TURBULENT FLOW OF WATER-BASED ALGORITHM IN TRUSS OPTIMIZATION

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ระเบียบวิธีการไหลวนของน้ำสำหรับการออกแบบที่เหมาะสมที่สุดของโครงถัก

น.ส.ซอ ธีรี คาย

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ซอ ธีรี คาย : ระเบียบวิธีการไหลวนของน้ำสำหรับการออกแบบที่เหมาะสมที่สุดของโครงถัก. (TURBULENT FLOW OF WATER-BASED ALGORITHM IN TRUSS OPTIMIZATION) อ.ที่ปรึกษาหลัก : เสวกชัย ตั้งอร่ามวงศ์

-วิทยานิพนธ์ฉบับนี้นำเสนอขั้นตอนระเบียบวิธีการไหลแบบปั่นป่วนของการเพิ่มประสิทธิภาพตามน้ำ (Turbulent Flow of Water-based Optimization) เพื่อการออกแบบโครงสร้างอย่างเหมาะสมที่สุดภายใต้แรงกระทำ ซึ่งกระบวนการนี้ได้รับแรงบันดาลใจมาจากธรรมชาติที่เกิดขึ้น และการตอบสนองแบบสุ่มของกระแสน้ำวนในลำธาร ทะเล รวมถึงมหาสมุทร โดยการออกแบบได้พิจารณาทางด้านความสิ้นเปลืองที่น้อยที่สุด ซึ่งรวมถึงน้ำหนัก และ ปริมาตรของโครงสร้าง โดยที่รูปร่างที่เหมาะสมที่สุดของโครงสร้างนี้ยังคงบรรลุวัตถุประสงค์ ในด้านความปลอดภัย และมีความสมบูรณ์ ซึ่งงานวิจัยชิ้นนี้ถูกวิเคราะห์ด้วยการเขียนโปรแกรมแบบไม่เชิงเส้นที่ท้าทายภายใต้เกณฑ์ความแข็งแรงและข้อกำหนดในด้านการใช้งานควบคู่กัน โดยเขียนรหัสโปรแกรมคอมพิวเตอร์ผ่านโปรแกรม MATLAB ในขณะที่โครงสร้างถูกแยกออกเป็นโครงถักมาตรฐานที่เชื่อมต่อกันด้วยพิน ซึ่งการออกแบบตามมาตรฐานอ้างอิงที่หลากหลายสามารถประสบความสำเร็จไปได้ด้วยวิธีการที่นำเสนอข้างต้น รวมถึงคำตอบของปัญหาที่มีความแม่นยำสามารถบรรลุผลได้ด้วยการใช้ทรัพยากรคอมพิวเตอร์เพียงเล็กน้อย และสามารถปฏิบัติตามข้อจำกัดที่มีได้ทั้งหมด

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Saw Thiri Khaing : TURBULENT FLOW OF WATER-BASED ALGORITHM IN TRUSS OPTIMIZATION. Advisor: Assoc. Prof. Dr. SAWEKCHAI TANGARAMVONG, Ph.D.

This thesis presents the novel Turbulent Flow of Water-based Optimization (TFWO) algorithm for the optimal design of structures under applied forces. The method has been inspired by the natural and random responses of vortices in rivers, seas and oceans. The design problem minimizes the total cost (viz., weight or volume) of the structure with the optimal distribution of its member sizes such that the safety and integrity of the resulting structure can be attained. It is formulated as the challenging nonlinear programming problem under the simultaneous ultimate strength and serviceability criteria. The proposed algorithm is encoded as a MATLAB code where the structure is discretized as a standard pin-connected truss. Various design benchmarks have been successfully processed by the TFWO approach. The accurate design solutions can be achieved at modest computing resources, where all constraints are strictly complied.

Field of Study:	Civil Engineering	Student's Signature ....
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CHAPTER 1

INTRODUCTION

1.1 Introduction

There are many kinds of structures, such as buildings, bridges, dams, and industries, that have an impact on our organization. Among them, buildings are the most important because millions of people spend most of their time in these buildings. The primary responsibility of structural engineers would be to produce safe and economical structures. Structural Optimization has become a well-known area of structural engineering research. Moreover, optimization has become the most important part of the design process these days because of its benefits to industries. The goal of optimization is to get the best result from a particular procedure while satisfying certain restrictions (RAPHAEL & HAFTKA, 1999). Therefore, the design system can be formulated as an optimization problem.

Structural optimization determines the optimal distribution of members and sizes assigned to the structure under the required strength and serviceability performance criteria. The problem is typically written in mathematical formulations aiming to compute the objective function (typically cost minimization) subjected to the constraints intrinsically describing the targeted design responses. The fast growth of recent computing technologies has encouraged the development of meta-heuristic methods that systematically perform iterative-type design procedures to find optimal solutions. A common way for solving optimization problems is the meta-heuristic method. Furthermore, many researchers are currently investigating the structural optimization problem with various meta-heuristic methods. On the other hand, one of the major drawbacks underlying is the return of local optimum, leading to premature solution convergence. The ability to obtain accurate optimal designs is largely problem dependent. The exploration of new and suitable methods is thus necessary for the specific structural design problems considered.

This study proposes the development of a so-called turbulent flow of water-based optimization (TFWO) to process the design of steel trusses under the required forces. The TFWO is inspired by the random behaviors in nature, for example, in rivers, seas, and oceans. It provides the optimal solutions of various complex problems with real-parameter benchmark functions for different dimensions. The specific problem considers the cost (total weight) minimization as the objective function subject to the constraints on the limited strength and serviceability responses of the design structures. The problem of structural optimization can be categorized into three parts: size, shape, and topology. Sizing Optimization is the most frequent approach in design optimization of truss structures. Therefore, the design algorithm's goal is to obtain the minimum weight truss structure by using size optimization.

1.2 Research Objective

The main objectives of this research and analysis are as follows:

i. To present an efficient and practical approach based on the Turbulent Flow of Water-based Algorithm (TFWO).

ii. To optimize the weight of the truss structure under the satisfied design constraints by using the Turbulent Flow of Water-based Optimization (TFWO) algorithm.

iii. To evidence the robustness and accuracy of the proposed method by comparing the obtained results with some other meta-heuristic algorithms in the literature.

1.3 Scope of Research

(1) The turbulent flow of water-based optimization algorithm was coded in the MATLAB program.

(2) The design of truss structures is analyzed using the finite element (direct stiffness method).

(3) The proposed TFWO approach is demonstrated by the sizing optimization of 2D and 3D steel trusses with strength and serviceability restrictions.

(4) The continuous variables and discrete variables are selected from the asserted design benchmark problems.

(5) Some benchmarks processed by various recent optimization algorithms were used in this study.

1.4 Structure of Report

There are five chapters in this study. Chapter 1 is an introduction, objective and scope of this research. Chapter 2 describes literature review (the background of optimal structural design). Chapter 3 presents the main principles of the turbulent flow of water optimization (TFWO) algorithm, which is used as an optimization method in this work. Chapter 4 contains analysis and design of steel truss structures by optimizing with the proposed algorithm and then it is illustrated through comparisons with some benchmarks processed by various recent optimization algorithms. Chapter 5 includes conclusion and recommendations based on the results of this work.

CHAPTER 2

LITERATURE REVIEW

2.1 Structural Optimization

Structural optimization is a class of optimization problem which has been widely used in the last decade as the most active branch of structural engineering. The loads or stresses applied to the structures (i.e., point loads or distributed loads), member deflections, lateral displacements, and buckling constraints will be considered in structural optimization.(Mirza, 2020). Nodal coordinates and cross-sectional areas are the two most important problems in structural optimization. Size, shape, and topology are the three basic types of structural optimization. The goal of all three categories is generally mass minimization with optional stress or displacement constraints. The reduction of structure weight obviously reduces material costs, resulting in lower total costs.

2.1.1 Sizing Optimization

The goal of sizing optimization problems may be to determine the optimal thickness distribution of plate and shell segments or the optimal member cross-sectional areas of bars in a truss structure. In sizing optimization, the design variable is the thickness of a plate or the cross-sectional area of a bar. The main feature of the sizing problem is that the layout of the structure and the state variables are prescribed and fixed throughout the optimization process (Eser, 2014). Fig. 1 shows the size optimization of a truss structure.

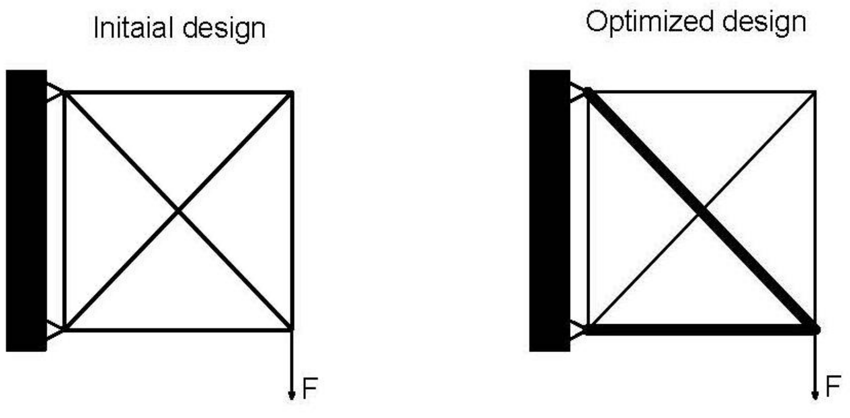


Figure 1. Sizing Optimization of a truss structure

2.1.2 Shape Optimization

The goal of shape optimization is to find the best location for the nodes. This can be accomplished numerically by minimizing an objective function. In shape optimization, the design variable is the location of finite element nodes (Al Rabadi, 2014). The shape optimization process consists of numerous steps. According to Yun Liang Ding (Ding & Structures, 1986), the steps of shape optimization are model description, selection of the objective function and shape variables, representation of boundary shape, finite element mesh generation & refinement, sensitivity analysis and solution methods. The shape optimized design for a cantilever truss is shown in Fig. 2.

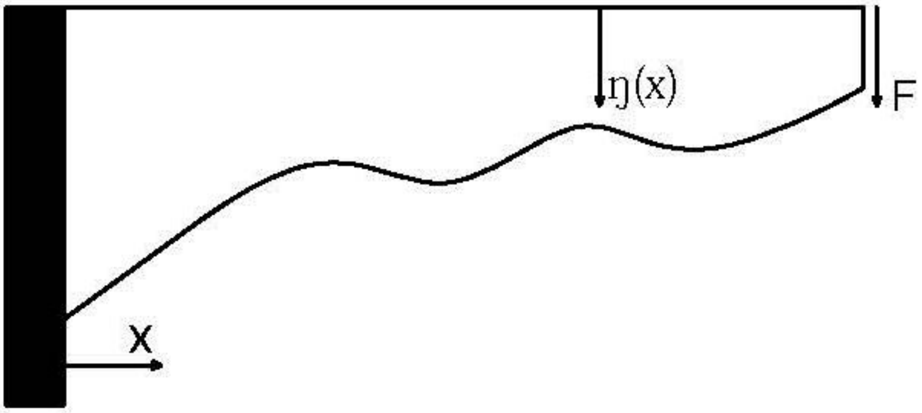


Figure 2. A shape optimization problem

2.1.3 Topology Optimization

Topology optimization is a mathematical method for maximizing system performance by optimizing material layout inside a particular design space for a given set of loads, boundary conditions, and restrictions. This method provides a minimum distribution of materials in the selected design space. It eliminates any unnecessary features or materials, reducing both waste and cost (Al Rabadi, 2014). Topology optimization improves with size and shape optimization, and the optimized structure has no limits (Kegl, 2002). Figure 3 depicts an example of a truss structure for topology design.

