R})] - 1 \}\right], \qquad (3.9)$$

where the density $\rho = N/S$. In the limit of $\rho \to \infty$, $v \to 0$ and $\rho v^2 \to$ finite, we can expand the exponential of v and then keep only the linear and quadratic terms. The average potential

$$E_o = \rho \int d\vec{R} v(\vec{r}(t) - \vec{R}), \qquad (3.10)$$

becomes infinite in the limit mentioned above. However, we are free to choose our energy origin as the average energy for removing the infinity. Now we consider the quadratic term

$$(-\rho/2\hbar^2) \int d\vec{R} \int_0^T \int_0^T dt d\sigma v(\vec{r}(t)-\vec{R}) v(\vec{r}(\sigma)-\vec{R})$$
$$= (-\rho/2\hbar^2) \int_0^T \int_0^T dt d\sigma W(\vec{r}(t) - \vec{r}(\sigma)), \qquad (3.11)$$

where the correlation function

$$W(\vec{r}(t) - \vec{r}(\sigma)) = \int d\vec{R} v(\vec{r}(t)-\vec{R}) v(\vec{r}(\sigma)-\vec{R}). \qquad (3.12)$$

From (3.9), (3.12) and after taking $E_o = 0$, we obtain

$$K(\vec{r}',\vec{r};\,T) = \int D[\vec{r}(t)]\,\exp[(im/2\hbar)\int_0^T(\dot{x}^2+\dot{y}^2+\Omega(x\dot{y}-y\dot{x}))dt - (\rho/2\hbar^2)\int_0^T\int_0^T dtd\sigma W(\vec{r}(t)-\vec{r}(\sigma)]. \qquad (3.13)$$

We note that the correlation function (3.12) depends explicitly on the scattering potential employed and (3.13) can be formally written in terms of an action S as

$$\bar{K}(\vec{r}'',\vec{r}';T) = \int D[\vec{r}(t)]\,e^{(i/\hbar)S[\vec{r}(t)]}, \qquad (3.14)$$

$$\text{where} \quad S[\vec{r}(t)] = (m/2)\int_0^T(\dot{x}^2+\dot{y}^2+\Omega(x\dot{y}-y\dot{x}))dt+(i\rho/2\hbar)\int_0^T\int_0^T dtd\sigma W(\vec{r}(t)-\vec{r}(\sigma)). \qquad (3.15)$$

First Cumulant Approximation

To compute the density of states (3.2), the propagator (3.14), must be known. However, the propagator (3.14) containing the correlation function $W(\vec{r}(t)-\vec{r}(\sigma))$ is very complicated, and is presently impossible to work out directly. Therefore we can evaluate the propagator only by approximating. To approximate such a propagator we begin with a zero order model, a free electron in the xy-plane under the influence of a perpendicular magnetic field B. The corresponding zero order action is

$$S_o = (m/2)\int_0^T(\dot{x}^2+\dot{y}^2+\Omega(x\dot{y}-y\dot{x}))dt. \qquad (3.16)$$

The zero order propagator is

$$K_o(\vec{r}'',\vec{r}';T) = \int D[\vec{r}(t)] \exp [(i/\hbar)S_o]. \qquad (3.17)$$

Then we express $\bar{K}$ in terms of K_o as

$$\bar{K}(\vec{r}'',\vec{r}';T) = K_o(\vec{r}'',\vec{r}';T) < \exp [(i/\hbar)\{S-S_o\}] >_o , \qquad (3.18)$$

and the average part is regularly defined by

$$< O >_o = \frac{\int \underline{D [\vec{r}(t)]O\exp[} (i/\hbar)S_o]}{\int D[\vec{r}(t)]\exp[(i/\hbar)S_o]} . \qquad (3.19)$$

The propagator K_o can be evaluated exactly (see Chapter I), and the average part can be evaluated approximately by the cumulant expansion, as was pointed out by Kubo (33),

$$< e^x > = \exp [<x> - (1/2!)\{ <x^2> - <x>^2 \}+.....]. \qquad (3.20)$$

Since we are only interested in the difference $(S-S_o)$, the kinetic and magnetic terms in S and S_o drop out and we only have to consider the correlation term,

$$(S-S_o) = (i\rho/2\hbar) \int_0^T \int_0^T W(\vec{r}(t)-\vec{r}(\sigma))dtd\sigma. \qquad (3.21)$$

Now we approximate (3.18) by the first cumulant, and (3.18) reduces to

$$\bar{K}_1(\vec{r}'',\vec{r}';T) = K_o(\vec{r}'',\vec{r}';T) \exp [(-\rho/2\hbar^2) \int_0^T \int_0^T <W(\vec{r}(t)-\vec{r}(\sigma)) >_o dtd\sigma]. \qquad (3.22)$$

