CHAPTER 7

ANALYSIS AND MODELING OF FADE-DURATION STATISTICS

7.1 Introduction

This chapter presents analytical results of fade-duration statistics in Southeast Asia over a three year period and proposes an empirical model to obtain fade-duration distribution in two ITU-R regions (zone N and zone P) which is useful for the design of Ku-band satellite link in Southeast Asia. Short-term statistics of fade durations, especially for fade of less than 10 seconds, is essential for the system design to meet the design requirement of the ITU-T recommendations. For example, ITU-T G 821 [1980] and G-826 [1994] defines the unavailable time when the link outage (BER<10^{-3}) occurs more than 10 seconds consecutive seconds. The knowledge of this fade duration distributions are essential for Ku-band satellite system design in Southeast Asia. Details of fade-duration statistic are also summarized in the open literature by R. Lekkla, K.S. McCormick, and D.V. Rogers [1998].

7.2 Data Analysis

Fade duration distribution is a conditional probabilities of rain fade exceeding a given duration, given that the attenuation has exceeded a chosen threshold. Two types of fade duration distribution are:

1) Event fade duration P_n(D/A) is a total number of fade that exceeds a given threshold (A) in a given duration (D) compared with all fade events.
2) Time fade duration P_t(D/A) is a total amount of time that exceeds a given threshold (A) in a given duration compared with all fading times.

In our analysis, we are only concerned with the event fade duration that is more useful for the system design. In general, the conditional probability of event fade duration can be expressed by:

$$P_n(D/A) \quad = \quad P(D > Dq/\ A > \alpha) = \frac{N(D>Dq,\ A>\alpha)}{Nt(A>\alpha)} \text{ --------------(7.1)}$$

where

D is the duration time (seconds),

Dq is the given threshold duration (seconds)

N(D > Dq, A > α) is the total number of fades that exceeds the threshold (α) in a given duration,

Nt(A > α) is the total number of fade exceeding the threshold.

Figure 7.1 shows an example of an event number (or episode) of fade duration obtained by the measured data in Bundung on 9 March 1993. There are two episodes that fade exceeds the threshold of 5.8 dB (200°K). The first episode has a duration D_1 while the second episode has a duration D_2.

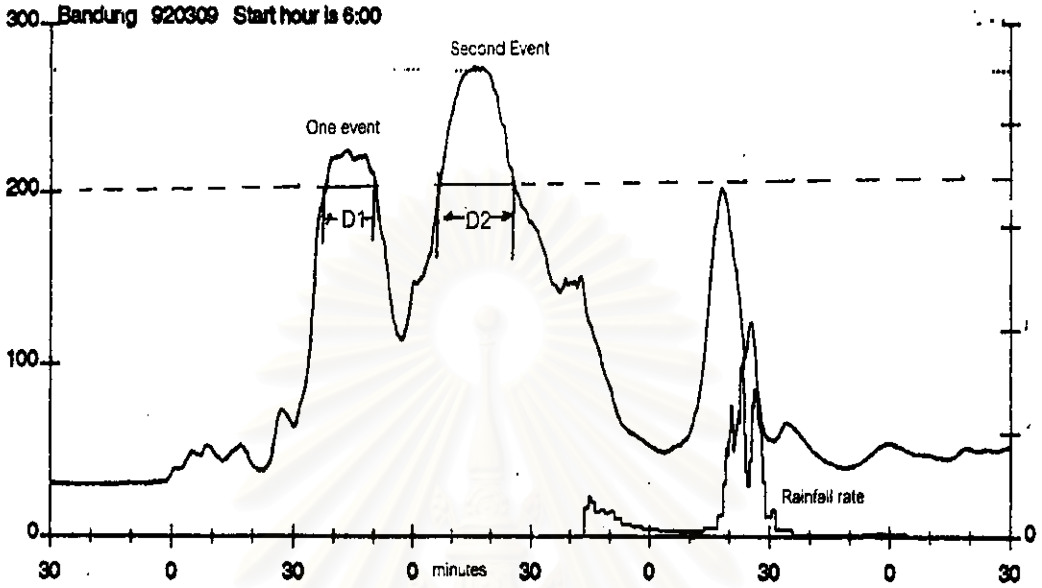


Figure 7.1 Daily measured data for Bandung on 9 March 1993 with two fade events (D_1 and D_2) exceeding 200° K.

Fade duration statistics along an earth-satellite path can be obtained by a beacon and a radiometric measurement. In trying to compute statistics from satellite beacons, there is a difficulty in that the recorded signal will vary due to non-precipitation effects, including receiver noise and atmospheric scintillation or non-rain fading unless some forms of low pass filtering is performed and the signal may fluctuate around a chosen threshold create short fade duration events not due to rain. When radiometer data are employed to compute the statistics, this problem is lessened since non-precipitation events do not affect the radiometer noise temperature.