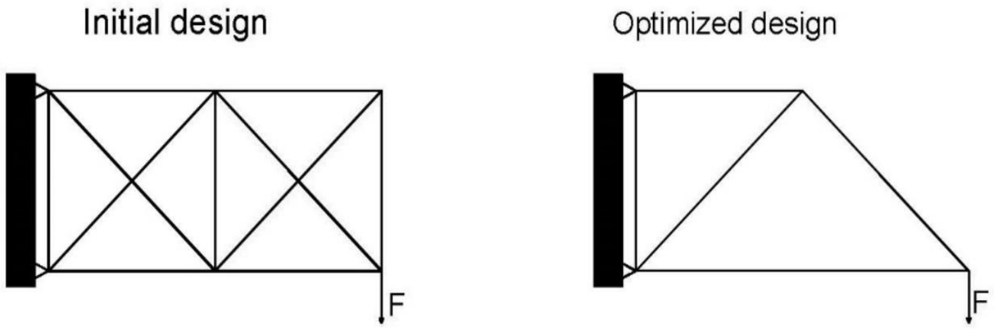


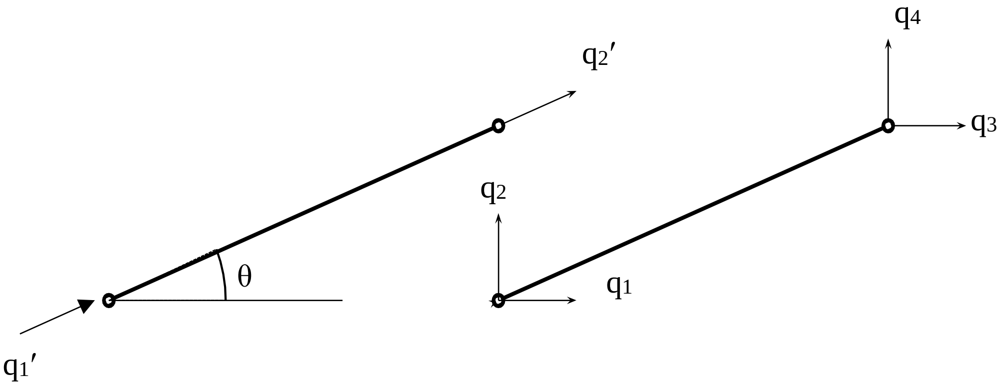
Figure 3. Topology optimization of a truss

2.2 Finite Element Analysis

The majority of structural optimization issues are handled repeatedly using a combination of finite element and mathematical programming techniques (Schoofs, 1988). Rather than being part of the optimization code, the cost function and constraints are considered part of the finite element specification (Jansen, 1988). The finite element model is a function of the optimization variables. There are a variety of methods for structural analysis problems, including (1) the direct method or direct equilibrium method, (2) variational methods consisting of among the subsets of energy methods and the principle of virtual work, and (3) weighted residual methods that can be used to derive the finite element equations.

2.2.1 Direct Stiffness method

The displacement (or) stiffness method is one of the direct techniques applied to solve truss problems. The method is frequently used in computer-assisted structural analysis of complicated structures, encompassing both statically determinate and indeterminate structures. The structural stiffness matrix can be synthesized from element stiffness (Logan, 2016).The local coordinate system is used to produce the element stiffness, which is then translated to the common global coordinate system (Logan, 2016). We begin by examining the beam or element shown in Fig. 4. Two coordinate systems are depicted in the diagram. One is a one-dimensional coordinate system that corresponds to the element's length. This will be referred to as the local coordinate system. The other is a non-aligned two-dimensional coordinate system. This will be used as the global coordinate system.



(a) Local coordinate (b) Global coordinate

Figure. 4. 2D truss element: (a) local and (b) global degrees of freedom

We will let q_1', q_2' represents displacement in local coordinate system and q_1, q_2, q_3, q_4 represents displacement in global coordinate system. The force applied to the beam has a linear relationship with the degree of deformation. We add potential energy to the beam when it is stretched or compressed. This energy is called strain energy. Strain energy is a type of energy that can be calculated using Hook's law. The force is directly proportional to the deformation, according to law.

$$F = k\Delta x \tag{1}$$

We can compute the energy by integrating over the deformation

$$u = k\int_0^Q x\,dx = \frac{1}{2}\,kQ^2 \tag{2}$$

where, k is the element stiffness, Q is the total change in length of the element that can be rewritten as:

$$Q = (q_2' - q_1') \tag{3}$$

Substituting Eq. (3) into Eq. (2) gives us

$$u = \frac{1}{2}\,k(q_2' - q_1')^2 \tag{4}$$

$$u = \frac{1}{2}\,k(q_2'^{\,2} - 2\,q_2'q_1' + q_1'^{\,2}) \tag{5}$$

We let this be expressed in vector form.

$$q' = \begin{Bmatrix} q_1' \\ q_2' \end{Bmatrix} \tag{6}$$

$$k' = \frac{EA}{\mathrm{Le}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{7}$$

where A is the cross sectional are of the element, L_e is the length of the element.

When we substitute Eq. (6) and (7) to Eq. (5), the equation can be written as follows

$$u = \frac{1}{2}\, q'^{T} k' q' \qquad (8)$$

We can transform the global coordinates to local coordinates with the following equations.

$$q_1' = q_1 \cos\theta + q_2 \sin\theta \qquad (9)$$

$$q_2' = q_3 \cos\theta + q_4 \sin\theta \qquad (10)$$

This can be rewritten in vector notation as:

$$q' = Lq \qquad (11)$$

where,

$$L = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \qquad (12)$$

where, $l = \cos\theta$, $m = \sin\theta$

Replacing Eq. (11) in Eq. (8), we obtain

$$u = \frac{1}{2}\, q^{T} [L^{T} k' L] q \qquad (13)$$

Now, the global stiffness of the truss becomes

$$K = L^{T} k' L \qquad (14)$$

and

$$K = \frac{EA}{\mathrm{Le}} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \qquad (15)$$

where, E = Young's modulus for the element material

A = the cross-sectional area of the element

L_e = the length of the element

$l = \cos\theta$, $m = \sin\theta$

2.2.2 Stress Computation

Weight, displacements, stresses, vibration frequencies, buckling loads, and cost, or any combination of these, can be employed as objective functions in structural optimization issues. Our design issues may also include reducing the truss mass and minimizing the stresses on its members (RAPHAEL & HAFTKA, 1999). The stress can be written as

$$\sigma = E\varepsilon \qquad (16)$$

where Ɛ is the strain, the change in length per unit of length. We can rewrite Eq (1) as:

$$\sigma = E\,\frac{q_2{}' - q_1{}'}{Le} \tag{17}$$

In vector form, we can write the equation as

$$\sigma = \frac{E}{Le}\begin{bmatrix}-1 & 1\end{bmatrix}\begin{Bmatrix}q_1{}' \\ q_2{}'\end{Bmatrix} \tag{18}$$

$$\sigma = \frac{E}{Le}\begin{bmatrix}-1 & 1\end{bmatrix} q' \tag{19}$$

When we substitute Eq. (11) to Eq. (19), the equation can be written as follows:

$$\sigma = \frac{E}{Le}\begin{bmatrix}-1 & 1\end{bmatrix} \mathrm{L}q \tag{20}$$

Now we multiply L matrix Eq. (12) by the vector

$$\sigma = \frac{E}{Le}\begin{bmatrix}-l & -m & l & m\end{bmatrix} q \tag{21}$$

2.3 Metaheuristic Algorithm

Manny metaheuristic algorithm have been employed to systematically capture the weight minima of practical structures without the need of mathematical programming implementations. These methods are generally inspired by the concept of observing nature-like collective birds and animal behaviors, e.g., genetic algorithm (GA) (Coello, Rudnick, & Christiansen, 1994), Firefly Algorithm (FFA) (Yang, 2010), Glowworm Swarm Optimization (GSO) (Krishnanand & Ghose, 2009), Differential Evolution (DE) (Storn & Price, 1997), Harmony Search (HS) optimizer (Lee, Geem, & engineering, 2005), a nature inspired meta-heuristic Water Wave Optimization (WWO)(Zheng, 2015), a modified new self-organizing hierarchical PSO with jumping time-varying acceleration coefficients (M Ghasemi, Aghaei, & Hadipour, 2017), Cuckoo Optimization Algorithm (COA) (Rajabioun, 2011), and Gradient Evolution (GE) (Kuo & Zulvia, 2015).etc. A wide class of these methods has been studied in the structural optimization. The metaheuristic algorithm requires two essential elements to obtain the solution: the exploitation phase and the exploration phase. Exploration is directly related to global search, and we utilize it to explore the whole search space of our solutions for finding good solutions globally. Exploitation is directly related to local search, and it is the process of updating solutions based on the best solutions aiming to enhance existing ones. They are standard features of all metaheuristic algorithms.

CHAPTER 3
OPTIMAL DESIGN OF STEEL TRUSSES

3.1 Introduction

In this chapter, the Turbulent Flow of Water-based optimization (TFWO) algorithm is discussed, and its method was used to solve numerical and structural truss size optimization problems. The main outlines of the TFWO algorithm have been presented by Ebrahim Akbari (2020) (Mojtaba Ghasemi et al., 2020). The number of design variables, size of search area, and number of design constraints are needed to find optimize designs. The design variables, objective function, and constraints are summarized in the following formulation of the optimization problem.

3.2 State Optimization Problem

The minimum weight design of the pin-connected steel truss structure can be mathematically described as follows:

$$\text{Minimize} \quad \mathrm{W(X)} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \rho_{\mathrm{i}} \mathrm{A}_{\mathrm{i}} \mathrm{L}_{\mathrm{i}} \tag{22}$$

$$\text{subject to} \quad \begin{cases} \sigma\mathrm{i} \ \le \ \sigma\mathrm{all},\, \mathrm{i} \ = \ 1{,}2, - - - - - -, \mathrm{n} \\ \delta\mathrm{j} \ \le \ \delta\mathrm{all},\, \mathrm{j} \ = \ 1{,}2, - - - - - -, \mathrm{n} \\ \mathrm{A}_{i}^{min} \ \le \ \mathrm{A}_{i} \ \le \ \mathrm{A}_{i}^{max}\,,\ \mathrm{i} \ = \ 1{,}2, - - -, \mathrm{n} \end{cases} \tag{23}$$

where W is the total weight of the designed structure; n is the number of all truss members; ρ_i is the material density of the i-th member for $\forall i \in \{1,\ldots,n\}$; A_i is the member cross-sectional area (defined as the design variables); L_i is the member length; σ_i is the member stress; σ_{all} is the allowable stress; δ_j is the nodal displacement for $\forall j \in \{1,\ldots,d\}$; and δ_{all} is the limited displacement at some j-th specified degree of freedom, and A_i^{min} and A_i^{max} are upper and lower bounds of the design variables.

To account for design infeasibility, a penalty function is applied to the structure weight. This approach exchanges a constrained optimization problem by an unconstrained one. The general form of the penalty function is defined below:

$$\mathrm{W} = \mathrm{W(X)}(1+\mathrm{C})^{\varepsilon} \tag{24}$$

where f is the penalized objective function, Ɛ is the penalty function exponent (viz., Ɛ is considered as 1in this study) and C is the parameter measuring the violation of penalty constraints:

$$C = \sum_{i=1}^{m} c_i \tag{25}$$

where m is the member of the constraint and c_i is the value of each constraint violation. c_i can be calculated as,

$$c_i(x) = \begin{cases} 0, & \text{if } \alpha_i \leq 0 \\ \alpha_i, & \text{if } \alpha_i > 0 \end{cases} \tag{26}$$

For stress constraints, α_i^{σ} is defined as:

$$\alpha_i^{\sigma} = \frac{|\sigma_i|}{|\sigma_i^a|} - 1 \tag{27}$$

where σ_i is the stress in element i and σ_i^a is the allowable stress in element i. For displacement constraints, σ_i^d is defined as:

$$\alpha_i^{d} = \frac{|d_i|}{|d_i^a|} - 1 \tag{28}$$

where d_i is the displacement at connection i and d_i^a is the limited displacement in element i.

3.2.1 Design Variables

Design variables are the parameters used in the formulation of the objective function to define the structural system. Design variables can be divided into two groups: continuous variables and discrete variables. Continuous design variables have a range of possible values and can take any value within that range. Discrete design variables can only take from a list of allowable values. In this work, solving an optimization problem with discrete variables is typically much more difficult than solving a problem with continuous design variables (RAPHAEL & HAFTKA, 1999). For the optimization process to be successful, the design variables must be consistent with the structural model and optimization algorithm. The optimal design procedure starts with determining the initial values of area variables (Eser, 2014). Design variables can be expressed mathematically as follows:

$$A_i = [x_1, x_2, x_3, \text{-------}, x_n],\ i = 1,2,\text{-------},n \tag{29}$$

3.2.2 Objective Function

In a typical structural design problem, the objective function is a simple function of the design variable (e.g., weight). Moreover, the value of the objective function is affected by a set of unknowns or variables. In a minimization problem, a function F chooses a small value because that is better than a large one. The goal of size optimization for truss structures is to obtain design variables x that minimizes a specific objective function F(x). The objective function may be written as follows:

$$\text{Minimize} \qquad F(x) = \sum_{i=1}^{n} \rho_i A_i L_i$$

3.2.3 Constraints

Constraints play an important role in structural optimization because they guarantee that the final structure is valid, viable, and safe, as well as not violating design criteria. In the design process, inequality constraints and equality constraints are utilized to minimize the weight of structure. In most structural optimizations, inequality constraints impose size, stress, and displacement limits, among other things. (RAPHAEL & HAFTKA, 1999). A constrained optimization problem is one that has a set of equality or inequality restrictions. An unconstrained optimization problem is one that does not have any equality or inequality restrictions (RAPHAEL & HAFTKA, 1999). By using penalty functions or other constraint handling methods, constrained optimization problems can sometimes be turned to unconstrained optimization problems. Constraints associated with stress and displacement are taken into account in this proposed method.