The Gaussian Random Model

In order to find (3.22), we assume the scattering potential due to impurities to be a gaussian form,

$$v(\vec{r}-\vec{R}) = (u/\pi l^2)\exp[-|\vec{r}-\vec{R}|^2/l^2], \qquad (3.23)$$

where u is the strength of the scattering potential . We find that for the potential (3.23), the correlation function becomes

$$W(\vec{r}(t)-\vec{r}(\sigma)) = (u^2/\pi L^2)\exp[-|\vec{r}(t)-\vec{r}(\sigma)|^2/L^2], \qquad (3.24)$$

where L denotes the correlation length of the random system related to l by

$$L^2 = 2l^2 . \qquad (3.25)$$

In order to calculate W, we write

$$W(\vec{x}) = (1/2\pi)^2 \int d^2k V(\vec{k})\exp[i\vec{k}.\vec{x}]. \qquad (3.26)$$

where $V(\vec{k})$ is the Fourier transform of $W(\vec{x})$. For a gaussian potential, $V(\vec{k})$ is given by

$$V(\vec{k}) = u^2 \exp[-\vec{k}^2L^2/4]. \qquad (3.27)$$

If ones inserts (3.27) into (3.26), one has

$$W(\vec{r}(t)-\vec{r}(\sigma)) = (1/2\pi)^2u^2\int d^2k \exp[\, -k^2L^2/4+i\vec{k}.(\vec{r}(t)-\vec{r}(\sigma))\,]\,. \quad (3.28)$$

After substituting (3.28) into (3.22), we obtain

$$\bar{K}_1(\vec{r}'',\vec{r}';T) = K_o(\vec{r}'',\vec{r}';T)\exp\Big[-\frac{\rho u^2}{2\hbar^2(2\pi)^2}\int_0^T\int_0^T d^2k\,dt\,d\sigma\exp[-k^2L^2/4] \cdot \langle e^{i\vec{k}.(\vec{r}(t)-\vec{r}(\sigma))}\rangle_o\Big]. \quad (3.29)$$

The evaluation of $\langle e^{i\vec{k}.(\vec{r}(t)-\vec{r}(\sigma))}\rangle_o$ can be performed exactly (see Appendix A), and after the integration of k is completed, one finds

$$\bar{K}_1(0,0;T) = K_o(0,0;T)\exp\Big[-\frac{\rho u^2}{2\hbar^2(2\pi)^2}\pi\int_0^T\int_0^T dt\,d\sigma[B(T,t,\sigma)]^{-1}\Big], \quad (3.30)$$

$$\text{where } B(T,t,\sigma) = \frac{L^2}{4}+\frac{i\hbar\sin[(\Omega/2)(T-(t-\sigma))]\sin[(\Omega/2)(t-\sigma)]}{(m\Omega\sin[\Omega T/2])}. \quad (3.31)$$

We note that the expression $B(T,t,\sigma)$ has the property

$$B(T,t,\sigma) = B(T,T-(t-\sigma)), \quad (3.32)$$

so that (3.31) reduces to

$$\bar{K}_1(0,0;T) = K_o(0,0;T)\exp\Big[-\Big(\frac{\rho u^2}{2\hbar^2(2\pi)^2}\Big)\pi T\int_0^T dx[B(T,x)]^{-1}\Big]. \quad (3.33)$$

$$\text{where } B(T,x) = \frac{L^2}{4}+\frac{i\hbar\sin[(\Omega/2)(T-x)]\sin[(\Omega/2)x]}{(m\Omega\sin[(\Omega/2)T])}. \quad (3.34)$$

The Density of States of a Two-Dimensional Random System

Before applying (3.2) to (3.34) to obtain the density of states, we follow Halperin and Lax (34) by defining $E_L = \hbar^2/(2mL^2)$ as the energy associated with localizing an electron within the correlation length, $\xi_L = \rho u^2/(\pi L^2)$ as the magnitude of the variance and $x = \hbar\Omega/E_L$. Then from (3.2) and (3.34) we obtain for both spins of an electron

$$n(E) = (S/\pi\hbar)\int_{-\infty}^{\infty} dT\, K_O(0,0;T)\exp[\, iET/\hbar - (\frac{\xi_L}{2\hbar^2})T\int_0^T dy\, [G(T,y)]^{-1}\,], \quad (3.35)$$

$$\text{where } G(T,y) = 1 + \frac{8i\sin[\Omega(T-y)/2]\sin[\Omega y/2]}{x\sin[\Omega T/2]}, \quad (3.36)$$

$$\text{and } K_O(0,0;T) = (\frac{m}{2\pi i\hbar T})(\frac{\Omega T}{2\sin(\Omega T/2)}).$$