For our radiometric measurement, various tests were carried out to examine the influence of the noise component of the radiometer due to small fluctuations of noise temperature along the threshold line. It was finally decided to process the data employing 'hysteresis". Once an attenuation threshold was exceeded, the duration of the fade (episode) did not end until the attenuation fell below the threshold by a certain amount. To obtain the event number of fade duration at relative short interval of 2 seconds to 4000 second, the fast fourier transform method was applied.

Figure 7.2 shows the results of two different analyses, one where a fixed hysteresis of 0.1 dB was used, and one where the hysteresis was calculated from a variation of antenna temperature of 4° K. This latter procedure gives varying values of hysteresis depending on the attenuation threshold, since the inferred attenuation is a logarithmic function. For the parameters used, the hysteresis for the 10 dB threshold was about 0.76 dB. Figure 7.2 also shows that small noise fluctuations appear to be largely removed by this larger value. All the data presented were processed using an assumed noise fluctuation of 4° K.

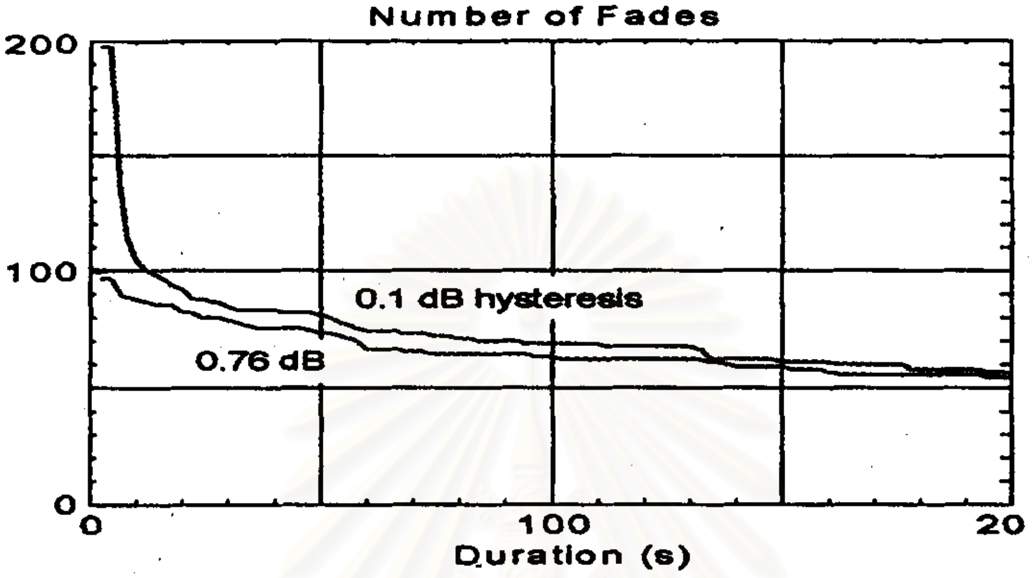


Figure 7.2 Measured data for Bangkok 1993 for a 10 dB threshold using different values of hysteresis in the data processing

For each of the four experimental sites, fade durations statistics were calculated for each month and for fade thresholds of 2, 4, 6, 8, and 10 dB. These statistics were combined on an annual basis, starting from March 1992 followed by the first few months for which data were available, and ending with February *1995*. Data referred to as "1993" actually include measurements from March 1993 to February 1994.

7.3 Analytical Results

Figure 7.3 shows an example of analytical result of Bundung data. The conditional probabilities of fade durations from 2 to 4000 seconds (66.67 minutes) are shown for fade thresholds of 2, 6 and 10 dB. These curves are typical to the curves found at all four locations, except that the probabilities of longer durations for Bangkok and Si-racha tend to be larger. *One of the important features of tropical rain attenuation, besides the severity, is the significant number of events of long duration.* For example, during the year 1993 in Bangkok, more than 50 fades exceeded 6 dB for at least 1000 seconds (17 minutes).

