CHAPTER 9

MODELLING OF THE DIURNAL VARIATION OF RAIN ATTENUATION

9.1 Introduction

In order to design a reliable Ku-band satellite link, system design must concern the outage time (unavailability) of the link that may be disconnected due to propagation effects, i.e. rain, with in a services period. Assuming the unavailability of the link is 0.01%, the system design may want to know how much the attenuation value (dB) will occupy for about 0.01% of the year, so that appropriate link margin can be provided. This information can be obtained by long-term measurement of cumulative distribution of attenuation or from the well-known prediction model i.e. ITU-R 618-4[1994]. However, this information does not give useful information about when attenuation will mostly occur during a day (diurnal variation). If the diurnal variation statistics of rain attenuation are known, system design can wisely allocate the most significant traffic to avoid the most attenuation period and/or provide some suitable compensation techniques only at that frequent attenuation hour. Therefore, satellite capacity can be used more efficiently than the conventional compensation techniques that occupy satellite capacity all the time.

In this Chapter, we propose the statistical model to predict rain attenuation distribution occurred at any given hour of each day from the knowledge of 1-hour rainfall statistics and the diurnal variations of rain attenuation described in chapter 8. The proposed model will be tested with measured data over 3 years. The result is found to be applicable to the design of a satellite system with link availability between 99% -99.9%.

9.2 Modeling approach

Hydrometoes i.e., rain, clouds, melting particles may attenuate radio wave above 10 GHz. Attenuation due to rain is the most essential factor for the Ku-band satellite link design. Therefore, this model is only concentrate on the attenuation due to rain. Our model is based on the statistics of the attenuation and rainfall data corrected in Southeast Asia during 1/March/92 - 2/February/1995. The model's objective is to obtain the amount of attenuation (dB) at a given fading time (0.1% - 1%) that occupied in each 2 hour daily. The model's approach is to predict all 12 curves of each 2-hour cumulative attenuation distribution $F_{Hi}(\alpha)$ using the measured rainfall data with one hour integration time and the log-normal distribution.

9.3 Data for the Prediction Model

Most today's rain attenuation prediction models apply the rainfall rate data from a quick response rain gauge with a short integration time and from a tipping-bucket rain gauge with variable integration time. However, high rainfall rate data from a short integration time is not available in many areas of the world. The 1-hour rainfall data is available in many meteorological stations. In our analysis, we found high correlation (>0.9) between our 1-hour rainfall data and the meteorological data (see Figure 8.1). Both are also correlated with the diurnal attenuation data (see Figure 8.15 - 8.18). For these reasons, the 1 hour rainfall data can be used to predict the cumulative attenuation distribution of every hour(s). Data for testing the model consists of:

(1) the measured diurnal attenuation or P(Hi\Y),
(2) the measured 1-hour rainfall data or P(Ri\Z),
(3) the fitted log-normal distribution or $\overline{F}_Y(\alpha)$,
(4) the measured attenuation distribution or $F_Y(\alpha)$.

Due to an unreliability of the meteorological rainfall data of Si-racha, Singapore and Bundung, we decide to use our measured rainfall data at four locations and the meteorological rainfall of Bangkok to test the model.

9.4 Model Analysis

The long-term cumulative distribution of attenuation $F_Y(\alpha)$ can be expressed by:

$$F_Y(\alpha) = P(Y \geq \alpha) \quad \text{------------(9.1)}$$

It should be noted that many probability and statistical text books use $F_Y(\alpha) = P(Y \leq \alpha)$ is the cumulative distribution. The cumulative attenuation occurred in the selected i'th hour (event Hi) can be expressed by

$$F_{Hi}(\alpha) = P(Hi \geq \alpha) \qquad 1 \leq i \leq 24 \quad \text{------------(9.2)}$$

Each event Hi is mutually exclusive then

$$H_m \cap H_n = \varnothing \qquad m \neq n = 1,2,\ldots,24$$

the joint probability $P(H_m \cap H_n) = 0$

The cumulative attenuation distribution $F_Y(\alpha)$ can be expressed in term of $F_{Hi}(\alpha)$ using the probability theory :

$$F_Y(\alpha) = \sum_{i=1}^{24} P\{Y \geq \alpha, Hi \geq \alpha i\}$$

$$= \sum F_{Y,H}(\alpha, \alpha i)$$

$$= \sum_{i=1}^{24} P(Y\backslash Hi)F_{Hi}(\alpha) \quad \text{------------}(9.3)$$

where

$F_{Y,H}(\alpha,\alpha i)$ is a joint probability distribution of attenuation in both event Y and event Hi

$P(Y\backslash Hi)$ is the conditional probability of event Y given that the event Hi is known.

In our proposed model, we need to find each $F_{Hi}(\alpha)$ by knowing a statistical property of $F_Y(\alpha)$ and the diurnal variation of attenuation or $P(Hi\backslash Y)$ and it can be expressed by:

$$P(Hi\backslash Y) = P(Hi,Y)/ P(Y) \quad \text{---------------}(9.4)$$

and

$$P(Y\backslash Hi) = P(Hi,Y)/P(Hi) \quad \text{---------------}(9.5)$$

Combined equation (9.4) and (9.5) then:

$$P(Hi) = \frac{[P(Hi\backslash Y)P(Y)]}{P(Y\backslash Hi)} \quad \text{----------------------------}(9.6)$$

From our analysis we found that there is a good correlation between the diurnal variation of attenuation $P(Hi\backslash Y)$ and the diurnal variation of rainfall $P(Ri\backslash Z)$ (see Figure 8.15 - 8.18). Then, we can assume that $P(Hi\backslash Y) \approx P(Ri\backslash Z)$. $P(Y\backslash Hi)$ is the conditional probability of event Y occurred given that event Hi is known. Due to all events Hi are subset of event Y and all events Hi are mutual exclusive, then $P(Y\backslash Hi)$ can be obtained by dividing each event Hi with all events Hi which is equal to 1/n where n is the total number of divided interval of the day. Substitution of $P(Hi\backslash Y)$ by $P(Ri\backslash Z)$ and $P(Y/Hi)$ by 1/n then Equation (9.6) can be expressed by:

$$P(Hi) \approx n\, P(Ri\backslash Z)P(Y) \quad \text{---------------------------}(9.7)$$