The integration of x in the exponential term of (3.35) can be evaluated exactly,then (3.35)becomes

$$n(E) = (S/\pi\hbar)\int_{-\infty}^{\infty} dT K_O(0,0;T)\exp[\, iET/\hbar - (\frac{\xi_L xT)\sin(\Omega T/2)}{2\hbar i\Omega\sqrt{a(x,T)^2-1}})\tan^{-1}\{\frac{\sqrt{a(x,T)-1}}{\sqrt{a(x,T)+1}}\tan(\Omega T/4)\}\,], \quad (3.37)$$

$$\text{where } a(x,T) = (x/4i)\sin(\Omega T/2) - \cos(\Omega T/2). \quad (3.38)$$

The density of states in (3.37) can not be evaluated analytically due to the complicated part in the exponential terms. A numerical method must be used in such a complicated integration. However, we now wish to find the density of states for the ground state energy. According to the uncertainty principle the limit of ground state energy implies the limit of large time. In this limit we make the following approximation

$$\sin(\Omega x/2) = \frac{e^{i\Omega x/2} - e^{-i\Omega x/2}}{2i} = (1/2i)e^{i\Omega x/2}(1-e^{-i\Omega x}) \rightarrow (1/2i)e^{i\Omega x/2}, \quad (3.39)$$

$$\cot(\Omega x/2) = i\frac{e^{i\Omega x/2} + e^{-i\Omega x/2}}{e^{i\Omega x/2} - e^{-i\Omega x/2}} = i\,\frac{(1+e^{-i\Omega x})}{(1-e^{-i\Omega x})} \rightarrow i, \quad (3.40)$$

$$\frac{\sin(\Omega x/2)\sin[(\Omega/2)(T-x)]}{\sin(\Omega T/2)} = (1/2i)\frac{(e^{i\Omega x/2} - e^{-i\Omega x/2})(e^{i\Omega(T-x)/2} - e^{-i\Omega(T-x)/2})}{e^{i\Omega T/2} - e^{-i\Omega T/2}}$$

$$= (1/2i)\frac{(1-e^{-i\Omega x})(1-e^{-i\Omega(T-x)})}{(1-e^{-i\Omega T})} \rightarrow (1/2i), \quad (3.41)$$

From (3.37), (3.38), and the large T limit, (3.37) becomes

$$n(E) = n_o\hbar\Omega(2\pi\Gamma^2)^{-1/2}\sum_{n=0}^{\infty} \exp\left[-(1/2)\frac{(E - E_n)^2}{\Gamma^2}\right], \quad (3.42)$$

where $n_o = m/\pi\hbar^2$, (3.43)

with $E_n = (n + 1/2)\hbar\Omega$, (3.44)

and the width parameter is $\Gamma^2(B, L) = \xi_L\left[\frac{x}{(4+x)}\right]$. (3.45)

The n(E) in (3.42) is clearly a sum of Gaussians centered at the Landau energies E_n. One can verify that (3.42) can be reduced to (3.43) when the magnetic field B goes to zero(see Appedix B). The width of the Gaussian Γ is a function of B, through $x = (2e/\hbar^2c)BL^2$ and of L through $\xi_L = \rho u^2/\pi L^2$. Clearly as $\Gamma \rightarrow 0$, n(E) reduces to a sum of delta functions, $n(E)=n_o\hbar\Omega\sum_n \delta(E-E_n)$. Typical observed values of Γ (6-13) are $\Gamma \simeq 1$ $(meVT^{-1/2})B^{1/2}$ or $\Gamma \simeq 2meV$ at B=5T (See Fig. 11).

Limits of Γ are interesting. For low B or short L so that $x \leq 4$, $\Gamma^2 \rightarrow \xi_L x/4$ and Γ is approximately proportional to $\sqrt{B}$, as observed. Since $\xi_L x$ is independent of L, the

magnitude of Γ is determined chiefly by magnitude of the potential fluctuation at moderate B. For large B and long L where x >> 4, $\Gamma \to \xi_L$ and Γ becomes independent of B. The present n(E) in (3.42) can display a substantial density of states between Landau levels depending upon the value of L. The value of L depends upon the origins of the disorder. If it is due to screened, charged impurities L will be approximately equal to the screening length . This is of the order of 100 Å in heterojunctions (2). The role of L here is to set the energy scale via $E_L = \hbar^2/2mL^2$. For L = 100 Å, $E_L \simeq 0.4$ eV. Let us assume, just to set scales, that E_L = 1 meV. It is convenient to represent n(E) in dimensionless units with all energies scaled by E_L, i.e. we define