3.2.3.1 Constraints for Member Stress

Member stresses restrictions are the most important criteria in structural engineering. The member stresses must not exceed a certain limit as following.

$$\sigma_i \le \sigma_{all} \tag{30}$$

where σ_i is the stress for member i; and σ_{all} is the allowable stress.

If the proposed truss satisfies Equation (30), the member stresses do not violate the stress constraint and the proposed truss will be accepted. Otherwise, If the member stresses do not comply with the stress restrictions, the proposed truss will be rejected.

3.2.3.2 Constraints for Nodal Displacements

In structural engineering, displacement constraints are frequently important. The structure is only allowed to deflect up to a particular amount, as shown below.

$$\delta i \le \delta all \tag{31}$$

where δ_i is the deflection for node i; and δ_{all} is the allowable deflection.

If the proposed truss satisfies Equation (31), the node deflections don't violate the deflection constraint and the proposed truss will be accepted. Otherwise, the nodes’ deflection violates the deflection constraint, and the proposed truss will be refused. On the contrary, stress and deflections constraints are necessary for any optimization category.

3.3 Turbulent Flow of Water-based Optimization

Similar to other metaheuristic algorithms, the idea of turbulent flow of water-based optimization (TFWO) algorithm is inspired from nature and based on the behavior of whirlpool occurs in rivers, seas and oceans.

3.3.1 Whirlpool Formation

In the beginning, the method divided the population into various whirlpool sets, where the best position in each whirlpool is set in the center of the whirlpool which generates the most traction strength. The following equation is used to obtain the random initial population.

$$X_{i,p} = X_p^{min} + rand * \left(X_p^{max} - X_p^{min}\right) \tag{32}$$

where, X_p^{max} and X_p^{max} are the upper and lower bounds of design variables, respectively. and rand is a uniformly distributed random number between 0 and 1.

3.3.2 A whirlpool’s influence on objects

Each whirlpool unifies the object positions to the whirlpool center by applying the centripetal force (i.e., $X_i = Wh_j$). Depending on the amount of object, other whirlpools lead to some deviations (ΔX_i) resulting in the new position of the object. The updated position are illustrated by the two simple equations of Eq. (33) and Eq. (34). Fig. 5 shows the proposed model of a whirlpool which affects objects.

$$\delta_i^{new} = \delta_i + \mathrm{rand}_1 * \mathrm{rand}_2 * \pi \tag{33}$$

$$\Delta_{\mathrm{t}} = \mathrm{f}(Wh_{\mathrm{t}}) \ * \ |Wh_{\mathrm{t}} - \ \mathrm{sum}(X_i)|^{0.5} \tag{34}$$

$$\Delta X_i = \Big(\cos(\delta_i^{new}) * rand\ (1, D) * \big(Wh_f - X_i\big) - \sin(\delta_i^{new}) * rand\ (1, D) * (Wh_w - X_i)\Big) * (1 + |\cos(\delta_i^{new}) * -\sin(\delta_i^{new})|) \tag{35}$$

$$X_i^{new} = \ Wh_j - \ \Delta X_i \tag{36}$$

where, δ_i is the i^th^ object's angle, $rand_1$ and $rand_2$ are random numbers, Δ_{t} is the distance between whirlpool and object, Wh_f is the whirlpool with minimum value of Δ_{t}, Wh_w is the whirlpool with maximum value of Δ_{t}, X_i^{new} is the new position of particles i, Wh_j is the position of whirlpool j.

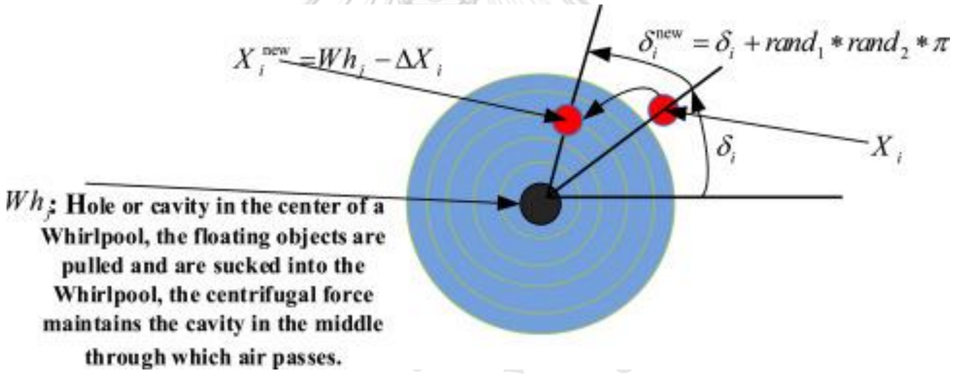


Figure. 5. The proposed model of whirlpool for optimization purposes

3.3.3 Force of Centrifugation

At variance with the centripetal force attracting the moving object toward its whirlpool, the centrifugal force pushes the object away the center. In the instance when the centrifugal force overcomes the centripetal counterpart as defined in Eq. (37), the object position transfers to the new position. The centrifugal force FE_i is described in Eq. (38) if it is greater than the random values. Fig. 6 depicts the various forces effects in a whirlpool.

$$FE_i = ((\cos(\delta_i^{new}))^2 * (\sin(\delta_i^{new}))^2)^2 \tag{37}$$

$$X_{i,p} = X_p^{min} + rand * (X_p^{max} - X_p^{min}) \tag{38}$$

where, FE_i is the centrifugal force and rand is a uniformly distributed random number between 0 and 1.

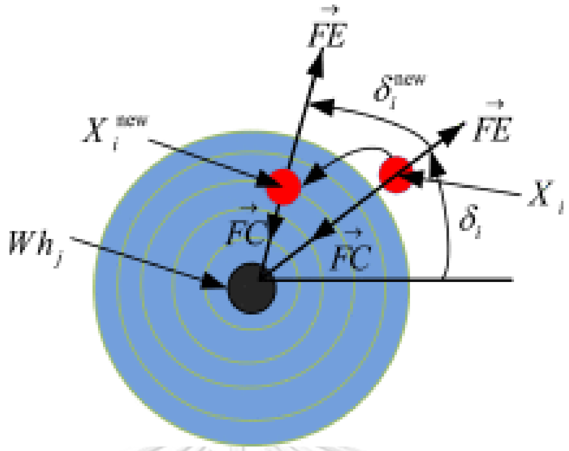


Figure. 6. The various types of forces in a whirlpool

3.3.4 Interaction between whirlpools

Moreover, the position of whirlpools can be influenced by the other whirlpool. If the solution given by the new whirlpool is better than its previous connecting whirlpool, the positions of whirlpool are exchanged. The whirlpool positions are updated as follows:

$$Wh_j^{new} = Wh_f - \Delta Wh_j \tag{39}$$

$$\Delta Wh_j = rand\ (1, D) * \left| cos(\delta_j^{new}) + sin(\delta_j^{new}) * (Wh_f - Wh_j) \right| \tag{40}$$

where δ_j is the value of the jth whirlpool hole's angle.

In the case where the best object among all members in the set is stronger than the whirlpool itself, the new whirlpool is updated by this best object for the consequent iteration. The analysis and design procedure are in repetition process until to reach the final acceptable results. Flow chart of TFWO algorithm is presented in Fig. 7.

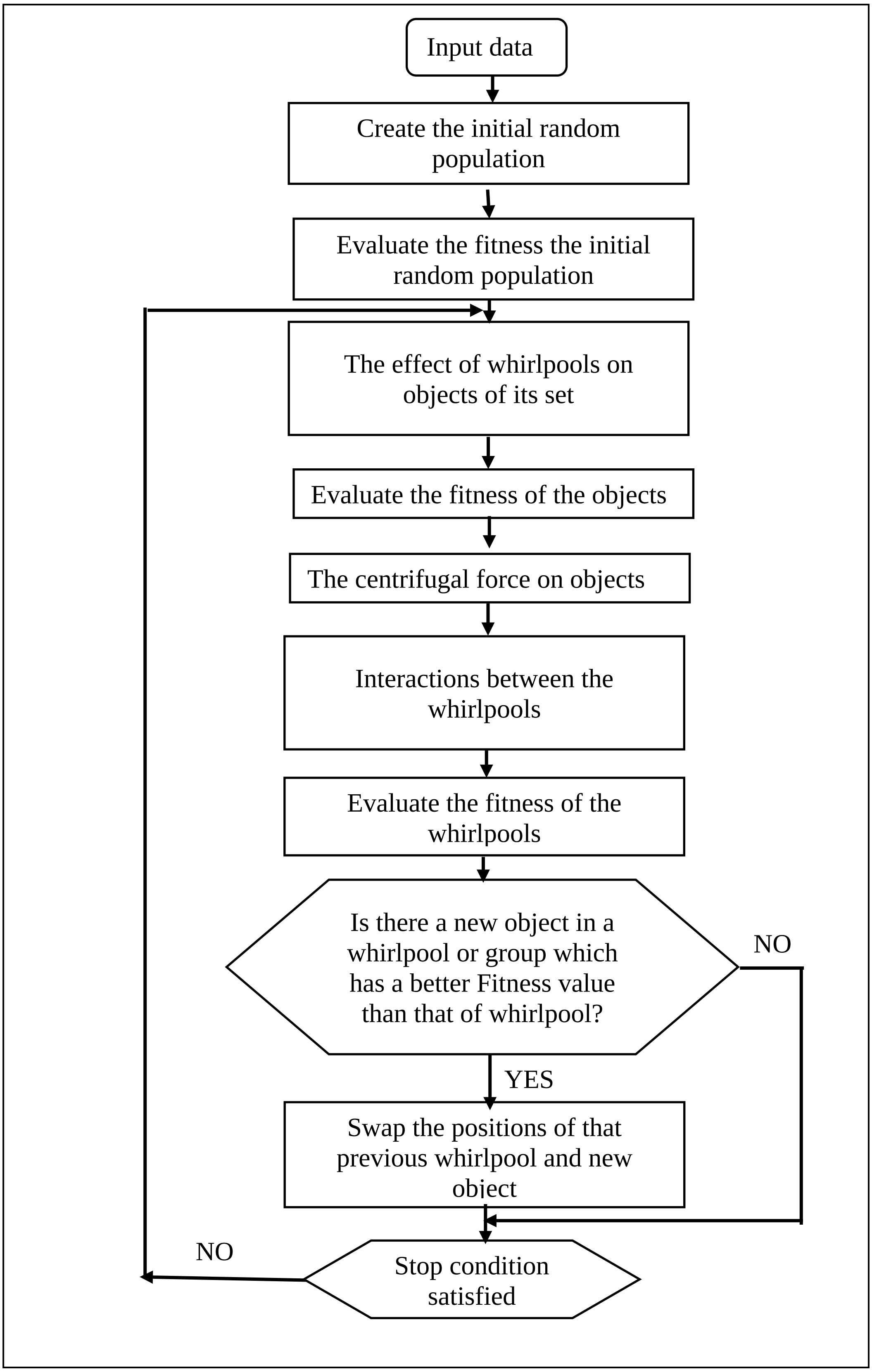


Figure 7. Flowchart of the proposed TFWO optimization algorithm

CHAPTER 4

RESULT AND DISCUSSION

4.1 Introduction

In this chapter, the application of the proposed TFWO method will be applied to solve continuous and discrete optimization benchmark problems of the truss structure to verify the efficiency and accuracy of the algorithm. The effectiveness of the TFWO in size optimization for trusses is demonstrated through the numerical examples of 10-bar truss and 25-bar truss with continuous variables and 200-bar truss, 160-bar truss, and 72-bar truss with discrete variables. The analyses of all trusses have been performed via the finite element method (FEM). The procedure of the optimization can be expressed step by step as follows.

1. The code begins with structural data such as node coordinates, element lengths, modulus of elasticity, and material density.
2. Input TFWO algorithm parameters: the number of particles (N), number of whirlpools (N_{wh}), and maximum number of populations (Np).
3. Initialize randomly all particles position $x_i \in A_i$, i = 1,2, ----,N.
4. Evaluate the fitness value of the truss structure following the structural analysis process.
5. Begin with the first iteration. (loop = 1)
6. According to a whirlpool's influence on objects, Eq. (36) is used to update particle's position.
7. Evaluate the fitness value of particle and
   assign $F(X_i) = W(Xi) = \min(W(X_1), W (X_2), ------, W (X_i))$, i = 1,2,3----, N
8. The centrifugal force, FE_i is determined by Eq. (25)
9. Update the particle positions according to Eq. (32).
10. Evaluate the objective function values as $W (X_i)$
11. According to the interaction between the whirlpools, Eq (39) and Eq (40) are used for updating the whirlpool's positions.
12. Find the best fitness value by following the process of structural analysis shown in Fig. 8.

13. The cross sectional areas and the weights of truss structure are the last results to be output.
14. Increase the number of iterations by loop = loop +1. The optimization will perform till the maximum number of iterations. The flow chart of structural analysis can be summarized as shown in Fig. 8.