From equation (9.7), the cumulative distribution of attenuation in each i hour interval $F_{Hi}(\alpha)$ can be obtained by:

$$F_{Hi}(\alpha) \approx n\, P(Ri\backslash Z)F_Y(\alpha) \quad \text{-------------------}(9.8)$$

Each $F_{Hi}(\alpha)$ can be obtained by knowing the number of intervals (n), the conditional probability of rainfall $P(Ri\backslash Z)$ that can be obtained from the measured rainfall data with 1 hour integration time, and the cumulative attenuation distribution $F_Y(\alpha)$ that can be obtained by the long-term measurement of attenuation or by many available prediction distributions i.e. ITU-R 618-2 model.

9.5 Calculation of A Cumulative Distribution of Rain attenuation in Each Interval ($F_{Hi}(\alpha)$)

In this section, we also provide a general procedure to calculate each P(Ai) that are described the calculation of cumulative distributions of rain attenuation in each-hours interval $F_{Hi}(\alpha)$ from an annual cumulative attenuation distribution $F_Y(\alpha)$.

When only the 1-hour rainfall data of the nearest meteorological station is available, it may be possible to obtain the cumulative distribution in each-hours interval (2,4,8,12) from the annual cumulative distribution suitable for that area by using the following procedures:

Step.1 The annual cumulative distribution $F_Y(\alpha)$ can be obtained by the long term measured data or many available prediction models that are well fitted to that particular region such as: the ITU-R 618-4 [1995], the Dissanayake & Allnutt model [1997].

Step 2 The diurnal variation of rainfall probability P(Ri\Z) can be obtained in equation (8.5). Due to it is highly dependent on the environmental condition, the measured rainfall with 1 hour integration time from the nearest meteorological station is strongly recommended.

Step 3 After P(Ri\Z) is obtained, each cumulative attenuation distribution curve $F_{Hi}(\alpha)$. can be calculated by applying equation (9.8) with n equal to a number of intervals, and $F_Y(\alpha)$ obtained from Step-1.

9.6 Comparison of the Measured and Predicted Cumulative Attenuation Distribution of Each 2-hour Intervals

Figures 9.1 - 9.4 illustrate results of measured attenuation distributions of each 2 hour interval ($F_{H1}(\alpha)$ - $F_{H12}(\alpha)$) compared among the predicted distributions by using equation (9.8). The P(Ri\Z) is obtained from the measured rainfall every 2 hours, the $F_Y(\alpha)$ is given by the log-normal distribution, and n = 12 (the number of intervals). The measured data are divided into 12 curves corresponding to the 2-hour intervals starting from 0700-0800LT (%0708LT) to 0500-0600LT (%0506LT). The measured curves are shown by the dotted line, while the predicted curves are shown by the solid line. The vertical scale is the percentage time that the measured attenuation exceeds the thresholds (the abscissa). It should be noted that attenuation higher than 12 dB shown in all Figures may error due to the limitation of the range of measurement.

Figure 9.1 shows a comparison between the measured and predicted distributions of Bangkok. The uppermost curve belongs to the %1718LT, %1920LT and the lowermost curve belongs to the %0708LT. When the diurnal rainfall P(Ri\Z) is used to predict $F_{Hi}(\alpha)$, they show a good fit to the measured data. However, some underestimation occurred due to incomplete correlation between P(Hi\Y) and P(Ri\Z).

In Figure 9.2, the measured curves of Si-racha are compared with the predicted curves. Corresponding to the diurnal attenuation P(Hi/Y), the uppermost curve belongs to the %1314LT while the lowermost curve belongs to the %0506LT. Due to some low correlations (0.5-0.6) between P(Hi\Y) and (Ri\Z) (see Figure 8.16), the predicted curves underestimate the measured curves especially at %1314LT.

Figure 9.3 shows a comparison between the measured distributions and the predicted distributions. All curves are corresponding to the diurnal attenuation and diurnal rainfall having the uppermost curve at %1516LT and the lowermost curve at %1920LT. In Figure 9.3, due to a good correlation (>0.95) between P(Hi\Y) and P(Ri\Z) (see Figure 8.17), the predicted curves $F_{Hi}(\alpha)$ agree well with the measured curves $F_{Hi}(\alpha)$.

Figure 9.4 shows a comparison between the measured and the predicted curves. The measured curves have the uppermost curves at %1314LT and the lowermost curve at %0506LT. The predicted curves show a good fit with the measured curves.