$$\xi'_L = (\xi_L)/(E_L^2),\ \nu = E/E_L,\ \Gamma'^2 = \xi'_L/(1+4/x) = \Gamma^2/E_L^2$$

so that

$$n(\nu) \quad = \quad n_0 x(2\pi\Gamma'^2)^{-1/2} \sum_{n=0}^{\infty} \exp\left[-(1/2)\{\underline{\nu - (n+1/2)x}\}^2 \right]. \qquad (3.46)$$
$$\Gamma'^2$$

With E_L = 1meV and for 1≤ B ≤ 10 T (corresponding to integer filling factors 8 < ν <1), x takes values 1≤x≤15. For $\Gamma \simeq$ 2 meV and E_L= 1 meV, Γ' and ξ'_L are of order 1 to 10.

The dependence of the density of states for adjacent Landau levels on x for ξ'_L = 1 is shown in Fig. 13. The density of states between Landau levels is essentially zero at x = 4 but increases substantially as x decreases to 2 (B≈1.5T). Basically, as the spacing between Landau levels decreases a significant density of states between Landau levels develops. In Fig. 14 the dependence of n(E) on ξ'_L for x = 5. As ξ'_L increases from 1 to 5 a substantial density of states between the Landau level develops.

We may make an approximate comparison with experiment by noting that the density of states in Fig. 13 for ξ'_L = 4 and x = 5 is very similar to that extracted from de Haas-van Alphen measurements by Eisenstein et al. (shown in Fig. (12)). At B = 5T

($\hbar\Omega$ = 8.7 meV) they obtain Γ = 2.2 meV. Using ξ'_L = 4 and the relation $\Gamma^2 = E_L^2\xi'_L/(1+4/x)$ we obtain a value of E_L≃1.3 meV, which corresponds to L≃50 Å, needed to reproduce experiment. Using a some what larger value of $x = \hbar\Omega/E_L$ in Fig. 14 (e.g. x = 8) would require a larger ξ'_L and a larger L to make the correspondence. Thus, we can reproduce Landau level broadening and the observed density of states between Landau levels from the simple model for physically reasonable values of L.

Recent experiments (35) have observed oscillations in the Landau level widths as the function of B. When the Fermi level, E_F, lies between two Landau levels, impurity charges are poorly screened and the potential fluctuations are large (14). When E_F lies on a Landau level, impurities are well screened and W(L) is small. This effect can be treated in the present model by representing v(r) by a screened coulomb potential (2) and determining, E_F, the screening v(r) consistently as Sritrakool et al.(23) have done for optical absorption in three dimensions.

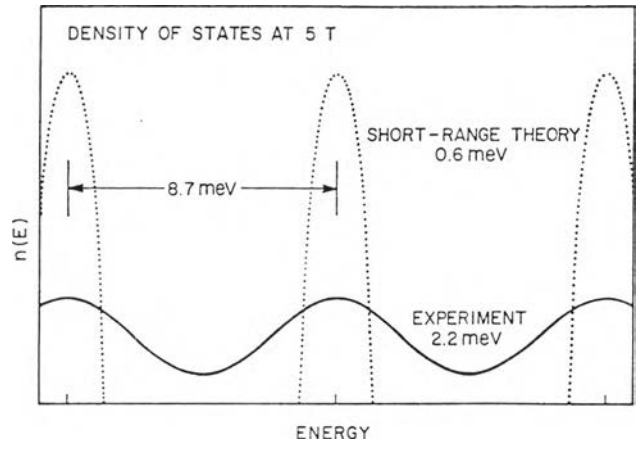

Fig.12 Comparison of the model density of states (solid line) which fits 2DES data for B = 5T and the SCBA short-range interaction theory (dashed line). At B = 5T, $\hbar\Omega$ = 8.7 meV. The rms half-widths of the Landau levels are shown. (Figure from Ref. 13)