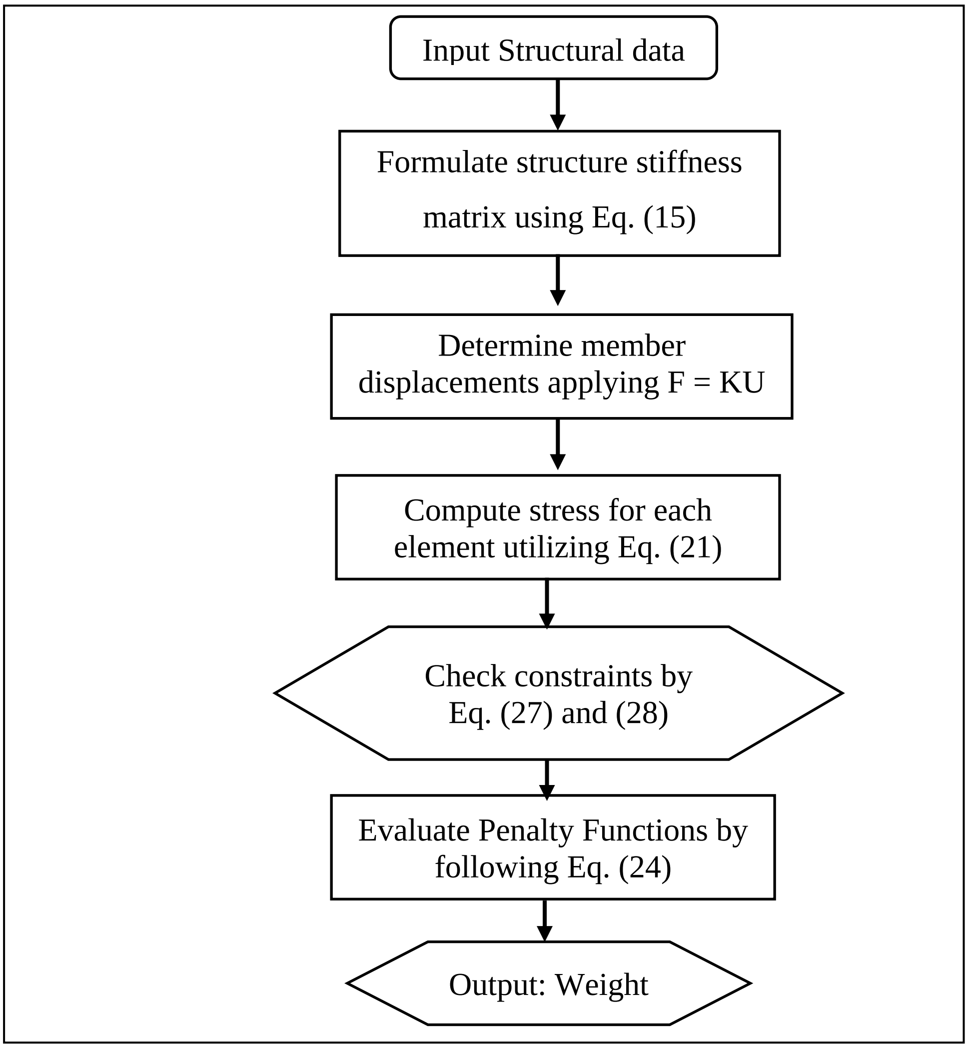


Figure 8. Flowchart of the Structural Analysis

4.2 10-Bar Planar Truss Structure

The size optimization for benchmark 10-bar plane truss shown in Fig. 9 is considered as the first numerical example. The vertical load affected nodes number 2 and 4 is equal to P = 10 kips. The material properties employed were the modulus of elasticity of 10^4 ksi, the material density of 0.1 lb.in^3, and the permissible tensile and compressive stresses of σ_i = 25 ksi for all members $i \in \{1,2, -------,10\}$.

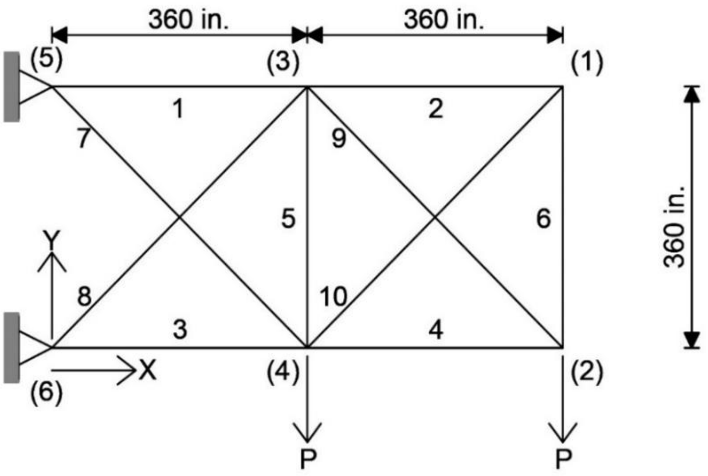


Figure. 9. Structure of benchmark 10 bar plane truss

The sizing optimization problem in Eq. (22) defined the design variables, namely the unknown member areas of W_A = [A_1, -----, A_{10}]. The cross-sectional area of each bar varies between 0.1 and 35 in^2. The displacement in each free node should not exceed 2 inches both horizontally and vertically. The TFWO method adopted 33 populations with the maximum number of 150 iterations. All imposed constraints were fully complied. The plot of solution convergence in Fig. 10 presents variations of the design variations of the design weights decreasing to the optimum over the increasing number of iterations. The algorithm took only 26 seconds to converge the optimum solution.

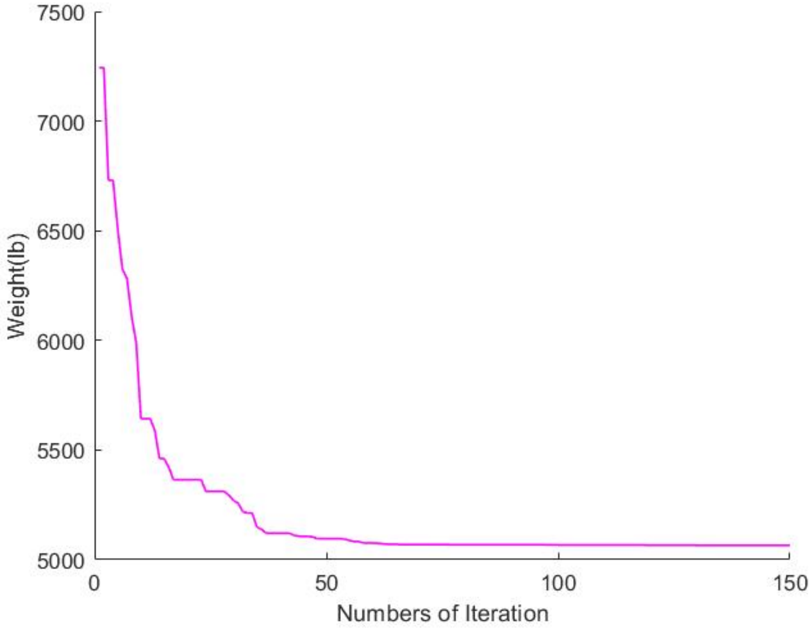


Figure. 10. Convergence histories for the 10-bar plane truss

The effect of TFWO algorithm is studied by using five different population as 22, 33, 44, 48, and 55. The number of iterations is taken as from 100 to 500 for all population size in this example. Table 1 shows the result of TFWO obtained by performing 25 independent optimization runs which is presented and compared with applying different algorithms. The best solution was observed in HS (Lee et al., 2005), PSO (Li, Huang, Liu, Wu, & Structures, 2007), and HPSACO (A Kaveh & Talatahari, 2009) but they are required more number of structural analyses than the proposed TFWO method except IHS (Lamberti & Pappalettere, 2009). Moreover, It is evidenced that the optimal design weight value of 5060.89 lb given by the present TFWO achieved better designs than those of PSO (Perez, Behdinan, & Structures, 2007), EHS and SAHS (S. J. C. Degertekin & Structures, 2012). The most important result is that the TFWO algorithm required significantly less structural analyses than EHS, SAHS (S. J. C. Degertekin & Structures, 2012), ABC-AP (Sonmez, 2011), PSOPC and HPSO (Dede, Bekiroğlu, & Ayvaz, 2011). Therefore, the results prove the competitive performance and robustness of TFWO algorithm compared to other state of the algorithm.

Table 1. Optimum results for various design methods.

Design variables A_i (in^2)	Lee and Geem (Lee et al., 2005)	Li et al. (Li et al., 2007)			Kaveh (A Kaveh & Talatahari, 2009)	Lamberti and Pappalettere (Lamberti & Pappalettere, 2009)	Sonmez (Sonmez, 2011)	Degertekin (S. J. C. Degertekin & Structures, 2012)		Present Study
	HS	PSO	PSOPC	HPSO	HPSACO	IHS	ABC-AP	EHS	SAHS	TFWO
A_1	30.15	33.469	30.569	30.704	30.307	30.5222	30.548	30.208	30.394	30.3815
A_2	0.102	0.11	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.100
A_3	22.71	23.177	22.974	23.167	23.434	23.2005	23.18	22.698	23.098	23.1858
A_4	15.27	15.475	15.148	15.183	15.505	15.2232	15.218	15.275	15.491	15.2419
A_5	0.102	3.649	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1000
A_6	0.544	0.116	0.547	0.551	0.5241	0.5513	0.551	0.529	0.529	0.5537
A_7	7.541	8.328	7.493	7.46	7.4365	7.4572	7.463	7.558	7.488	7.4600
A_8	21.56	23.34	21.159	20.978	21.079	21.0367	21.058	21.559	21.189	21.0973
A_9	21.45	23.014	21.156	21.508	21.229	21.5288	21.501	21.491	21.342	21.5595
A_{10}	0.1	0.19	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1000
Weight(lb)	5057.9	5529.5	5061	5060.9	5056.56	5060.82	5060.88	5062.39	5061.42	5060.89
Average Weight(lb)	N/A	N/A	N/A	N/A	N/A	N/A	N/A	5063.73	5061.95	5065.10
No of Structural analyses	20,000	150,000	150,000	125,000	10,650	1,350	500,000	9,791	7,081	4,950

4.3 25-Bar Space Truss Structure

The design of 25-bar space truss in Fig. 11 is chosen as the second size optimization example in this work. Nodal coordinates and lay out of the members of this truss are given in Appendix. The design variables defined the member areas categorized into 8 different groups as follows (1) A_1, (2) A_2-A_5, (3) A_6-A_9, (4) A_{10}-A_{11}, (5) A_{12}-A_{13}, (6)A_{14}-A_{17}, (7) A_{18}-A_{21}, (8) A_{22}-A_{25}. The structure is subject to the loading condition is given in Table 2.

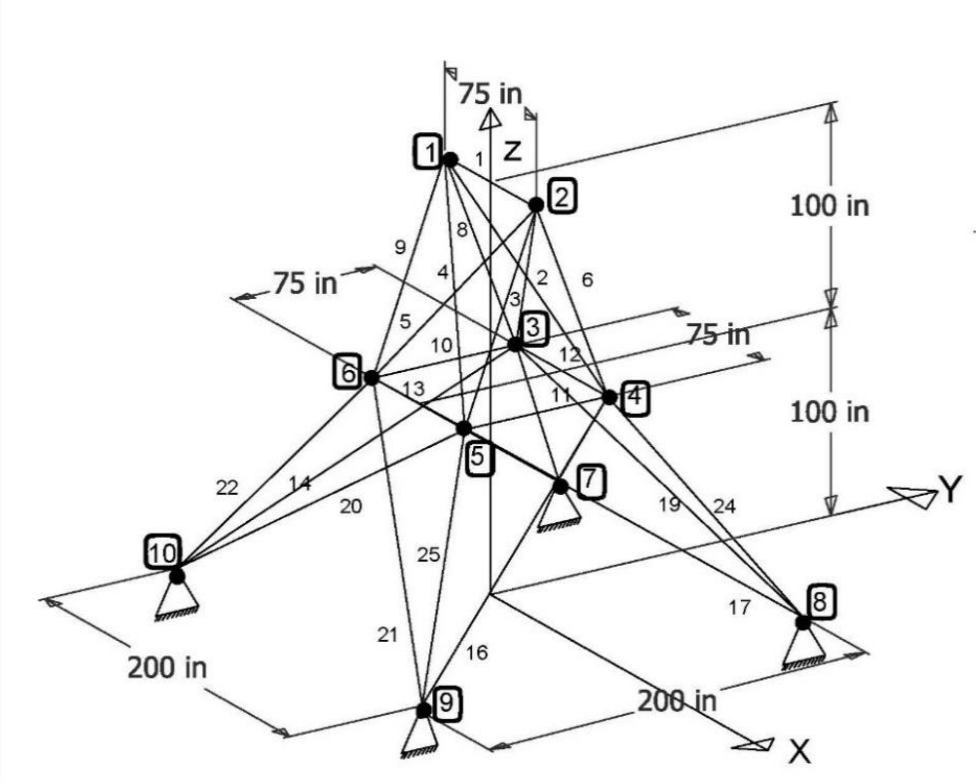


Figure 11. 25-bar space truss structure.