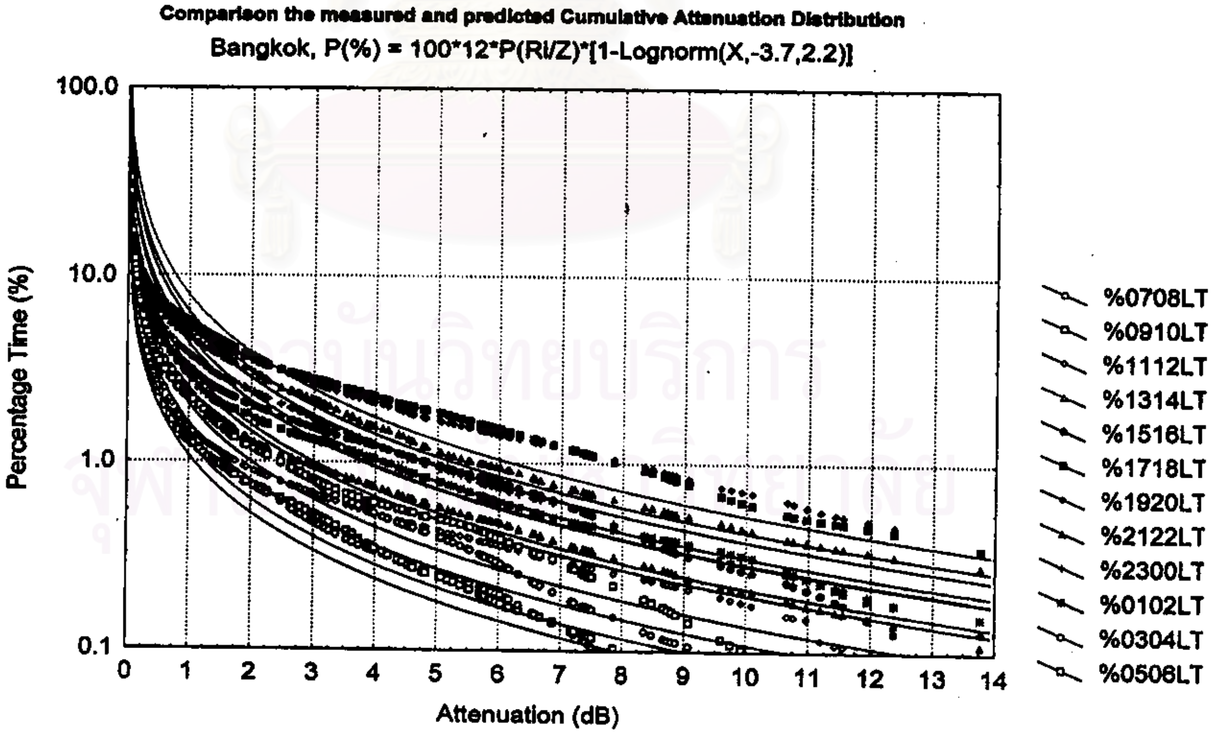

Figure 9.1 Comparison between the measured and the predicted 2-hour cumulative attenuation distribution in Bangkok over a three year period.

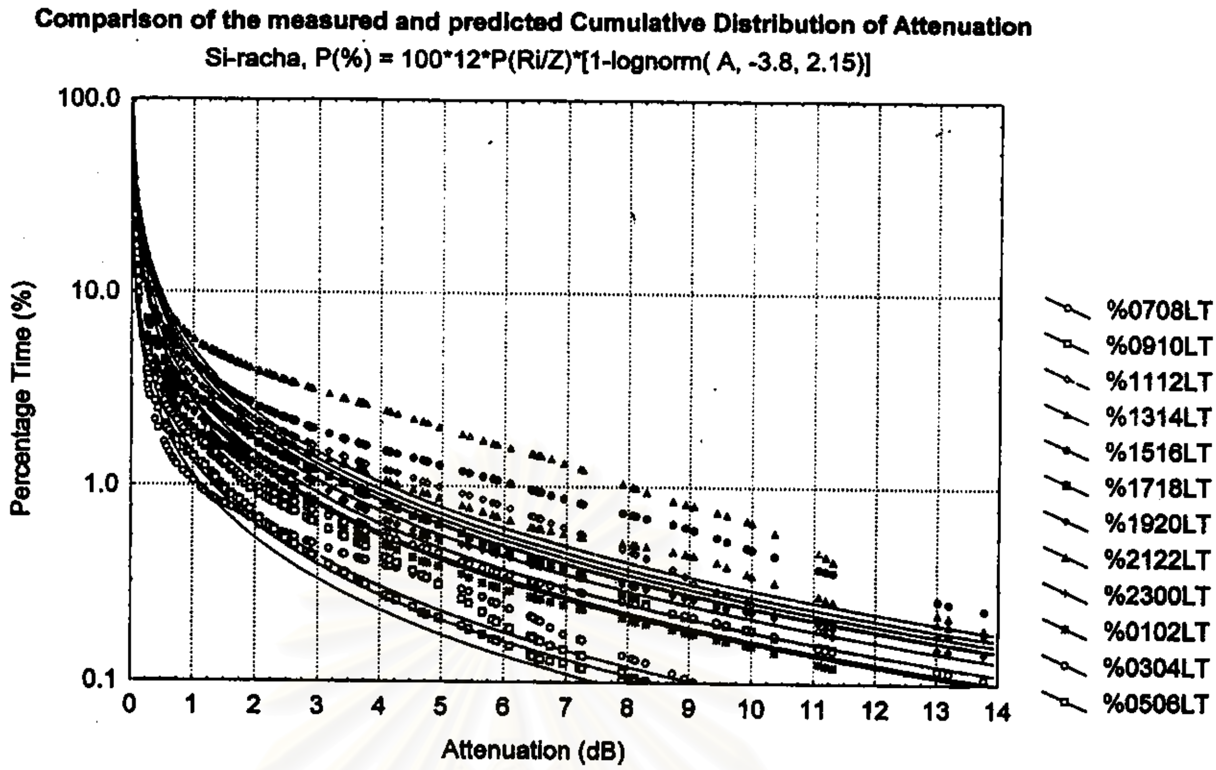

Figure 9.2 Comparison between the measured and the predicted 2-hour cumulative attenuation distribution in Si-racha Thailand over a three year period.