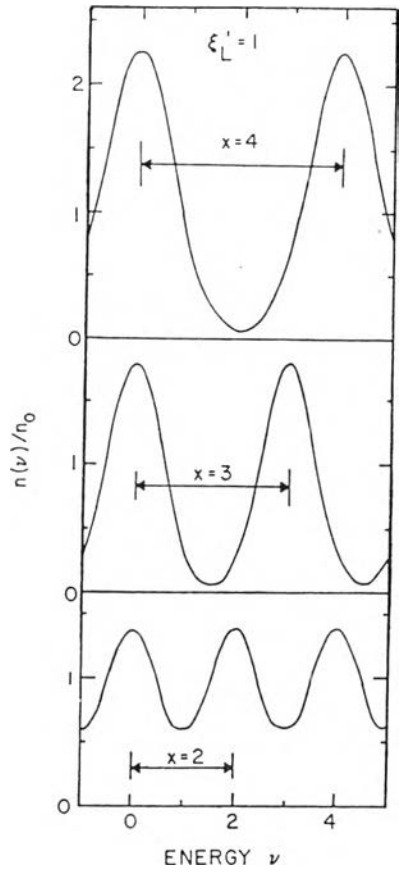

Fig.13 Density of states from Eq.(3.46) for $\xi'_L = 1$ and $2 \leq x \leq 4$.

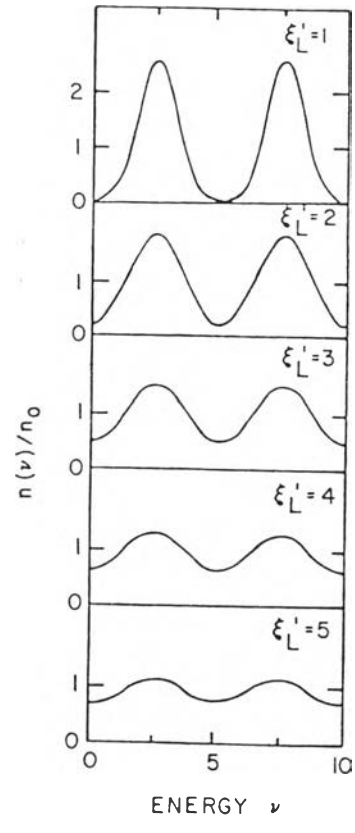

Fig.14 Density of states from Eq.(3.46) for x = 5 and $1 \leq \xi'_L \leq 5$.

Numerical Results and Comparisons with Experiments

As we have seen in the preceding section, for large-T approximation, it is found that the density of states of a 2D electron placed in a transverse magnetic field constitutes a sum of gaussians centered at the Landau energies E_n with a constant width parameter as

$$\Gamma^2 = \xi_L \frac{x}{x+4} = \xi_L \frac{1}{1+2\ell^2/L^2}, \qquad (3.47)$$

where ℓ and L are the magnetic length and the correlation length respectively. As shown in Fig. 12-14, we are able to reproduce the density of states lying between Landau levels using some appropriate gaussian variance and correlation length L. However, the width and height of the peak of every Landau level is equal which seems to contradict the direct measurement of Kukushkin and Timofeev as shown in Fig.9. In their experimental results, the width and height of the peaks of the Landau levels depend on the quantum number N and the filling factor v. For integer filling of electrons, the width Γ may be in the form (36, 37)

$$\Gamma^2 = \frac{Q^2}{1+(4N+2)\ell^2/d^2}, \qquad (3.48)$$

where Q is the fluctuation amplitude, d is the linear scale of the fluctuation and N represents the Landau index. Certainly, the Q^2 appearing in (3.48) corresponds to ξ_L in (3.47) and the linear scale d in (3.48) is identical to the correlation L in our expression. Clearly, (3.48) becomes (3.47) when N = 0, this means that the n(E) expression in (3.41) strictly describes only the lowest few Landau levels. It is almost certainly a result of using the long time limit that we obtain the value Γ which is independent of the Landau index N. To avoid such an unreasonable approximation , a numerical method can be used in the complicated integration of (3.37), and then the comparison with experiment must be done to justify our n(E) expression.

It was known that there are various methods used to determine the density of states under condition of complete filling of the Landau levels (when the quatum Hall effect is observed(18)). However, all methods may be classified into two different experiments, the thermally activated magnetoconductances(6-13) and magnetooptics(16). It must be borne in mind that different quantities are studied by these two methods. Whereas optical spectroscopy of 2D electrons makes it possible to

study the entire n(E) dependence and yields $n(E_F)$, in all other experiments one determies only thermodynamic DOS, i. e., the quantity dn/dE_F, which is in general not equal to $n(E_F)$. At the same time it can be expected that in the case when E_F lands at the minimum of $n(E_F)$, the values of $n(E_F)$ and dn/dE_F should be equal (38), so that it is more reasonable to compare our numerical result with the magnetooptic experiment.