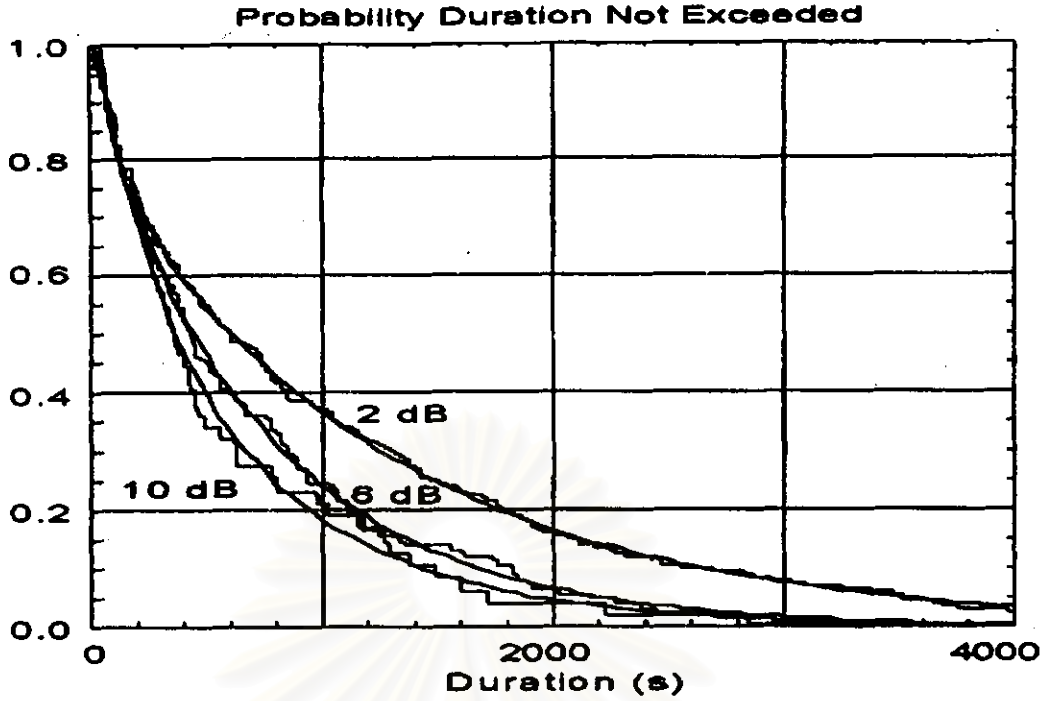


Figure 7.3 Measured data in Bundung 1993 and a double exponential curve of 2 dB, 6 dB, 10 dB.

Figures 7.4 - 7.7 illustrate a double exponential curve that is well fitted with the measured data in Bangkok, Si-racha, Singapore and Bundung respectively. Results of statistical analysis was found that the double exponential curve has the excellent fit with event fade duration data having the following form:

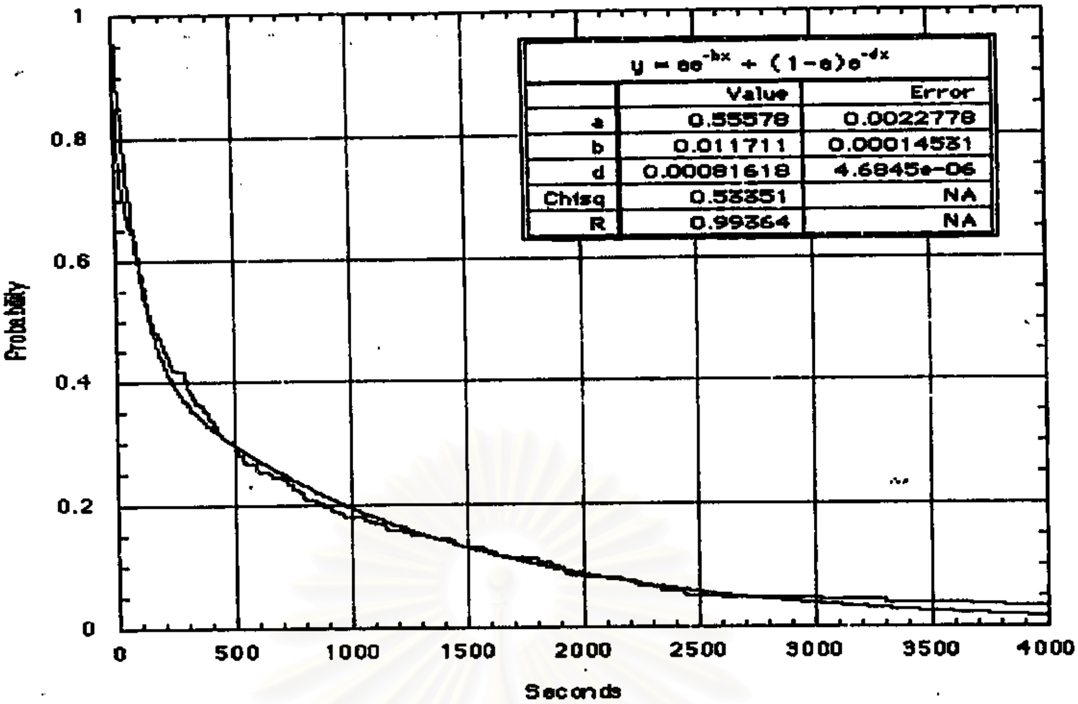
$$P(D>Dq \,/\, A>\alpha) = (a)\exp(-b(D-2)) + (1-a)\exp(-d(D-2)) \text{ ------------------(7.2)}$$