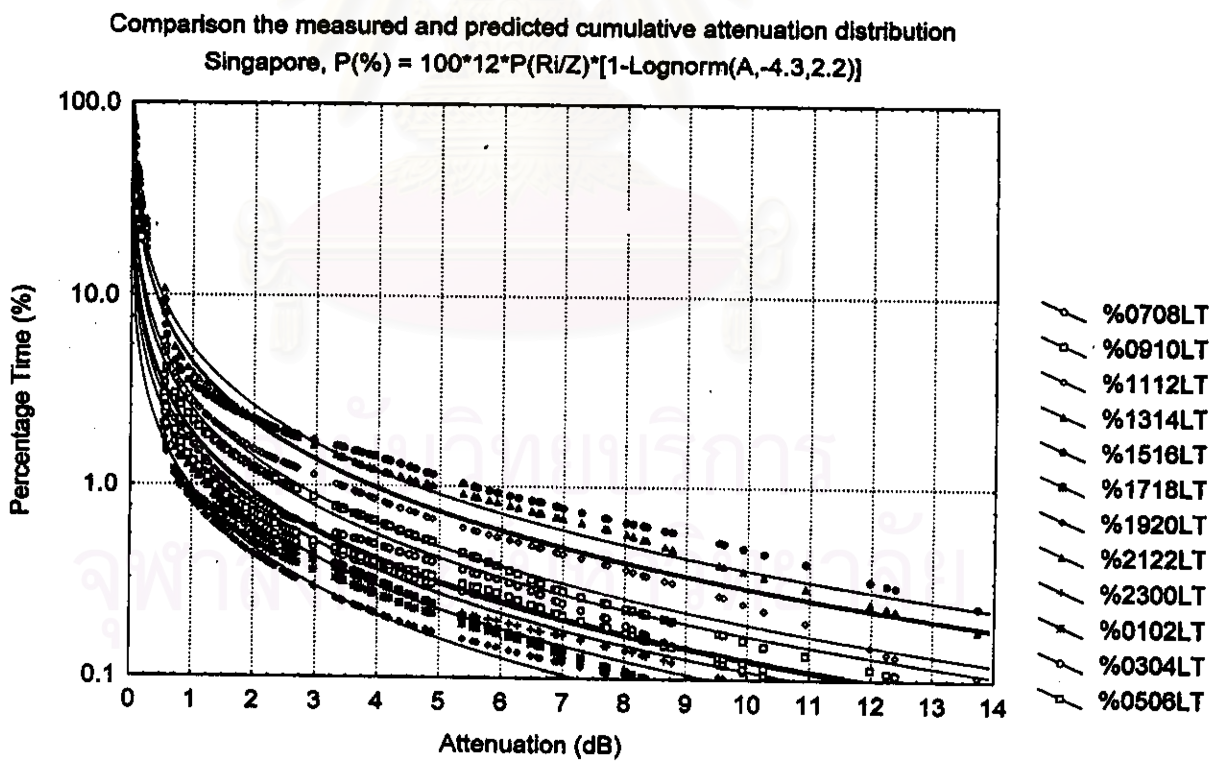

Figure 9.3 Comparison between the measured and the predicted 2-hour cumulative attenuation distribution in Singapore over 3 years.

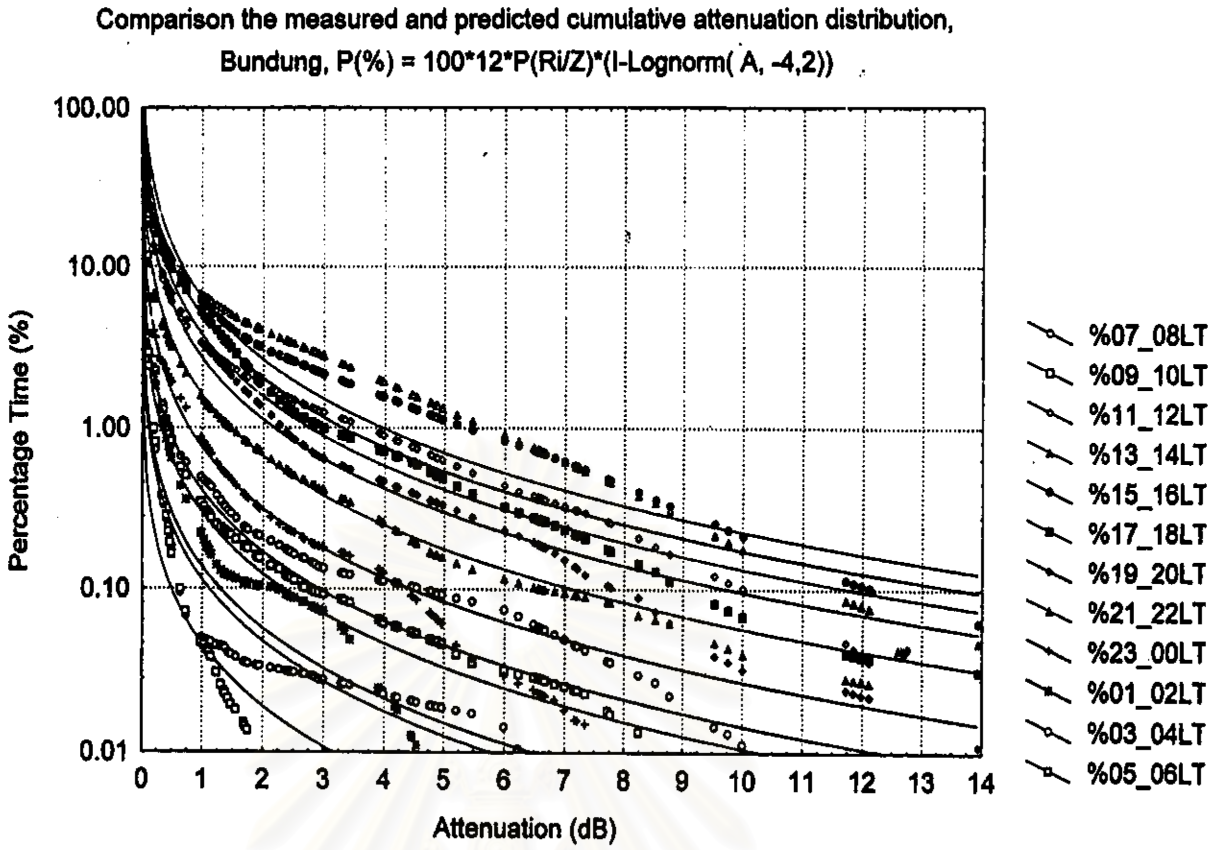

Figure 9.4 Comparison between the measured and the predicted 2-hour cumulative attenuation distribution in Bundung over 3 years.