This magnetooptic experiment was performed by Kukushkin and Timofeev (16), they studied ordinary metal-oxide-semiconductor transistors fabricated on the (100) surface of p-type silicon with a boron concentration of 8.3×10^{14} cm^{-3}. The structures have an annular geometry (they are Carbino disks), the thickness of the insulator is 1300 Å, and the maximum mobility of the 2D electron is $\mu = 3$ $m^2/(V.s)$ at $n = 2.7 \times 10^{12}$ cm^{-2} and T = 1.6 K. For the study they used an optical cryostat with a solenoid (B up to 8 T). The spectral instrument is a double monochromator with a dispersion of 10 Å/mm in the working region. The emission is observed in the Voigt geometry and is detected under photon-counting conditions with a subsequent built up of the signal. Furthermore, on their experiments, all the spectroscopic and magnetotransport measurements were carried out simultaneously.

We now turn to the quantitative determination of n(E) from the radiative recombination spectra. This method is based on a comparison of the radiation spectrum measured in a magnetic field for complete filling of the N Landau levels, on the one hand, and the emission spectrum measured at the same electron density n and B = 0. The energy distribution of the radiation intensity I(E) is determined by the convolution of the electron distribution function,

$$F_e(E) = n(E)f(E), \quad (f(E) = \theta(E-E_F)) \qquad (3.49)$$

with that of the holes $F_h(E)$, and the width of $F_h(E) \cong 10$ K, which is much less than the characteristic electron energies E_F. We have therefore

$$I(E) = An(E). \qquad (3.50)$$

To determine the constant that relates the intensity with the DOS, we recall that at B = 0 the shape of the 2D line is that of a step function of the energy, a reflection of the constancy of the DOS of 2D electrons at $n(E) = \frac{4m}{2\pi\hbar^2} = n_o$. This circumstance enables us to establish on the intensity scale an absolute value of the DOS, equal to n_o, and then determine quantitatively the function n(E), by comparing the radiation spectrum obtained in a magnetic field with the spectrum measured at the same 2D-electron density but at B = 0. The spectra compared must have equal integrated intensities, since the value of n is the same in both cases.

To calculate the density of states, from (3.35) and (3.36), we write (see Appendix C)

$$n(E) = n_o(2/\pi)\sum_{n=0}^{\infty}\int_0^{\infty} dt\, \mathrm{Re}\, e^{2i(v/x-(n+1/2))t+f'(t)}, \qquad (3.51)$$

where

$$f'(t) = -\frac{t\sin t}{2ix}\xi'_L\int_0^t dy \frac{1}{[(x/4i)\sin t - \cos t + \cos y]}, \qquad (3.52)$$

with $\xi'_L = \xi_L/E_L^2$ and $v = E/E_L$. The integration in (3.52) can be performed analytically. We take $t = \pi N + \theta$ $(-\pi/2 \leq \theta \leq \pi/2)$, so that (3.52) becomes

$$n(E) = n_o \sum_{n=0}^{\infty} K(v - x(n+1/2)), \qquad (3.53)$$

where

$$K(\upsilon) = (2/\pi)\int_0^{\infty} dt \, \mathrm{Re}\, e\,[2i\upsilon t/x + f'(t)], \qquad (3.54)$$

with

$$f'(t) = -(\pi N+\theta)\xi'_L \frac{\sin\theta}{2ix\sqrt{a^2-1}}\left[\pi N + 2\tan^{-1}\left(\frac{\sqrt{a-1}}{\sqrt{a+1}}\tan(\theta/2)\right)\right], \qquad (3.55)$$

and

$$a = (x/4i)\sin\theta - \cos\theta, \quad |a - \sqrt{a^2-1}| < 1. \qquad (3.56)$$

The time integral is then performed by Gaussian quadrature (42). The computer programs will be seen in Appendix D.

Fig, 15a shows the recombination-radiation spectra found for $n=2.7\times10^{12}cm^{-2}$ in the absence (curve 1) of a magnetic field and in the presence (curve 2) of a magnetic field B = 7T. At B=0, the emission spectra reflect the constancy of the DOS and have a width equal to the Fermi energy of the 2D electrons with the density $n=2.7\times10^{12}cm^{-2}$. In a transverse magnetic field B = 7T and $n = 2.7\times10^{12}cm^{-2}$ ($\upsilon = 16$), we clearly see four equidistant line of spectrum, corresponding to four Landau levels split by the cyclotron energy $\hbar\Omega$ = 4 meV, which corresponds to a cyclotron mass $m^* = 0.20m$. We see that the spin and valley splittings of the levels are not resolved under these conditions. To demonstrate the two-dimensional nature of the electron system under study, they used the method of rotation of the magnetic field. The angle (θ) is the deviation of the magnetic field from the normal to the 2D layer, with $\theta = 60°$, and B=7T, the filling factor exactly doubles, and eight Landau levels turn out to line under the Fermi surface. Fig. 15b (solid line) shows the spectrum of the recombination radiation found under these conditions. In this spectrum we can clearly see eight Landau levels, whose splitting is half that in Fig. 15a. This is unambiguous proof that the DOS of 2D electron is being seen in the recombination spectra.