where

$P(D>Dq \,/\, A>\alpha)$ is the conditional probability of event fade duration,

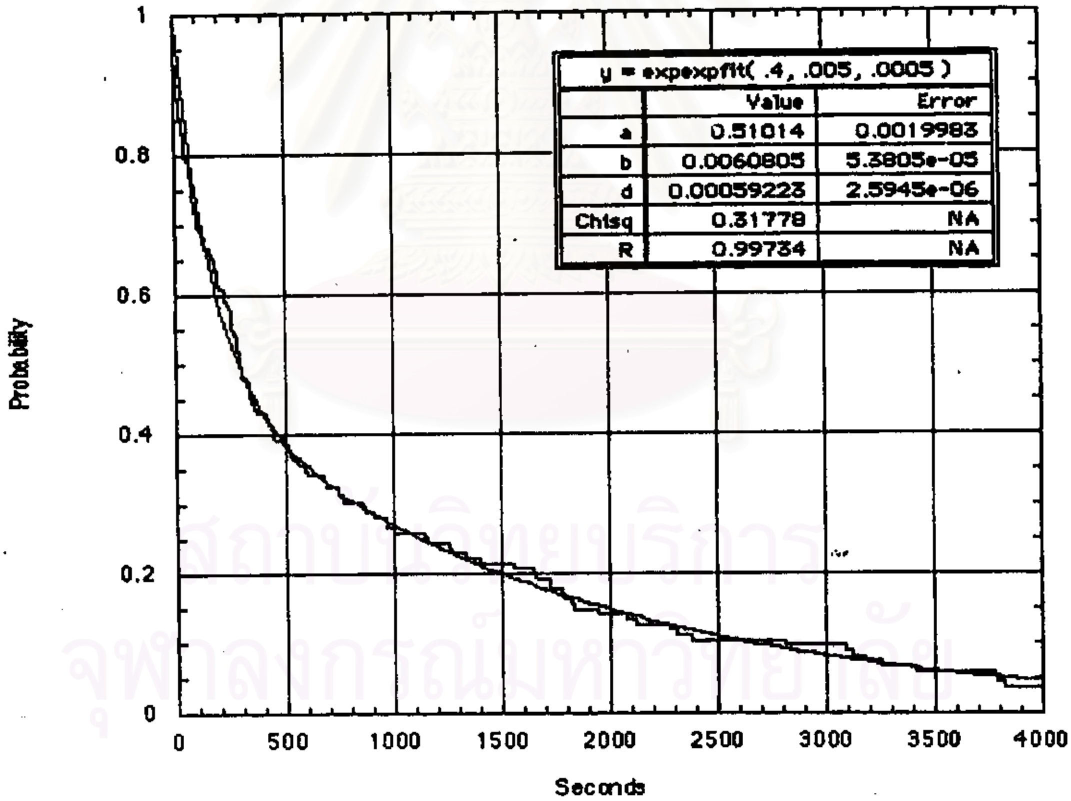
a, b, d is the constant that are fitted to the equation.

In equation (7.2), the use of amplitude terms "(a)" and "(1-a)" - the shape parameter - force the fit through the ordinate value 0 - 1, and the term (D-2) is the shortest measurable fades at 2 seconds.



$y = ae^{-bx} + (1-a)e^{-dx}$	Value	Error
a	0.55578	0.0022778
b	0.011711	0.00014531
d	0.00081618	4.6845e-06
Chisq	0.53351	NA
R	0.99364	NA