Results of comparisons can be concluded that the proposed model reasonably agree with the measured data especially when there is a high correlation between the diurnal rainfall probability P(Ri\Z) and the diurnal attenuation probability P(Hi\Y). The shape of the predicted curve is likely dependent on P(Hi\Y) and P(Ri\Z). It should be noted that no attempt to be made to adjust the model to have the minimum rms error

9.7 Model Evaluation

An evaluation of the predicted model can be illustrated in Figure 9.5 - 9.8 by calculating all attenuation distribution ($F_{H1}(\alpha) - F_{H12}(\alpha)$) of each percentage time between 0.1% - 5% and plots each attenuation value which is fallen in each 2 hour interval. It should be noted that attenuation higher than 12 dB and percent attenuation lower than 0.1% can not be comparable. In Figure 9.5 - 9.8, it is clearly indicated that the predicted diurnal attenuation curves (the dashed curves) are similar to the measured diurnal rainfall curves P(Ri\Z) (see Figure 8.5 - 8.8) that are mainly influenced on the accuracy of the prediction. The predicted and the measured curves show a good fit.

The evaluation of the model can be expressed by the percentage prediction error :

$$e_{i,j}(\%) = \frac{100\,[Ap_{i,j} - Am_{i,j}]}{Am_{i,j}} \qquad \text{------------------(9.9)}$$

where

i,j is the given percent probability level (0.1%, 0.5% , 1% ,5%) of interval i

$Ap_{i,j}$ is the predicted attenuation at i,j

$Am_{i,j}$ is the measured attenuation at i.j

$e_{i,j}$ is the percent prediction error. at i,j

The percent probability levels (j) of 0.1%, 0.5%, and 1% are used for this comparison. Table 9.1 shows the measured attenuation values (Am_i(0.1%), Am_i(0.5%) , Am_i(1%), Am(5%)), the predicted attenuation values (Ap_i(0.1%), Ap_i(0.5%), Ap_i(1%)) and the percent error prediction of e_i(0.5%). Table 9.2 shows results comparison between Model-I (the worse case) and Model-II. It includes a mean error (e_{mean}), a rms error (e_{rms}), and a Standard deviation (STD). In Model-I, n equals 12, the measured diurnal variation of rainfall P(Ri\Z), and the log-normal distribution P(Y) are applied. In Model II, n equals 12, the measured diurnal variation of attenuation P(Hi\Y) and the measured attenuation distribution P(Y) are used. As shown in Table 9.2, Model - I shows a large rms error at all four sites, it is due to the correlation of the P(Ri\Z) and the P(Hi\Y), some errors due to (1) the log-normal distribution may not well fit with the measured data at the low probability of occurrence, (2) insufficient of the measured data to be represent a cumulative distribution at very small interval (≤ 2 hours).

In Model II, it is remarkably indicated that when the diurnal attenuation P(Hi\Y) is applied to predict the $F_{Hi}(\alpha)$, Singapore provides the smallest rms error (16.3%) while Bundung shows the largest rms error. It is due to the log-normal distribution of each interval especially at the low probability of rainfall between 0102LT - 0910LT that seem not to fit with the measured distribution (see Figure 9.8). When the distribution of %0304LT, %0506LT and %0708LT are neglected, the rms error can be improved to 20% At Bangkok, the rainfall data from the Bangkok meteorological station are also applied. Increasing rms error of about 3 - 5 % due to some low correlations between P(Ri\Z) and P(Hi\Y).

The accuracy of this prediction model is highly dependent on the measured diurnal rainfall P(Ri\Z), it is strongly recommended to use the rainfall data at the nearest experimental site. It should be noted that this model that predict attenuation every 2 hour interval (12 intervals) is supposed to be the most critical prediction (the worst case scenario), improvement of error prediction can be made by reducing the number of predicted interval (2,4,8).

Table 9.2 Comparison between Model-I and Model-II

MODEL-I (n=12, P(Ri\Z), $F_Y(\alpha)$ = Log-normal Distribution, e(0.5%))				MODEL-II (n=12, P(Hi\Y), $F_Y(\alpha)$ = Measured attenuation, e(0.5%)).		
Site Name	rms error (%)	STD. (%)	Mean error (%)	rms error (%)	STD. (%)	Mean error (%)
Bangkok	33.8 (36*)	28.72	-19.7	25.2 (28.5*)	20	-16.5
Si-racha	34.7	33.1	-16.5	28.3	24	-16.5
Singapore	30.4	33	18.4	16.3	11	-12.3
Bundung	35.8	30	-21	35.1 (20.2**)	55	-31.8

* Using rainfall data from the Meteorological Department of Thailand.

**The attenuation distribution of %0304LT, %0506LT, %0708LT are neglected.

9.8. Concluding Remarks

24-year rain attenuation and rainfall data in Southeast Asia were analyzed to obtain the diurnal variation of attenuation P(Hi\Y) and diurnal variation of rainfall P(Ri\Z). It was found that there was a good correlation between the diurnal attenuation and diurnal rainfall distributions. In addition, their diurnal distributions likely rely on the environmental conditions (e.g., a geographical location) but independent on the distance. The intensive prediction model (every 2 hours) was introduced to predict the amount of attenuation that fall in each hour(s) interval $F_{HI}(\alpha)$ using the knowledge of the statistical of the measured 1-hour rainfall and the log-normal distribution. The proposed model was tested with the measured data between the percent probability of 0.1% - 1% and attenuation up to 12 dB. Result evaluation was reasonably agreed with the measured data.

Although more error are found in the model comparison, due to the low correlation of the diurnal rainfall probability P(Ri\Z), and the diurnal attenuation probability P(Hi\Y). In addition, the predicted distribution may not well correlated with the cumulative distributions especially at the interval that has relatively low probability of occurrence. It is noted that no attempt has been made to adjust the model to minimized the rms error.

Finally, results of this analysis and the prediction model are found useful for the design of the Ku-band satellite communication system with link availability between 99% -99.9% and the results may lead to further development of more-efficient and cost-effective rain compensation techniques.