Table 2 Loading Conditions for the 25-bar space truss

Node	F_x (kips)	Fy (kips)	Fz (kips)
1	1	10	-5
2	0	10	-5
3	0.5	0	0
6	0.5	0	0

The material properties employed were elastic modulus of 10,000 ksi (68,950 MPa) and uniform material density of 0.1 lb.in^3 (2767.99 kg.m^3). The cross-sectional areas were selected within the range between 0.01 in^2 and 3.4 in^2. The allowable displacements of each node were limited to the variation of 0.35 in at x- and y-directions. The maximum stress limits in all compression and tension members are 40 ksi. The optimal design of the steel space truss was successfully performed by the proposed TFWO method within 50 analysis iterations. The solution (total weight) convergence with the number of analysis 100 iterations is clearly depicted in Fig. 12. More explicitly, the minimum weight of 482.026 lbs was computed at the 42th iteration and took only 23 seconds. The optimal results, including the total weight and designed member areas, are reported in Table 3, and agree well with those from benchmarks (Bekdaş, Nigdeli, & Yang, 2015), (Camp & Bichon, 2004), (Cao, 1997), (Li et al., 2007), (Camp & Farshchin, 2014) and (Camp, 2007). In essence, the present TFWO approach provides the most minimum weight solution with the satisfaction of all constraints.

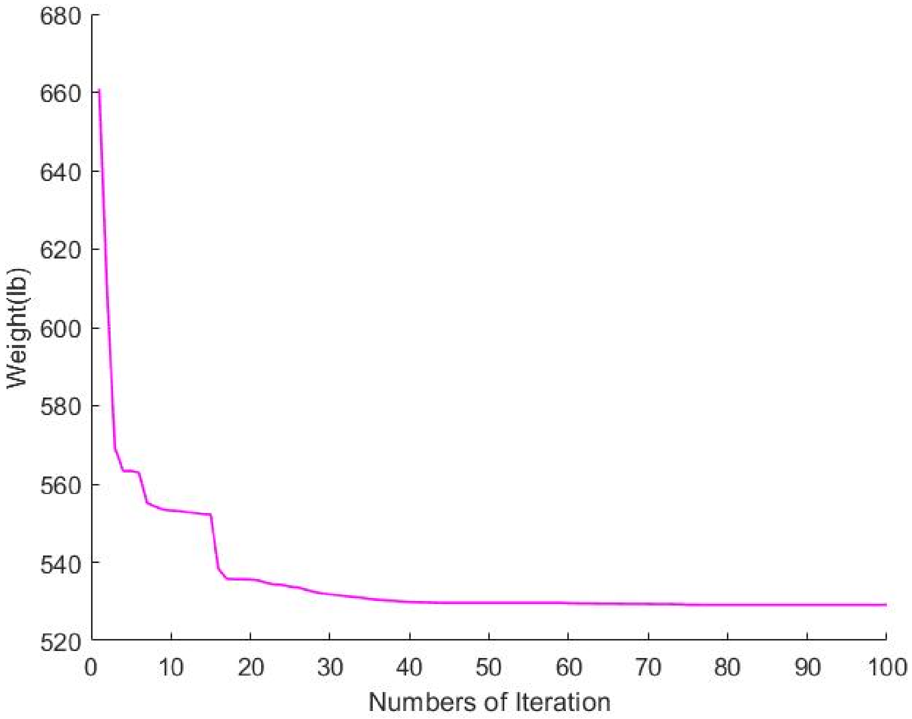


Figure 12. Solution convergence of 25-bar truss structure by TFWO method

Table 3. Optimum area and weight solutions computed various design method

Design Variables	Coello et al. (Coello et al., 1994)	Li et al. (Li et al., 2007)	TLBO (Camp & Farshchin, 2014)	Camp (Camp, 2007)	Kaveh and Shojaee (Camp & Bichon, 2004)	FPA (Bekdaş et al., 2015)	Present Study
	GA	HPSO		BB-BC	ACO		TFWO
A_1	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100
A_2	2.0119	1.9700	1.9878	2.0920	2.0000	1.8300	1.6569
A_3	2.9493	3.0160	2.9914	2.9640	2.9660	3.1834	2.7232
A_4	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100
A_5	0.0295	0.0100	0.0100	0.0100	0.0120	0.0100	0.0100
A_6	0.6838	0.6940	0.6828	0.6890	0.6890	0.7017	0.8169
A_7	1.6798	1.6810	1.6764	1.6010	1.6790	1.7266	1.1771
A_8	2.6759	2.6430	2.6656	2.6860	2.6680	2.5713	3.3095
Weight (lb)	545.80	545.19	545.18	545.38	545.53	545.16	524.35
Average Weight (lb)	-	-	545.48	545.78	546.34	545.73	529.784
No of Structural Analyses	-	125,000	12,199	20,566	16,500	8,149	3,300

4.4 200-Bar Truss Structure

The third design example regards the planar 200 bar truss structure shown in Fig. 13. All design member areas were categorized into 29 design groups. Nodal coordinates and end nodes of the members are listed in Appendix. The density of the material is 0.283 lb/in^3 and the modulus of elasticity is 30,000 ksi. The allowable stress for all member of the structure is ± 10 ksi in both tension and compression. The truss is subjected to three loading cases: (1) 1 kip acting in the positive x-direction at node 1, 6, 15, 20, 29, 34, 43, 48, 57, 62, and 71; (2) 10 kips acting in the negative y-direction at nodes 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 70, 71, 72, 72, 73, 74, and 75; (3) Load cases (1) and (2) acting together.

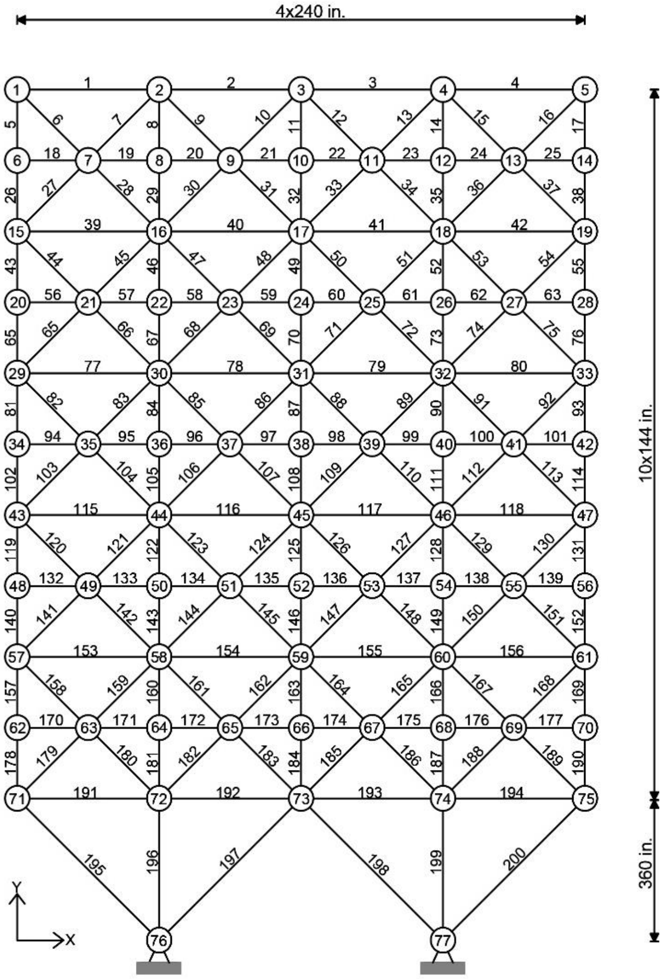


Figure 13. 200-bar truss structure.

The areas variables are selected from the set available sections consisting of the discrete areas of A = [0.1, 0.347, 0.44, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.8, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.3, 10.85, 13.33, 14.29, 17.17, 19.18, 23.68, 28.08, 33.7] (in^2). The optimal sizing design of this truss structure was successfully performed by the TFWO method with the total of 20 independent runs and the population of 63 particles. The optimal solutions see Fig. 14, converged at early number of analysis iterations. The TFWO found the best solution before the 180 number of iterations.

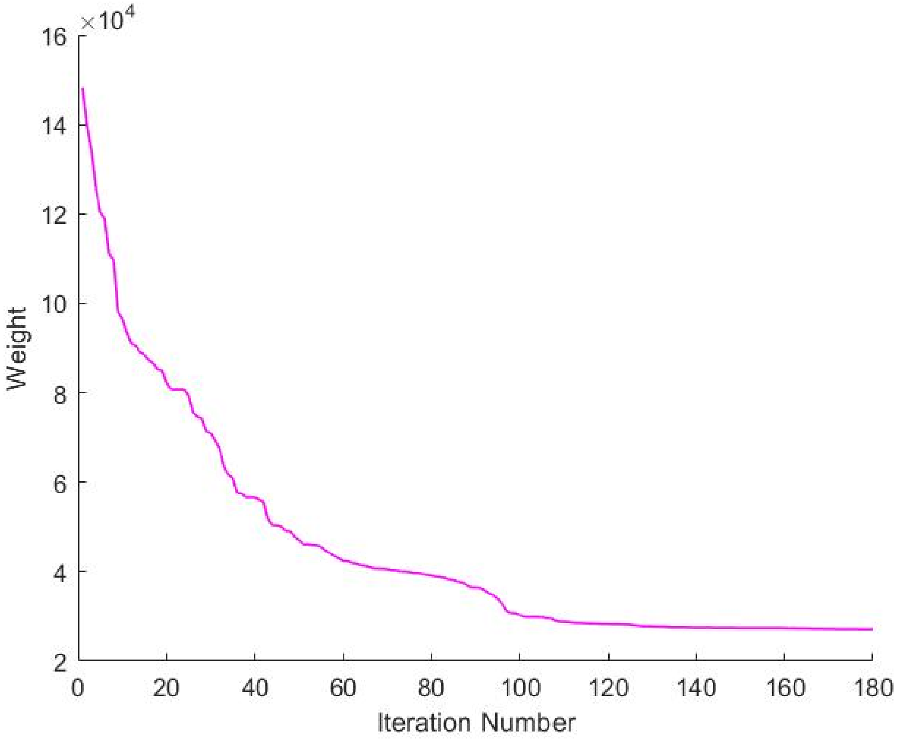


Figure 14. Solution convergence of 200-bar truss structure by TFWO method

The resulting member sizes of all 29-design groups and all the total weight of W = 27129.849lb are reported in Table 4, where the solutions from various analysis method are also compared. It is proved that the optimal design result computed by the present method achieved the most minimum as compared to all other benchmarks, namely 28075.4 lb in EASS (Azad & Hasançebi, 2014), 28544.014 lb in GA (Toğan & Daloğlu, 2008), 27163.59 lb HHS (Cheng, Prayogo, Wu, & Lukito, 2016), and 27282.57 lb in DAJA (S. Degertekin, Lamberti, & Ugur, 2019). More explicitly, the result of GA, HHS and DAJA obtains a mean weight of 28544.014, 28470.1lb and 28425.8 lb which are around 2% higher than that of proposed solution).