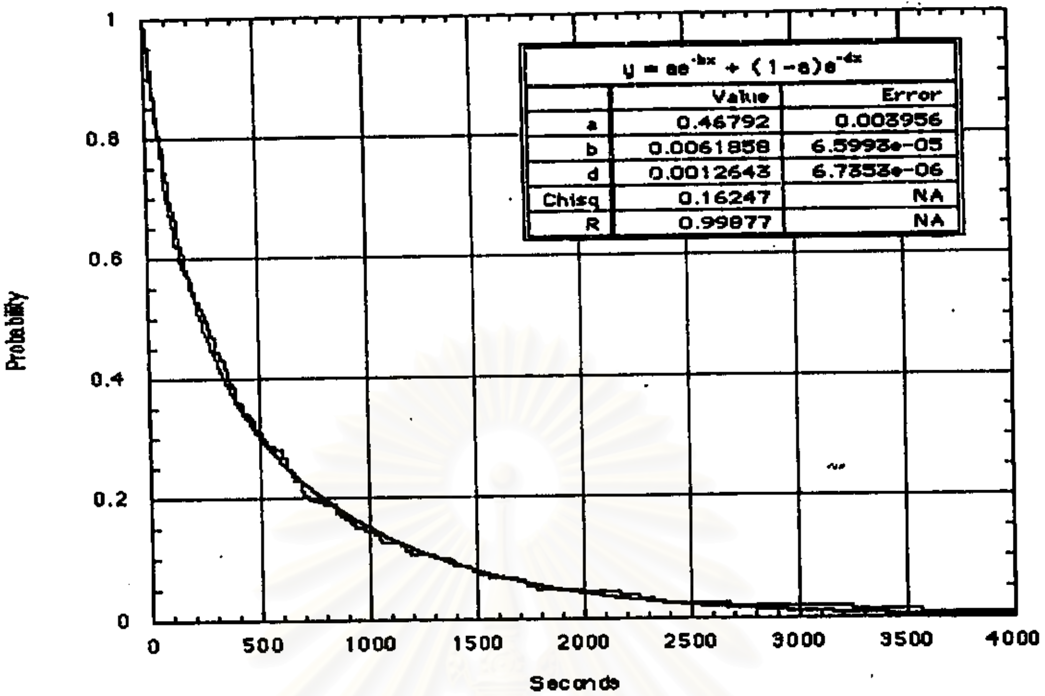
Figure 7.4 Comparison of measured Bangkok data with a double exponential curve and the fitted parameter "a, b, d" with a correlation coefficient >0.993 shown in the table.



y = expexpfit(.4, .005, .0005)	Value	Error
a	0.51014	0.0019983
b	0.0060805	5.3805e-05
d	0.00059223	2.5945e-06
Chisq	0.31778	NA
R	0.99734	NA

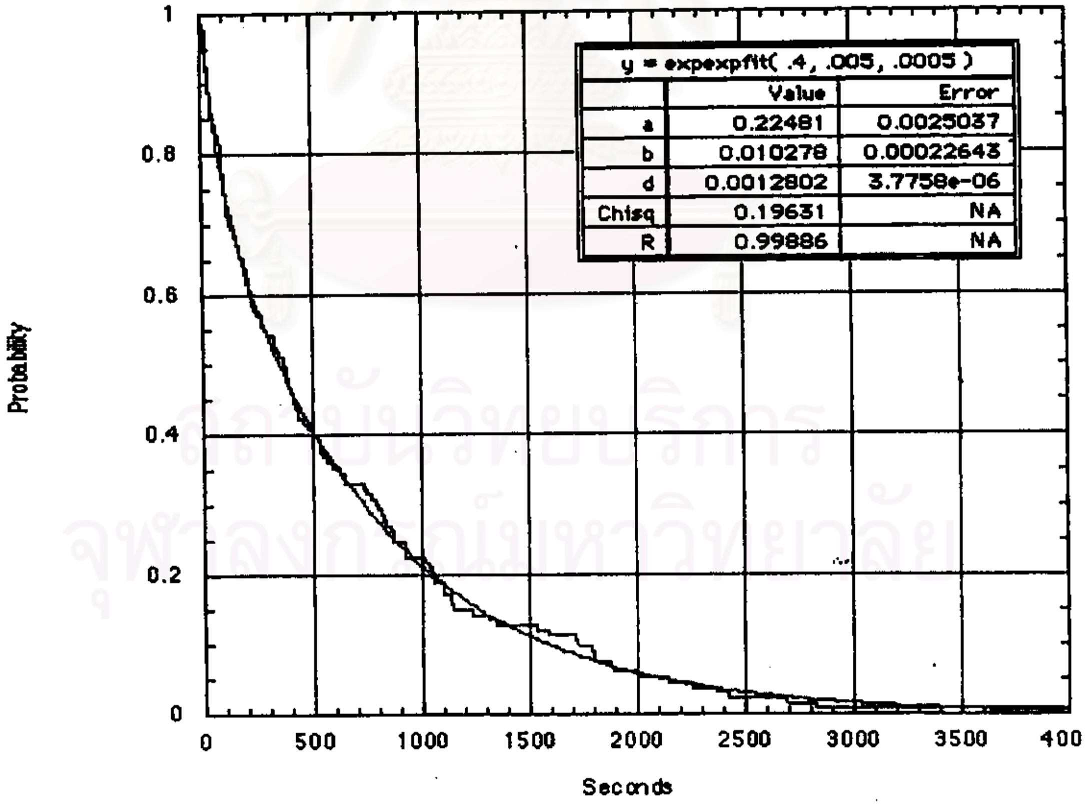
Figure 7.5 Comparison of measured Si-racha data with a double exponential curve and the fitted parameter "a, b, d" with a correlation coefficient >0.997 shown in the table.