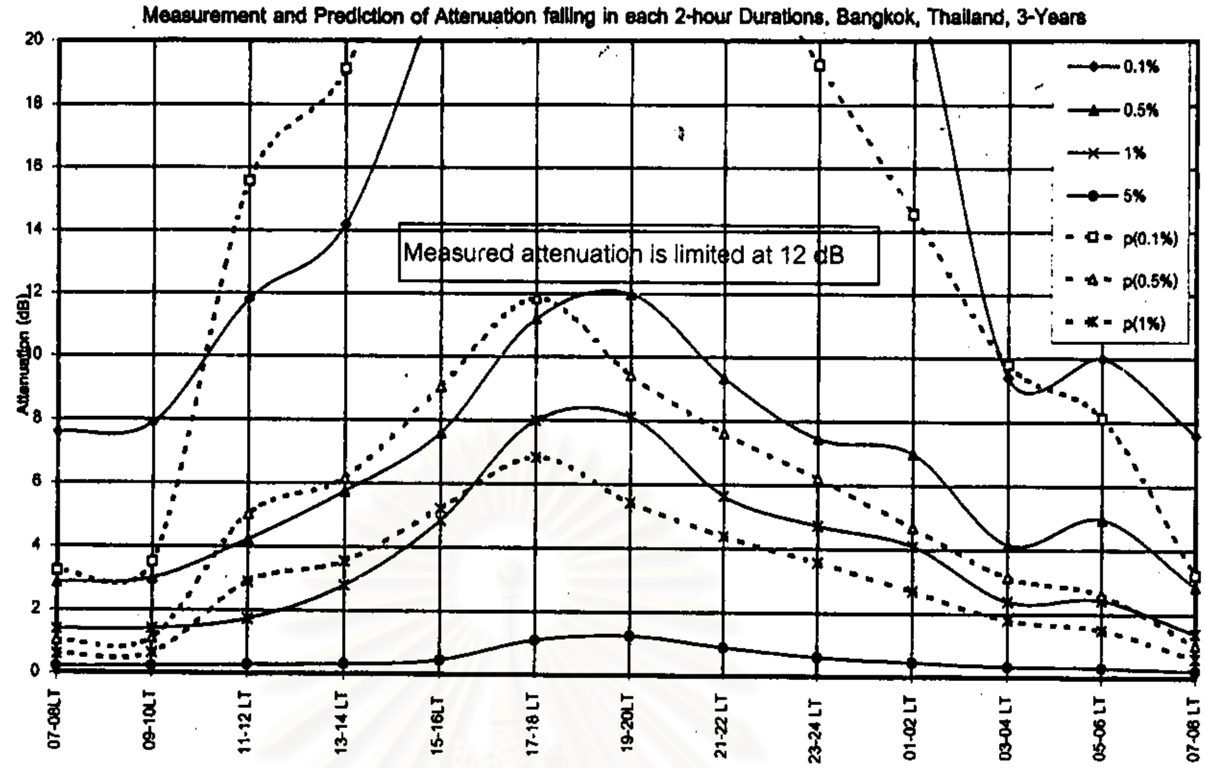

Figure 9.5. Comparison of measured and predicted attenuation occupying each 2-hour interval in Bangkok. It is noted that the measured attenuation is limited up to 12 dB.

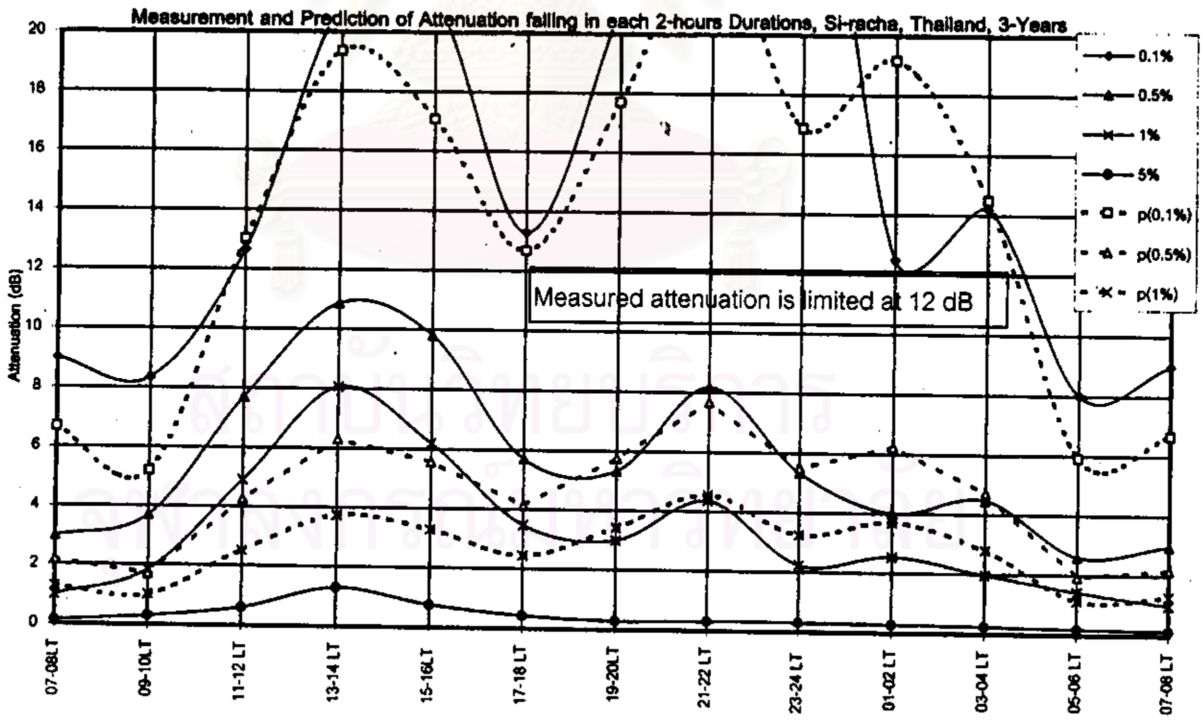

Figure 9.6. Comparison of measured and predicted attenuation occupying each 2-hour interval in Si-racha. It is noted that the measured attenuation is limited up to 12 dB.