Table 4 Comparisons of optimization results for various analysis approaches

Design Variables	EASS (Azad & Hasançebi, 2014)	GA (Toğan & Daloğlu, 2008)	HHS (Cheng et al., 2016)	DAJA (S. Degertekin et al., 2019)	Present Study (TFWO)
A_{1}	0.1	0.347	0.1	0.1	0.1
A_{2}	0.954	1.081	0.954	0.954	0.954
A_{3}	0.1	0.1	0.1	0.347	0.1
A_{4}	0.1	0.1	0.1	0.1	0.1
A_{5}	2.142	2.142	2.142	2.142	2.697
A_{6}	0.347	0.347	0.347	0.347	0.1
A_{7}	0.1	0.1	0.1	0.1	0.1
A_{8}	3.131	3.565	3.131	3.131	2.8
A_{9}	0.1	0.347	0.1	0.1	0.539
A_{10}	4.805	4.805	4.805	4.805	3.813
A_{11}	0.347	0.44	0.44	0.44	0.539
A_{12}	0.1	0.44	0.347	0.347	0.1
A_{13}	5.952	5.952	5.952	5.952	4.805
A_{14}	0.1	0.347	0.347	0.347	0.1
A_{15}	6.572	6.572	6.572	6.572	6.572
A_{16}	0.44	0.954	0.954	0.954	0.539
A_{17}	0.539	0.347	0.347	0.1	2.142
A_{18}	7.192	8.525	8.525	8.525	7.192
A_{19}	0.44	0.1	0.1	0.539	0.1
A_{20}	8.525	9.3	9.3	9.3	8.525
A_{21}	0.954	0.954	1.081	0.954	1.488
A_{22}	1.174	1.764	0.347	0.1	0.347
A_{23}	10.85	13.33	13.33	13.33	10.85
A_{24}	0.44	0.347	0.954	0.1	0.954
A_{25}	10.85	13.33	13.33	13.33	13.33
A_{26}	1.764	2.142	1.764	0.954	1.488
A_{27}	8.525	4.805	3.813	5.952	4.805
A_{28}	13.33	9.3	8.525	10.85	9.3
A_{29}	13.33	17.17	17.17	14.29	14.29
Best Weight (lb)	28075.4	28544.014	27163.59	27282.57	27129.849
Average Weight (lb)	N/A	28470.1	28425.8	28780.12	28087.1
No. of Structural Analyses	11,156	51,360	5,000	4,693	9,450

4.5 160-Bar Truss Structure

The design of 160-bar space truss structure, shown in Fig. 15. which was considered as a large problem to illustrate the capability of the TFWO algorithm. The members' linkage and the nodal coordinate of this truss is given in Appendix. The truss material density and Young's modulus are ρ= 0.00785 kg/ cm^3 and E = 2.047 x10^6 kgf/cm^2, respectively. The design variables are the cross-sectional areas of the truss member which are linked into 38 groups. The areas variables are selected from the 42 prescribed discrete sections set of A = {1.84, 2.26, 2.66, 3.07, 3.47, 3.88, 4.79, 5.27, 5.75, 6.25, 6.84, 7.44, 8.06, 8.66, 9.40, 10.47, 11.38, 12.21, 13.79, 15.39, 17.03, 19.03, 21.12, 23.20, 25.12, 27.50, 29.88, 32.76, 33.90, 34.77, 39.16, 43.00, 45.65, 46.94, 51.00, 52.10, 61.82, 61.90, 68.30, 76.38, 90.60, 94.13} cm^2. Eight independent load cases assumed are shown in Table 5.

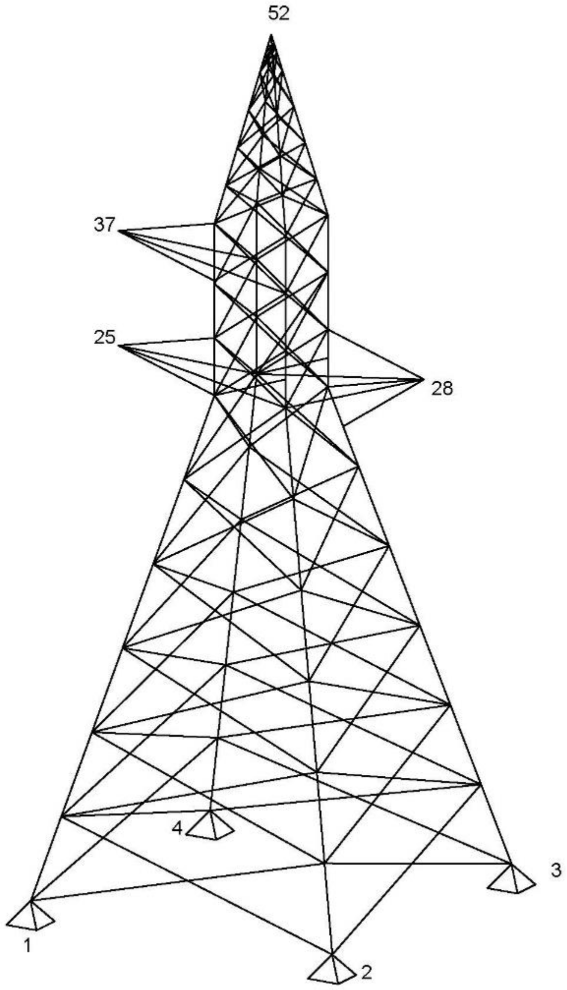


Figure 15. 160-bar transmissions tower truss structure.

Table 5 Eight Cases of Load distribution on nodes for the 160-bar truss

Load Case	Node	Px (N)	Py (N)	Pz (N)	Load Case	Node	Px (N)	Py (N)	Pz (N)
1	52	-868	0	-491	5	52	-917	0	-491
	37	-996	0	-546		37	-951	0	-546
	25	-1091	0	-546		25	-1015	0	-546
	28	-1091	0	-546		28	-636	1259	-428
2	52	-493	1245	-363	6	52	-917	0	-491
	37	-996	0	-546		37	-572	1303	-428
	25	-1091	0	-546		25	-1015	0	-546
	28	-1091	0	-546		28	-1015	0	-546
3	52	-917	0	-491	7	52	-917	0	-491
	37	-951	0	-546		37	-951	0	-546
	25	-1015	0	-546		25	-1015	0	-546
	28	-1015	0	-546		28	-636	1303	-428
4	52	-917	0	-546	8	52	-498	1460	-363
	37	-572	1259	-428		37	-951	0	-546
	25	-1015	0	-546		25	-1015	0	-546
	28	-1015	0	-546		28	-1015	0	-546

The truss members are subjected to the stress limits of ± 1500 kg/cm^2 (tension and compression members). Moreover, the constraints for all members under compressive stress, the buckling stress limitation are taken into account. Buckling stress is evaluated as follows:

$$\sigma_b = \begin{cases} 1300 - \frac{(\frac{kl}{r})^2}{24} & if \ \frac{kl}{r} \le 120 \\ \frac{10^7}{(\frac{kl}{r})^2} & if \ \frac{kl}{r} > 120 \end{cases} \tag{41}$$

where l is the length of the member, r is the corresponding radius of gyration, and k is the effective length factor fixed as 1 for all members. The corresponding radius of gyration for the prescribed discrete solutions are r = {0.47, 0.57, 0.67, 0.77, 0.87, 0.97, 0.97, 1.06, 1.16, 1.26, 1.15, 1.26, 1.36, 1.46, 1.35, 1.36, 1.45, 1.55, 1.75, 1.95, 1.74, 1.94, 2.16, 2.36, 2.57, 2.35, 2.56, 2.14, 2.33, 2.97, 2.54, 2.93, 2.94, 2.94, 2.92, 3.54, 3.96, 3.52, 3.51, 3.93, 3.92, 3.92}cm (Groenwold & Stander, 1997).

Fig. 16 shows the weight convergence history of the 160-bar truss structure. It found the maximum number of iterations is 200 and the number of structural analyses is 16800 for TFWO algorithm.

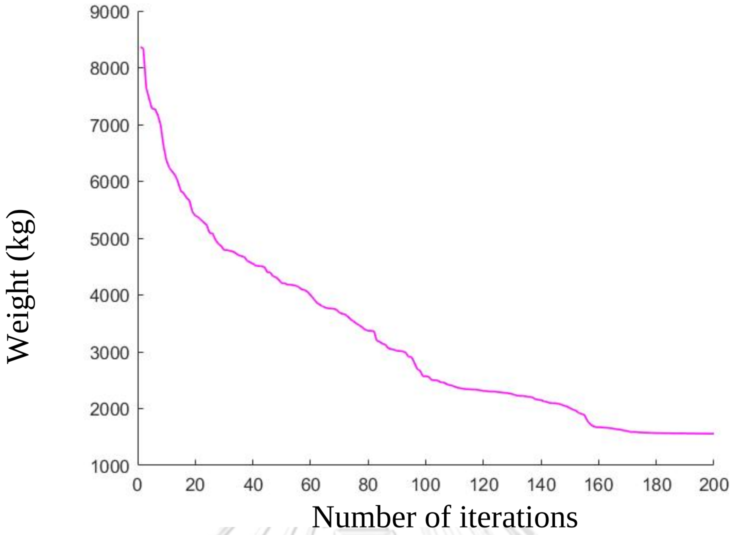


Figure 16. Convergence histories for the 160-bar planar truss

The present TFWO method is successfully designed in optimal solutions which reported in Table 6 include the best, worst, and mean solutions of the 160-bar truss structure. Table 6 also represents the direct comparison to some other benchmarks: RGA (Groenwold, Stander, & Snyman, 1999), RBAS (Capriles, Fonseca, Barbosa, & Lemonge, 2007), aeDE (Ho-Huu, Nguyen-Thoi, Vo-Duy, & Nguyen-Trang, 2016), EFA (Le, Bui, Ngo, Nguyen, & Nguyen-Xuan, 2019), Jaya (R. Rao, 2016), and SAMP-Jaya (R. V. Rao & Saroj, 2017). The minimum weight is obtained 1,337.69 lb from the proposed method, similar to the results of other algorithm, which is 0.1 % lighter than Rank-based Ant Colony algorithms (RBAS) (Capriles et al., 2007), Regional Genetic Algorithm (RGA) (Groenwold et al., 1999). In detail, comparing the best solutions of IS-Jaya design variables with optimal sets of design variables obtaining from the proposed TFWO, there are a few different design variables: A_{19}, A_{20}, A_{21}, and A_{31}. While comparing the number of structural analyses, the TFWO is considerably more efficient than other algorithms except IS-Jaya.

Table 6 Comparison of optimal design for the 160-bar truss

Design Variables	RGA	RBAS	aeDE	EFA	Jaya	SAMP-Jaya	IS-Jaya	Present Study
A_1	19.03	19.03	19.03	19.03	19.03	19.03	19.03	19.03
A_2	5.27	5.27	5.27	5.27	5.27	5.27	5.27	5.27
A_3	19.03	19.03	19.03	19.03	19.03	19.03	19.03	19.03
A_4	5.27	5.27	5.27	5.27	5.27	5.27	5.27	5.27
A_5	19.03	19.03	19.03	19.03	19.03	19.03	19.03	19.03
A_6	5.75	5.75	5.75	5.75	5.75	5.75	5.75	5.75
A_7	15.39	15.39	15.39	15.39	15.39	15.39	15.39	15.39
A_8	5.75	5.75	5.75	5.75	5.75	5.75	5.75	5.75
A_9	13.79	13.79	13.79	13.79	13.79	13.79	13.79	13.79
A_{10}	5.75	5.75	5.75	5.75	5.75	5.75	5.75	5.75
A_{11}	5.75	5.75	5.75	5.75	5.75	5.75	5.75	5.75
A_{12}	13.79	12.21	12.21	12.21	12.21	12.21	12.21	12.21
A_{13}	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25
A_{14}	5.75	5.75	5.75	5.75	5.75	5.75	5.75	5.75
A_{15}	2.66	3.47	3.88	3.88	3.88	3.47	3.88	3.88
A_{16}	7.44	7.44	7.44	7.44	7.44	7.44	7.44	7.44
A_{17}	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
A_{18}	8.66	9.4	8.66	8.66	8.66	8.66	8.66	8.66
A_{19}	2.66	2.66	2.66	2.66	2.66	2.66	2.66	3.07
A_{20}	3.07	3.47	3.07	3.07	3.07	3.07	3.07	2.66
A_{21}	2.66	3.07	2.66	2.66	2.66	2.66	3.47	1.84
A_{22}	8.06	8.06	8.06	8.06	8.06	8.06	8.06	8.06
A_{23}	5.27	5.27	5.75	5.75	5.75	5.75	5.75	5.75
A_{24}	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25
A_{25}	5.75	5.75	5.75	6.25	6.25	5.75	5.75	6.25
A_{26}	1.84	2.26	2.26	1.84	1.84	2.26	2.26	2.26
A_{27}	4.79	4.79	4.79	4.79	4.79	4.79	4.79	4.79
A_{28}	2.66	3.07	2.66	2.66	2.66	2.66	2.66	2.66
A_{29}	3.47	3.47	3.47	3.47	3.47	3.47	3.47	3.47
A_{30}	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
A_{31}	2.26	3.88	2.26	2.26	2.26	2.26	2.26	3.88
A_{32}	3.88	3.88	3.88	3.88	3.88	3.88	3.88	3.88
A_{33}	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
A_{34}	1.84	2.26	1.84	1.84	1.84	1.84	1.84	1.84
A_{35}	3.88	3.88	3.88	3.88	3.88	3.88	3.88	3.88
A_{36}	1.84	2.26	1.84	1.84	1.84	1.84	1.84	1.84
A_{37}	1.84	3.47	1.84	1.84	1.84	1.84	1.84	2.26
A_{38}	3.88	3.88	3.88	3.88	3.88	3.88	3.88	3.88
Best Weight (kg)	1337.44	1348.90	1336.63	1336.70	1336.70	1337.04	1336.63	1337.93
Average Weight (kg)	N/A	1367.52	1355.875	1372.55	1356.54	1355.33	1342.81	1350.4
No. of Structural Analyses	N/A	90,000	23,925.00	16,870	18,160	17,780	11,740	16,800

4.6 72-Bar Truss Structure

The last example is a 72-bar space truss structure shown in Fig. 17. Model data for 72-bar truss can be seen in Appendix. This problem has been considered in many researchers including Wu et al. (Wu & Chow, 1995), Kaveh et al. (A Kaveh & Talatahari, 2009), Li et al. (Li et al., 2007), Sadollah et al. (Sadollah, Bahreininejad, Eskandar, & Hamdi, 2012), Kaveh et al. (A Kaveh & Mahdavi, 2014), (Ho-Huu et al., 2016), Sadollah et al. (Sadollah et al., 2012), (Le et al., 2019), (Le et al., 2019) and etc. The material density is 0.1 lb/in3 and the modulus of elasticity is 104 ksi. The truss members are subjected to the stress limits of ± 25,000 psi. All nodes are subjected to the displacement limits of ±0.25 in. The design variables are categorized into 16 groups: (1) A1-A4, (2) A5-A12, (3) A13-A16, (4) A17-A18, (5)A19-A22, (6) A23-A30, (7) A31-A34, (8) A35-A36, (9) A37-A40, (10) A41-A48, (11) A49-A52, (12) A53-A54, (13) A55-A58, (14) A59-A66, (15) A67-A70, and (16) A71-A72. The design variables are chosen from Table 7. The design forces applied were 5 kips at node 17 in both positive x and y direction and in the negative z-directions.