As shown in Figures 7.4 - 7.7, the values of the fitted parameters "a, b, d" are also indicated associated with a correlation coefficient in the range 0.996 to 0.999, which is typical of those obtained for all the data. The parameter "b" is a scale parameter associated with the exponential curve that decays most quickly, in which "a' is larger than "1-a".



$y = ae^{-bx} + (1-a)e^{-dx}$	Value	Error
a	0.46792	0.003956
b	0.0061858	6.5993e-05
d	0.0012643	6.7353e-06
Chisq	0.16247	NA
R	0.99877	NA

Figure 7.6 Comparison of measured Singapore data with a double exponential curve and the fitted parameter "a, b, d" with a correlation coefficient >0.996 shown in the table



y = expexpfit(.4, .005, .0005)	Value	Error
a	0.22481	0.0025037
b	0.010278	0.00022643
d	0.0012802	3.7758e-06
Chisq	0.19631	NA
R	0.99886	NA

Figure 7.7 Comparison of measured Bundung data with a double exponential curve and the fitted parameter "a, b, d" with a correlation coefficient >0.997 shown in the table

7.4 Modeling of Fade-Duration

Assuming that "1/b" and "1/d" are referred to "characteristic durations" having units of seconds. The plots of value "1/b" of each year corresponding to each threshold (2, 4, 6, 8, 10 dB) are illustrated in Figure 7.8 for ITU-R zone-N (Bangkok, Si-racha) and Figure 7.9 for ITU-R zone P(Singapore, Bundung). The plot of values "1/d" of each year corresponding to each threshold are shown in figure 7.10 for ITU-R Zone-N and Figure 7.11 for ITU-R zone-P.

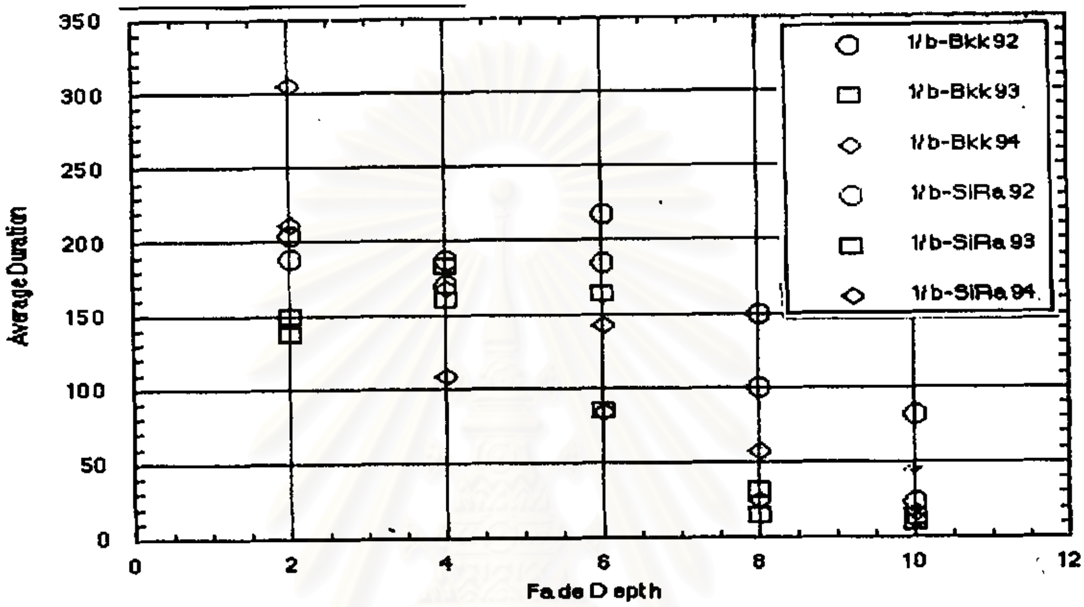


Figure 7.8 Value of the characteristics duration "1/b" in Bangkok and Si-racha (ITU-R zone -N)