From Eq. (3.51), by choosing an appropriate parameter ξ_L = 6.8 meV^2 with L = 97 Å corresponding to $E_L = \frac{\hbar^2}{2m^*L^2}$ = 2 meV (using m*≃0.20m as in the experiment of Kukushkin and Timofeev on the MOS inversion layer), our numerical results of the DOS are in good agreement with experiments as shown in Fig. 15 (dashed line).The numerical result show that the width Γ tends to decrease while increasing the Landau index N, the density of states between Landau levels is not exponentially small and is an appreciable fraction of n(E) at B = 0. Although our result of the DOS between Landau Levels seems to be a little lower than that obtained by experiments from the magnetooptic method, it can be acceptable because the tendency of peak-height increasing and peak-width decreasing when the Landau index increases, are in good agreement.

The points in the upper part of Fig.15a shows the energy positions of the Landau level in magnetic fields of 7, 5.6, and 4 T, at which respectively four, five, and seven levels are completely filled at n = 2.7×10^{12} cm^{-2}. The extreme points in the splitting fan determines the positions of the Fermi energy E_F and the band bottom E_O.

In summary, we are able to reproduce the density of states observed in a typical two-dimensional electron gas, such as shown in Fig. 14 using a simple model of disorder having gaussian variance ξ_L with finite correlation length L ≃ 100 Å.