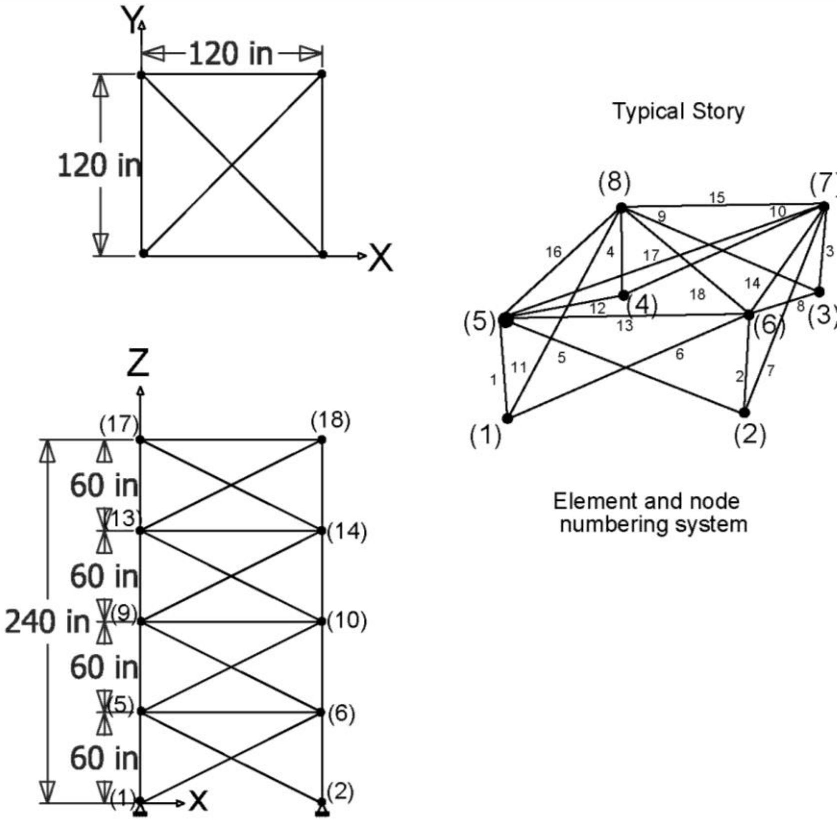


Figure 17. 72-bar space truss structure

Table 7 The cross-sectional areas values from AISC code

No.	in^2	No.	in^2	No.	in^2	No.	in^2
1	0.111	17	1.563	33	3.840	49	11.500
2	0.141	18	1.620	34	3.870	50	13.500
3	0.196	19	1.800	35	3.880	51	13.900
4	0.250	20	1.990	36	4.180	52	14.200
5	0.307	21	2.130	37	4.220	53	15.500
6	0.391	22	2.380	38	4.490	54	16.000
7	0.442	23	2.620	39	4.590	55	16.900
8	0.563	24	2.630	40	4.800	56	18.800
9	0.602	25	2.880	41	4.970	57	19.900
10	0.766	26	2.930	42	5.120	58	22.000
11	0.785	27	3.090	43	5.740	59	22.900
12	0.994	28	3.130	44	7.220	60	24.500
13	1.000	29	3.380	45	7.970	61	26.500
14	1.228	30	3.470	46	8.530	62	28.000
15	1.266	31	3.550	47	9.300	63	30.000
16	1.457	32	3.630	48	10.850	64	33.500

The optimal sizing design of the truss was successfully performed by the proposed TFWO method with the total independent 20 runs and the population of 33 objects. The convergence rate of the 72-bar truss can be seen in Fig. 18, the proposed method obtained the best solution before 80 iterations. All solution of (SGA) Wu et al. (Wu & Chow, 1995), (DHPSACO) Wu et al. (Wu & Chow, 1995), (HPSO) Li et al. (Li et al., 2007), (MBA) Sadollah et al. (Sadollah et al., 2012), (CBO) Kaveh et al. (A Kaveh & Mahdavi, 2014), (aeDE) (Ho-Huu et al., 2016) , (IMBA) Sadollah et al. (Sadollah et al., 2012), (EFA) (Le et al., 2019), (IS-Jaya) (Ali Kaveh, Hosseini, & Zaerreza, 2021), and TFWO are shown in Table 8. Based on the results obtained in Table 8, the total weight of W = 377.886 lb outperformed the best optimal design by proposed method after 2640 structural analyses. It is also seen that DHPSACO, MBA, CBO, aeDE, IMBA, EFA, and IS-Jaya (Ali Kaveh et al., 2021) outweighed TFWO algorithm by more than 3% weights solution. In this example, HPSO shows the worst performance which is about 60% heavier than the TFWO method.

Table 8 Optimum results for various design method

Design Variables	Wu et al. (Wu & Chow, 1995)	Kaveh et al. (A Kaveh & Talatahari, 2009)	Li et al. (Li et al., 2007)	Sadollah et al. (Sadollah et al., 2012)	Kaveh et al. (A Kaveh & Mahdavi, 2014)	(Ho-Huu et al., 2016)	Sadollah et al. (Sadollah et al., 2012)	(Le et al., 2019)	(Ali Kaveh et al., 2021)	Present Study
	SGA	DHPSACO	HPSO	MBA	CBO	aeDE	IMBA	EFA	IS-Jaya	TFWO
A_1	0.196	1.800	4.970	0.196	1.620	1.990	1.990	1.990	1.990	2.130
A_2	0.602	0.442	1.228	0.563	0.563	0.563	0.442	0.563	0.563	0.442
A_3	0.307	0.141	0.111	0.442	0.111	0.111	0.111	0.111	0.111	0.111
A_4	0.766	0.111	0.111	0.602	0.111	0.111	0.111	0.111	0.111	0.111
A_5	0.391	1.228	2.888	0.442	1.457	1.228	1.228	1.228	1.228	1.130
A_6	0.391	0.563	1.457	0.442	0.442	0.442	0.563	0.442	0.563	0.442
A_7	0.141	0.111	0.141	0.111	0.111	0.111	0.111	0.111	0.111	0.111
A_8	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111
A_9	1.800	0.563	1.563	1.266	0.602	0.563	0.563	0.563	0.563	0.602
A_{10}	0.602	0.563	1.228	0.563	0.563	0.563	0.563	0.563	0.442	0.563
A_{11}	0.141	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111
A_{12}	0.307	0.250	0.196	0.111	0.111	0.111	0.111	0.111	0.111	0.111
A_{13}	1.563	0.196	0.391	1.800	0.196	0.196	0.196	0.196	0.196	0.111
A_{14}	0.767	0.563	1.457	0.602	0.602	0.563	0.563	0.563	0.563	0.563
A_{15}	0.141	0.442	0.766	0.111	0.391	0.391	0.391	0.391	0.391	0.307
A_{16}	0.111	0.563	1.563	0.111	0.563	0.563	0.563	0.563	0.563	0.602
Weight (lb)	427.030	393.380	933.090	390.730	391.070	389.334	389.334	389.334	389.334	377.886
Average Weight (lb)	-	-	-	395.432	403.710	390.913	389.823	395.112	389.936	398.423
No of Structural Analyses	60000	125000	50000	11600	4500	4160	6250	3740	2680	2640

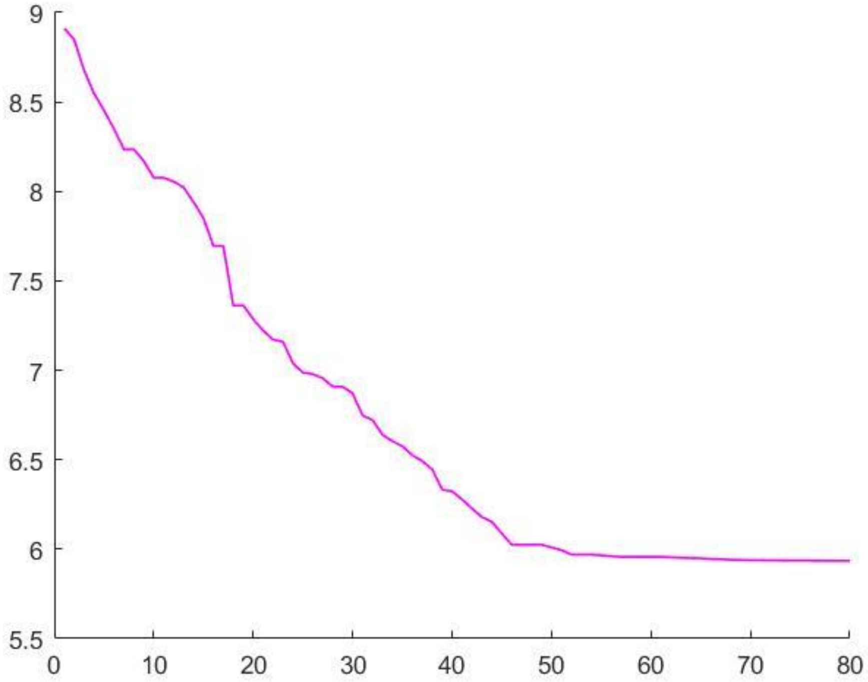


Figure 18 Solution convergence in the TFWO process of 72 bar truss

CHAPTER 5
CONCLUSIONS

5.1 Conclusions

This research presented a new and effective grouping optimization algorithm, namely the Turbulent Flow of Water-based Optimization (TFWO) algorithm. In the present study, the proposed algorithm has been presented for the optimal sizing design of steel truss structures under applied forces. This strategy is being investigated to solve benchmark optimization problems. The present algorithm applied a penalty method, convergence criteria based on the absolute deviation of the best and the mean objective function values of the population, and a method for handling discrete variables to solve five optimization problems of truss structure. These examples include 10-bar truss, 25-bar truss, 200-bar truss, 160-bar truss and 72-bar truss structures. In 10-bar truss, 25-bar truss, and 72-bar truss examples, the numerical results indicated that the TFWO obtained the best optimum weight solutions with less iterations than other metaheuristic algorithms. Furthermore, the TFWO algorithm requires a smaller number of structural analyses than other methods. This demonstrated the proposed TFWO's robustness, not only in terms of accuracy, but also in terms of convergence speed. In 200-bar design examples, the present algorithm can achieve the best optimum weight among other methods. However, in the 200-bar truss design, the proposed TFWO algorithm required a few higher structural analyses than HHS and DAJA. In the 160-bar truss example, the proposed algorithm achieved the second lowest weight when compared with other algorithms, although it required more analyses than IS-Jaya. In general, the TFWO results proved that the proposed scheme has competitive performance and robustness when compared to other optimization algorithms. In essence, the minimum weight of all members employed as illustrated in this paper is evidenced. All the above demonstrate that the proposed TFWO is an effective, robust, and reliable optimization algorithm for dealing with different constrained optimization problems of truss structures with continuous and discrete design variables. This study can provide efficient and effective solutions to many kinds of purposefully arranged optimization problems.

5.2 Recommendation

Based on the work in the present thesis and the results obtained, the following may be valid points for future research:

1. This proposed method should be investigated to determine its performance in more advanced structural optimization such as shape and topology optimization.
2. The algorithm can be extended to various engineering optimization problems, such as optimization of frames, composite plates/shell structures, and stiffened plates and shell structures.