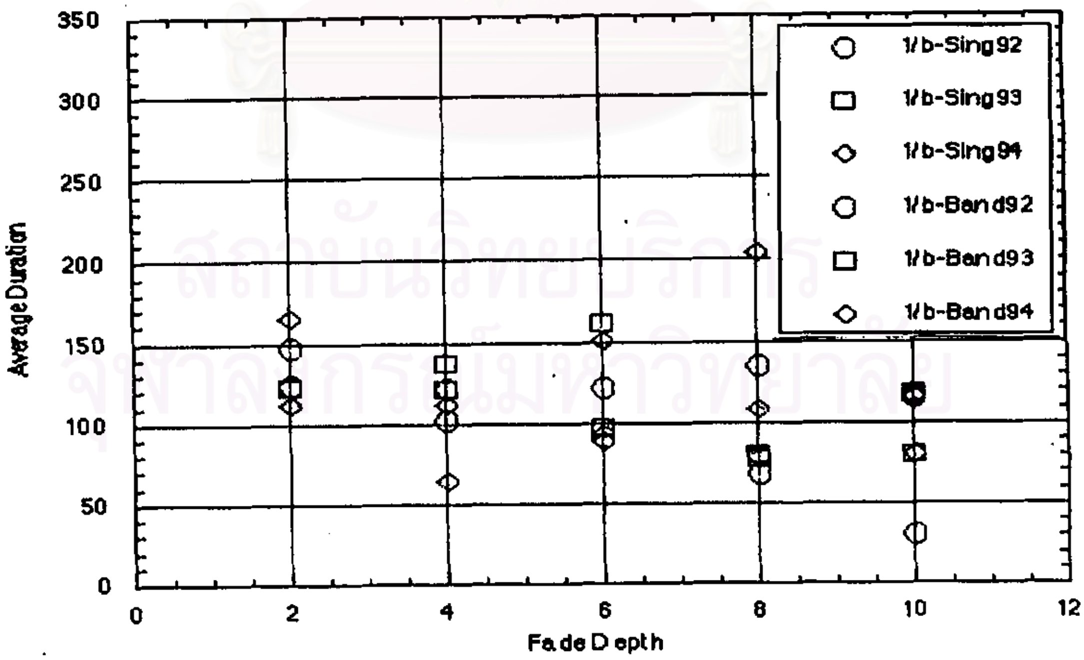


Figure 7.9 Value of the characteristics duration "1/b" in Singapore and Bundung (ITU-R zone -P)

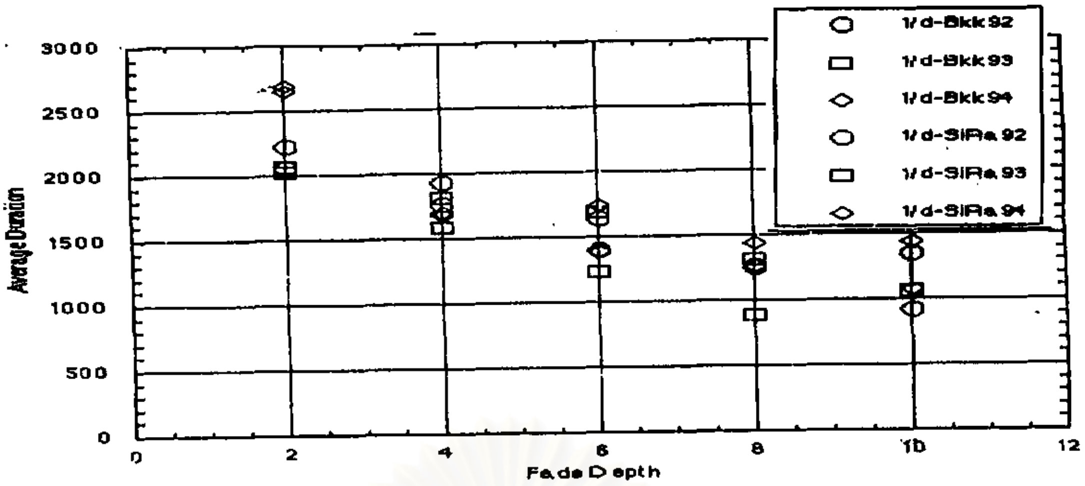


Figure 7.10 Value of the characteristics duration "1/d" of Bangkok and Si-racha (ITU-R zone -N)

As shown in Figures 7.8 - 7.9, the "1/b" implies the intense rain that its duration is relatively short (1/b decrease). Data of Singapore and Bundung (Figure 7.9) are tighter and smaller than Bangkok and Si-racha In Figures 7.10 - 7.11, All data group are tighter but again Singapore and Bundung tend to be tighter than Bangkok and Si-racha. It implies that a convective rain, having short rainfall duration, more frequently occurs in the ITU-R zone P while the stratiform rain having long rainfall duration and the convective rain are often occurred in the ITU-R zone N.