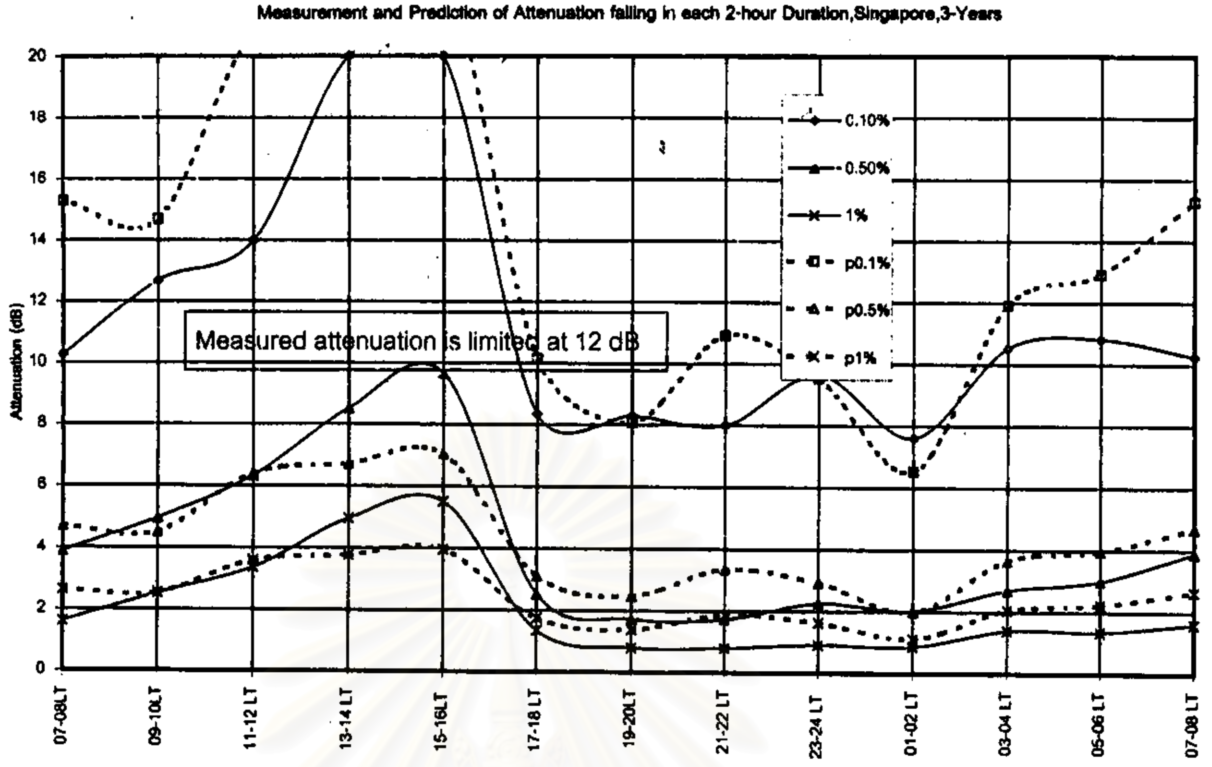

Figure 9.7. Comparison of measured and predicted attenuation occupying each 2-hour interval in Singapore. It is noted that the measured attenuation is limited up to 12 dB.