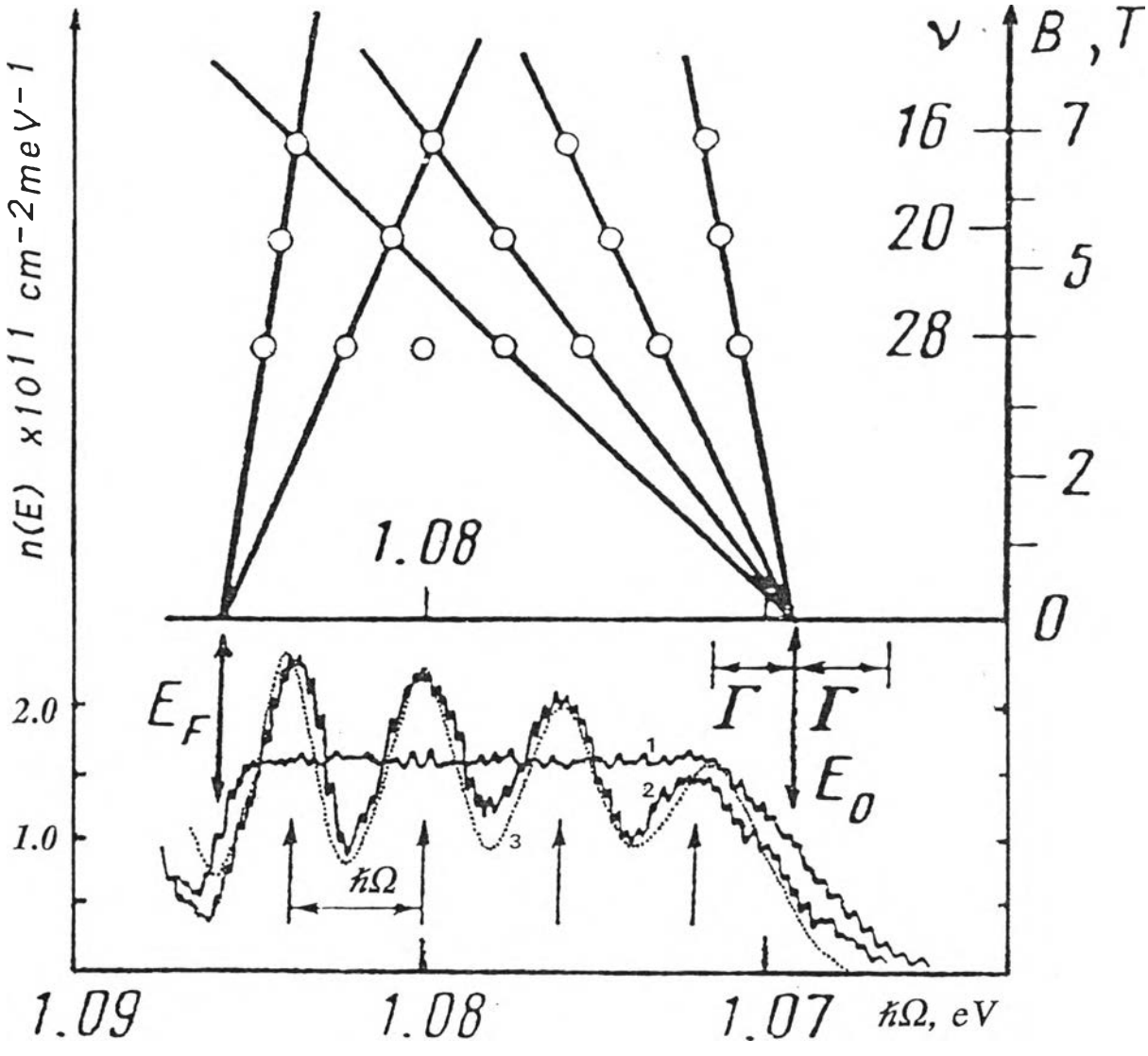

Fig. 15 a) Curves 1 and 2 show the emission spectra of 2D electrons found for T = 1.6K and B = 0 (spectrum 1) or B = 7T, $\hbar\Omega$ = 4 meV(spectrum 2). Curve 3 (dashed line) shows numerical result for the DOS using ξ_L= 6.8 meV^2 with L = 97 Å.The magnitude of the DOS at B = 0, n_o = $1.6x10^{11}cm^{-2}meV^{-1}$, was found by equating the integrated emission intensity in spectra 1 and 2.

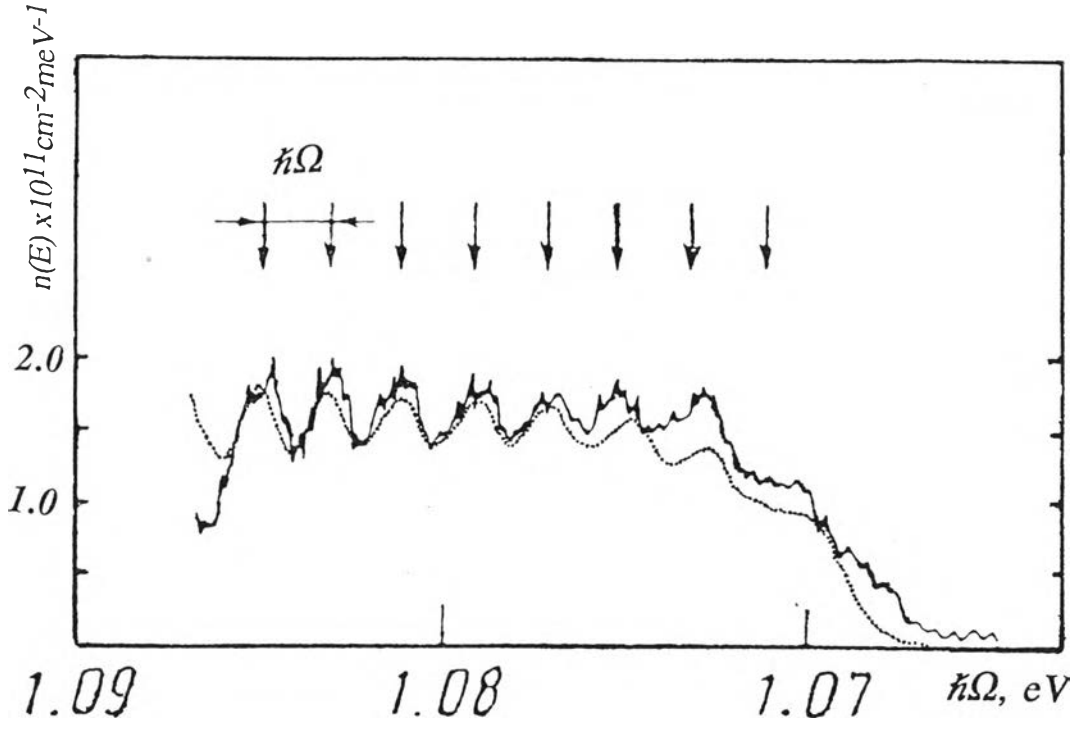

Fig. 15 b) Emission spectrum found at the angle between the magnetic field and the normal to the plane of the 2D layer is 60°($\hbar\Omega$ = 2 meV). The dashed line shows numerical result for the DOS with ξ_L = 6.8 meV^2 and L = 97Å.