APPENDIX

Model Data for Truss Structures

25 bar Truss: element data

Elem. No.	Nodes		Elem. No.	Nodes	
	1st	2nd		1st	2nd
1	1	2	14	3	10
2	1	4	15	6	7
3	2	3	16	4	9
4	1	5	17	5	8
5	2	6	18	3	8
6	2	5	19	4	7
7	2	4	20	6	9
8	1	3	21	5	10
9	1	6	22	3	7
10	3	6	23	4	8
11	4	5	24	5	9
12	3	4	25	6	10
13	5	6			

25 Truss: Nodal coordinates data

Node	X-axis	Y-axis	Z-axis
1	37.5	0	200
2	37.5	0	200
3	-37.5	37.5	100
4	37.5	37.5	100
5	37.5	-37.5	100
6	-37.5	-37.5	100
7	-100	100	0
8	100	100	0
9	100	-100	0
10	-100	-100	0

72-bar Truss: element data

No. of bars	Node 1	Node 2	No. of bars	Node 1	Node 2	No. of bars	Node 1	Node 2
1	1	5	25	7	10	49	13	14
2	2	6	26	6	11	50	14	15
3	3	7	27	7	12	51	15	16
4	4	8	28	8	11	52	16	13
5	1	6	29	8	9	53	13	15
6	2	5	30	5	12	54	14	16
7	3	6	31	9	10	55	13	17
8	2	7	32	10	11	56	14	18
9	3	8	33	11	12	57	15	19
10	4	7	34	12	9	58	16	20
11	4	5	35	9	11	59	13	18
12	1	8	36	10	12	60	14	17
13	5	6	37	9	13	61	15	18
14	6	7	38	10	14	62	14	19
15	7	8	39	11	15	63	15	20
16	8	5	40	12	16	64	16	19
17	5	7	41	9	14	65	16	17
18	6	8	42	10	13	66	13	20
19	5	9	43	11	14	67	17	18
20	6	10	44	10	15	68	18	19
21	7	11	45	11	16	69	19	20
22	8	12	46	12	15	70	20	17
23	5	10	47	12	13	71	17	19
24	6	9	48	9	16	72	18	20

72-bar Truss: Nodal coordinates data

0	X-coordinate	Y-Coordinate	Z-Coordinate
1	0	0	0
2	120	0	0
3	120	120	0
4	0	120	0
5	0	0	60
6	120	0	60
7	120	120	60
8	0	120	60
9	0	0	120
10	120	0	120
11	120	120	120
12	0	120	120
13	0	0	180
14	120	0	180
15	120	120	180
16	0	120	180
17	0	0	240
18	120	0	240
19	120	120	240
20	0	120	240

200-bar Truss: Nodal coordinates data

Node	X-axis	Y-axis	Node	X-axis	Y-axis	Node	X-axis	Y-axis
1	0	1800	27	840	1368	53	600	792
2	240	1800	28	960	1368	54	720	792
3	480	1800	29	0	1224	55	840	792
4	720	1800	30	240	1224	56	960	792
5	960	1800	31	480	1224	57	0	648
6	0	1656	32	720	1224	58	240	648
7	120	1656	33	960	1224	59	480	648
8	240	1656	34	0	1080	60	720	648
9	360	1656	35	120	1080	61	960	648
10	480	1656	36	240	1080	62	0	504
11	600	1656	37	360	1080	63	120	504
12	720	1656	38	480	1080	64	240	504
13	840	1656	39	600	1080	65	360	504
14	960	1656	40	720	1080	66	480	504
15	0	1512	41	840	1080	67	600	504
16	240	1512	42	960	1080	68	720	504
17	480	1512	43	0	936	69	840	504
18	720	1512	44	240	936	70	960	504
19	960	1512	45	480	936	71	0	360
20	0	1368	46	720	936	72	240	360
21	120	1368	47	960	936	73	480	360
22	240	1368	48	0	792	74	720	360
23	360	1368	49	120	792	75	960	360
24	480	1368	50	240	792	76	240	0
25	600	1368	51	360	792	77	720	0
26	720	1368	52	480	792			

200-bar Truss: element data

Elem. No.	Nodes		x_j	Elem. No.	Nodes		x_j	Elem. No.	Nodes		x_j	Elem. No.	Nodes		x_j
	1^{st}	2^{nd}			1^{st}	2^{nd}			1^{st}	2^{nd}			1^{st}	2^{nd}	
1	1	2	1	43	20	15	8	111	46	40	15	147	59	53	21
2	2	3	1	46	22	16	8	114	47	42	15	148	53	60	21
3	3	4	1	49	24	17	8	82	29	35	16	150	60	55	21
4	4	5	1	52	26	18	8	83	35	30	16	151	55	61	21
5	6	1	2	55	28	19	8	85	30	37	16	153	57	58	22
8	8	2	2	57	21	22	9	86	37	31	16	154	58	59	22
11	10	3	2	58	22	23	9	88	31	39	16	155	59	60	22
14	12	4	2	59	23	24	9	89	39	32	16	156	60	61	22
17	14	5	2	60	24	25	9	91	32	41	16	157	62	57	23
19	7	8	3	61	25	26	9	92	41	33	16	160	64	58	23
20	8	9	3	62	26	27	9	103	43	35	16	163	66	59	23
21	9	10	3	64	29	20	10	104	35	44	16	166	68	60	23
22	10	11	3	67	30	22	10	106	44	37	16	169	70	61	23
23	11	12	3	70	31	24	10	107	37	45	16	171	63	64	24
24	12	13	3	73	32	26	10	109	45	39	16	172	64	65	24
18	6	7	4	76	33	28	10	110	39	46	16	173	65	66	24
25	13	14	4	44	15	21	11	112	46	41	16	174	66	67	24
56	20	21	4	45	21	16	11	113	41	47	16	175	67	68	24
63	27	28	4	47	16	23	11	115	43	44	17	176	68	69	24
94	34	35	4	48	23	17	11	116	44	45	17	178	71	62	25
101	41	42	4	50	17	25	11	117	45	46	17	181	72	64	25
132	48	49	4	51	25	18	11	118	46	47	17	184	73	66	25
139	55	56	4	53	18	27	11	119	48	43	18	187	74	68	25
170	62	63	4	54	27	19	11	122	50	44	18	190	75	70	25
177	69	70	4	65	29	21	11	125	52	45	18	158	57	63	26
26	15	6	5	66	21	30	11	128	54	46	18	159	63	58	26
29	16	8	5	68	30	23	11	131	56	47	18	161	58	65	26
32	17	10	5	69	23	31	11	133	49	50	19	162	65	59	26
35	18	12	5	71	31	25	11	134	50	51	19	164	59	67	26
38	19	14	5	72	25	32	11	135	51	52	19	165	67	60	26
6	1	7	6	74	32	27	11	136	52	53	19	167	60	69	26
7	7	2	6	75	27	33	11	137	53	54	19	168	69	61	26
9	2	9	6	77	29	30	12	138	54	55	19	179	71	63	26
10	9	3	6	78	30	31	12	140	57	48	20	180	63	72	26
12	3	11	6	79	31	32	12	143	58	50	20	182	72	65	26
13	11	4	6	80	32	33	12	146	59	52	20	183	65	73	26
15	4	13	6	81	34	29	13	149	60	54	20	185	73	67	26
16	13	5	6	84	36	30	13	152	61	56	20	186	67	74	26
27	15	7	6	87	38	31	13	120	43	49	21	188	74	69	26
28	7	16	6	90	40	32	13	121	49	44	21	189	69	75	26
30	16	9	6	93	42	33	13	123	44	51	21	191	71	72	27
31	9	17	6	95	35	36	14	124	51	45	21	192	72	73	27
33	17	11	6	96	36	37	14	126	45	53	21	193	73	74	27
34	11	18	6	97	37	38	14	127	53	46	21	194	74	75	27
36	18	13	6	98	38	39	14	129	46	55	21	195	71	76	28
37	13	19	6	99	39	40	14	130	55	47	21	197	76	73	28
39	15	16	7	100	40	41	14	141	57	49	21	198	73	77	28
40	16	17	7	102	43	34	15	142	49	58	21	200	77	75	28
41	17	18	7	105	44	36	15	144	58	51	21	196	76	72	29
42	18	19	7	108	45	38	15	145	51	59	21	199	77	74	29

160-bar Truss: element data

Elem. No.	Nodes		x_j	Elem. No.	Nodes		x_j	Elem. No.	Nodes		x_j	Elem. No.	Nodes		x_j
	1st	2nd			1st	2nd			1st	2nd			1st	2nd	
1	1	5	1	41	13	18	8	81	25	31	17	121	36	40	29
2	2	6	1	42	14	17	8	82	28	32	17	122	38	41	29
3	3	7	1	43	14	19	8	83	28	33	17	123	39	42	29
4	4	8	1	44	15	18	8	84	25	34	17	124	35	43	29
5	1	6	2	45	15	20	8	85	26	31	18	125	40	41	30
6	2	5	2	46	16	19	8	86	27	32	18	126	41	42	30
7	2	7	2	47	16	17	8	87	29	33	18	127	42	43	30
8	3	6	2	48	13	20	8	88	30	34	18	128	43	40	30
9	3	8	2	49	17	21	9	89	26	32	19	129	35	36	31
10	4	7	2	50	18	22	9	90	27	31	19	130	36	38	31
11	4	5	2	51	19	23	9	91	29	34	19	131	38	39	31
12	1	8	2	52	20	24	9	92	30	33	19	132	39	35	31
13	5	9	3	53	17	22	10	93	27	33	20	133	40	44	32
14	6	10	3	54	18	21	10	94	29	32	20	134	41	45	32
15	7	11	3	55	19	24	10	95	30	31	20	135	42	46	32
16	8	12	3	56	20	23	10	96	26	34	20	136	43	47	32
17	5	10	4	57	18	23	11	97	26	29	21	137	40	45	33
18	6	9	4	58	19	22	11	98	27	30	21	138	41	46	33
19	6	11	4	59	20	21	11	99	31	35	22	139	42	47	33
20	7	10	4	60	17	24	11	100	32	36	22	140	43	44	33
21	7	12	4	61	21	26	12	101	33	38	22	141	44	45	34
22	8	11	4	62	22	27	12	102	34	39	22	142	45	46	34
23	8	9	4	63	23	29	12	103	33	39	23	143	46	47	34
24	5	12	4	64	24	30	12	104	32	35	23	144	44	47	34
25	9	13	5	65	21	27	13	105	31	36	23	145	44	48	35
26	10	14	5	66	22	26	13	106	34	38	23	146	45	49	35
27	11	15	5	67	23	30	13	107	32	38	24	147	46	50	35
28	12	16	5	68	24	29	13	108	33	36	24	148	47	51	35
29	9	14	6	69	22	29	14	109	34	35	24	149	45	48	36
30	10	13	6	70	23	27	14	110	31	39	24	150	46	49	36
31	10	15	6	71	24	26	14	111	37	35	25	151	47	50	36
32	11	14	6	72	21	30	14	112	37	39	25	152	44	51	36
33	11	16	6	73	26	27	15	113	37	40	26	153	48	49	37
34	12	15	6	74	27	29	15	114	37	43	26	154	49	50	37
35	12	13	6	75	29	30	15	115	35	40	27	155	50	51	37
36	9	16	6	76	30	26	15	116	36	41	27	156	48	51	37
37	13	17	7	77	25	26	16	117	38	42	27	157	48	52	38
38	14	18	7	78	27	28	16	118	39	43	27	158	49	52	38
39	15	19	7	79	25	30	16	119	35	38	28	159	50	52	38
40	16	20	7	80	29	28	16	120	36	39	28	160	51	52	38

160-bar Truss: Nodal coordinate data

Node	X-axis	Y-axis	Z-axis	Node	X-axis	Y-axis	Z-axis
1	-105	-105	0	27	40	-40	1027.5
2	105	-105	0	28	214	0	1027.5
3	105	105	0	29	40	40	1027.5
4	-105	105	0	30	-40	40	1027.5
5	-93.929	-93.929	175	31	-40	-40	1105.5
6	93.929	-93.929	175	32	40	-40	1105.5
7	93.929	93.929	175	33	40	40	1105.5
8	-93.929	93.929	175	34	-40	40	1105.5
9	-82.859	-82.859	350	35	-40	-40	1256.5
10	82.859	-82.859	350	36	40	-40	1256.5
11	82.859	82.859	350	37	-207	0	1256.5
12	-82.859	82.859	350	38	40	40	1256.5
13	71.156	-71.156	535	39	-40	40	1256.5
14	71.156	-71.156	535	40	-40	-40	1346.5
15	71.156	71.156	535	41	40	-40	1346.5
16	-71.156	71.156	535	42	40	40	1346.5
17	-60.085	-60.085	710	43	-40	40	1346.5
18	60.085	-60.085	710	44	-26.592	-26.592	1436.5
19	60.085	60.085	710	45	26.592	-26.592	1436.5
20	-60.085	60.085	710	46	26.592	26.592	1436.5
21	-49.805	-49.805	872.5	47	-26.592	26.592	1436.5
22	49.805	-49.805	872.5	48	-12.737	-12.737	1526.5
23	49.805	49.805	872.5	49	12.737	-12.737	1526.5
24	-49.805	49.805	872.5	50	12.737	12.737	1526.5
25	-214	0	1027.5	51	-12.737	12.737	1526.5
26	-40	-40	1027.5	52	0	0	1615

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