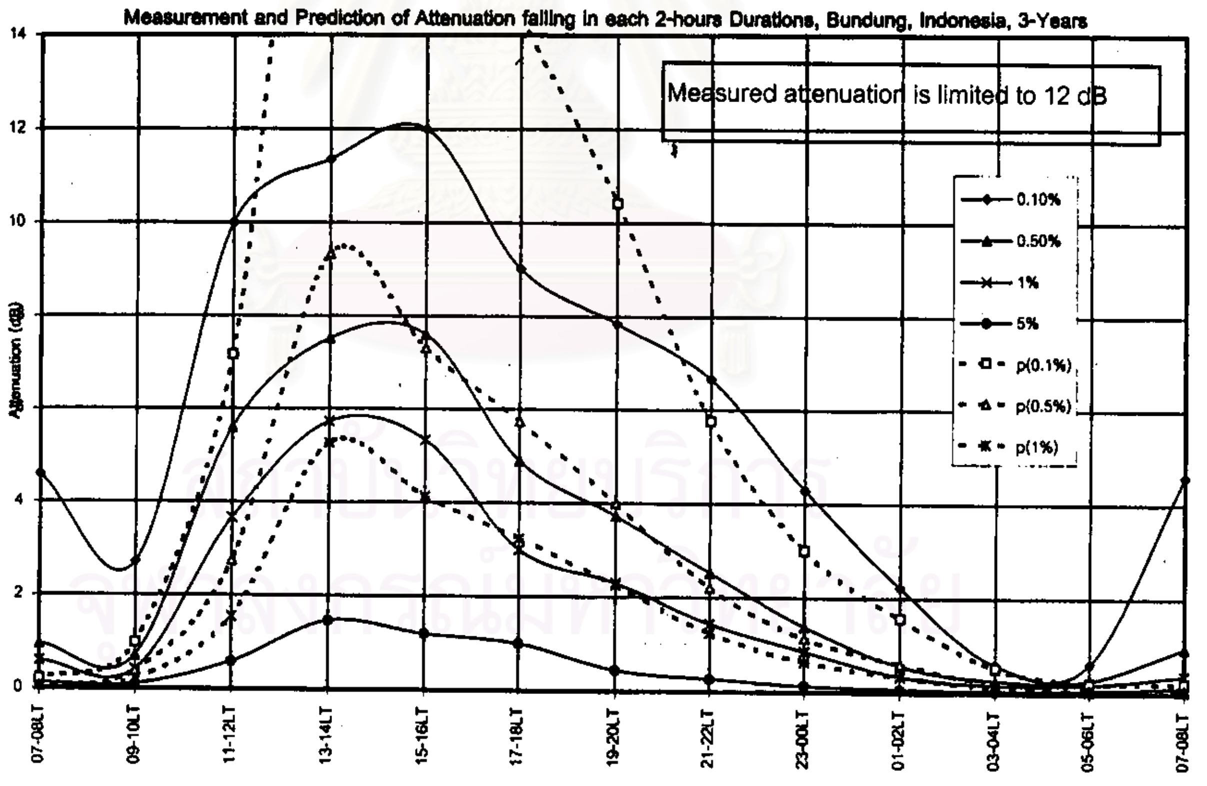

Figure 9.8. Comparison of measured and predicted attenuation occupying each 2-hour interval in Bundung, Indonesia. It is noted that the measured attenuation is limited up to 12 dB.

Table 9.1 Comparison between the measured and the predicted attenuations

Bangkok	07-08LT	09-10LT	11-12 LT	13-14 LT	15-16LT	17-18 LT	19-20LT	21-22 LT	23-24 LT	01-02 LT	03-04 LT	05-06 LT
Am(0.1%)	7.60	7.80	11.60	14.20	n/a	n/a	n/a	n/a	n/a	n/a	9.40	10.00
Am(0.5%)	2.87	3.00	4.23	5.75	7.60	11.20	12.00	9.35	7.46	7.00	4.17	4.96
Am(1%)	1.41	1.43	1.74	2.80	4.64	8.00	8.12	5.65	4.73	4.11	2.40	2.44
Ap(0.1%)	3.23	3.51	15.58	19.11	28.08	36.72	28.20	23.61	19.24	14.56	9.80	8.13
Ap(0.5%)	1.04	1.13	5.03	8.17	9.05	11.85	9.42	7.62	6.21	4.69	3.16	2.62
Ap(1%)	0.60	0.65	2.90	3.88	5.22	6.83	5.43	4.39	3.58	2.70	1.82	1.51
e(0.5%)	-63.75	-62.31	18.80	7.23	19.09	5.76	-21.52	-18.55	-16.82	-32.97	-24.17	-47.09
Si-racha	07-08LT	09-10LT	11-12 LT	13-14 LT	15-16LT	17-18 LT	19-20LT	21-22 LT	23-24 LT	01-02 LT	03-04 LT	05-06 LT
Am(0.1%)	9.00	8.35	12.67	n/a	n/a	13.30	n/a	n/a	n/a	12.50	14.20	8.01
Am(0.5%)	2.96	3.70	7.70	10.86	9.82	5.71	5.34	8.19	5.34	4.02	4.48	2.60
Am(1%)	1.00	1.84	4.90	8.06	6.20	3.48	3.00	4.36	2.24	2.86	1.97	1.43
Ap(0.1%)	6.68	5.21	13.04	19.36	17.08	12.69	17.70	23.57	18.87	19.17	14.49	5.92
Ap(0.5%)	2.16	1.70	4.26	6.31	5.57	4.14	5.77	7.66	5.50	6.28	4.72	1.93
Ap(1%)	1.29	1.01	2.52	3.74	3.30	2.45	3.42	4.56	3.26	3.71	2.80	1.14
e(0.5%)	-26.97	-54.06	-44.80	-41.89	-43.31	-27.56	8.01	-6.07	3.00	55.46	5.44	-25.80
Singapor	07-08LT	09-10LT	11-12 LT	13-14 LT	15-16LT	17-18 LT	19-20LT	21-22 LT	23-24 LT	01-02 LT	03-04 LT	05-06 LT
Am(0.1%)	10.28	12.68	14.00	n/a	n/a	8.35	5.30	8.00	9.65	7.61	10.83	10.82
Am(0.5%)	3.90	4.96	6.33	8.52	9.67	2.53	1.73	1.73	2.26	2.03	2.70	3.00
Am(1%)	1.62	2.82	3.37	4.95	5.50	1.35	0.80	0.80	0.92	0.87	1.40	1.36
Ap(0.1%)	15.29	14.89	20.83	21.87	22.93	10.15	8.08	10.91	8.83	6.53	11.92	12.93
Ap(0.5%)	4.70	4.51	6.40	6.72	7.04	3.12	2.48	3.35	2.93	2.00	3.66	3.97
Ap(1%)	2.64	2.54	3.60	3.76	3.98	1.75	1.39	1.88	1.64	1.13	2.06	2.23
e(0.5%)	20.45	-8.84	1.06	-21.14	-27.16	23.27	43.49	93.80	29.60	-1.26	35.64	32.41
Bundung	07-08LT	09-10LT	11-12 LT	13-14 LT	15-16LT	17-18 LT	19-20LT	21-22 LT	23-24 LT	01-02 LT	03-04 LT	05-06 LT
Am(0.1%)	4.60	2.72	10.00	11.36	12.00	9.03	7.85	6.67	4.30	2.23	0.60	0.63
Am(0.5%)	0.96	0.72	5.60	7.52	7.61	4.90	3.73	2.53	1.40	0.57	0.20	0.10
Am(1%)	0.60	0.42	3.66	5.73	5.34	3.00	2.30	1.45	0.87	0.35	0.17	0.16
Ap(0.1%)	0.22	0.99	7.17	24.31	19.05	14.94	10.42	5.76	3.01	1.58	0.53	0.22
Ap(0.5%)	0.10	0.35	2.56	6.68	6.80	5.33	3.72	2.06	1.07	0.56	0.19	0.08
Ap(1%)	0.05	0.22	1.55	8.27	4.13	3.24	2.26	1.25	0.65	0.34	0.11	0.05
e(0.5%)	-89.55	-50.84	-54.31	16.43	-10.64	8.85	-0.21	-18.70	-23.32	-1.32	-5.55	-21.32