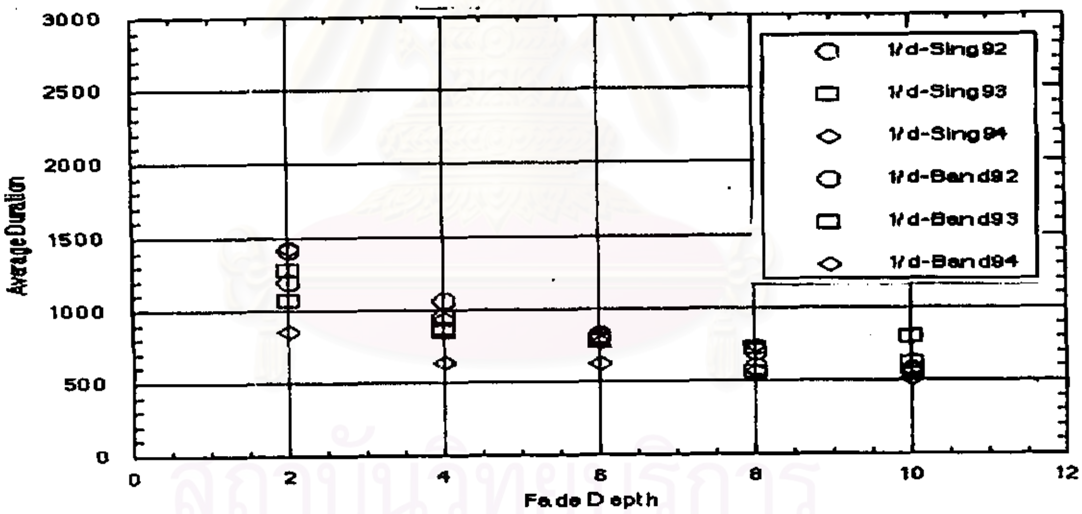


Figure 7.11 Value of the characteristics duration "1/d" in Singapore and Bundung (ITU-R zone - P)

1) Modeling "b" parameter

Figure 7.12 combines the characteristics duration (1/b) of two rain climate zones from Figures 7.8 and 7.9. There are two outlying points in the upper right quadrant, arising from the Bandung 1994 data. That year, there were relatively few 8 and 10 dB fades of durations greater than 2000 seconds, and the data could possibly have been fitted by a single exponential curve. These two points were omitted from further consideration. The white circles in Figure 7.12 show the mean values at each threshold (α). The vertical bars indicate one standard deviation about the mean and the line is a linear fit through these points. The linear equation of the fitted line is:

$$1/b = 181 - 6.50\alpha \quad \text{------------------} \quad (7.3)$$

where

α is the attenuation threshold (2,4,6,8,10 dB),

2) Modeling "d" parameter

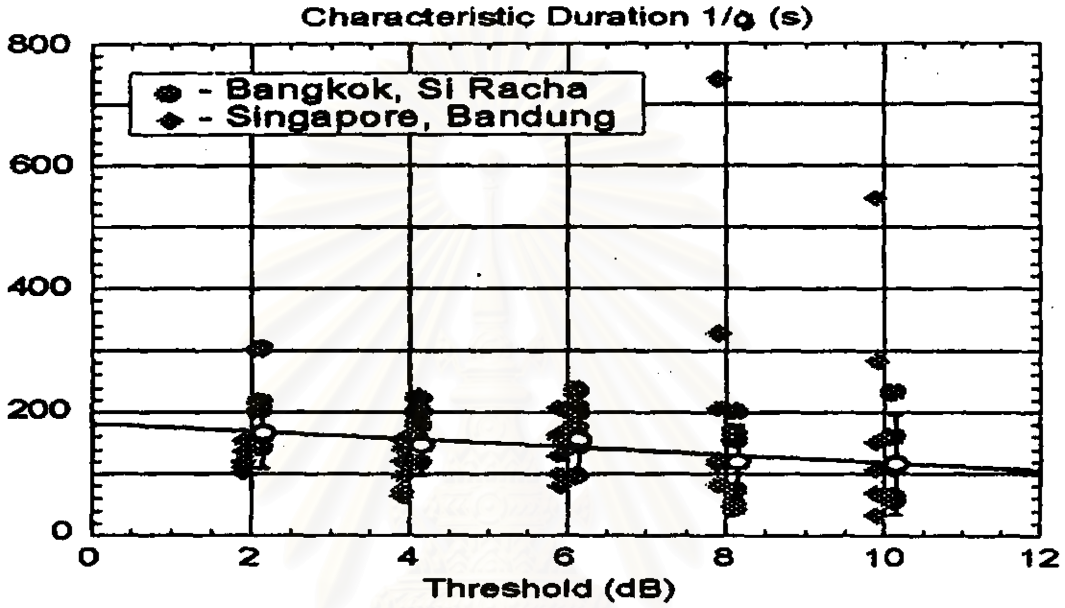


Figure 7.12 Values of the characteristic duration "1/b" derived from all twelve station years of data.

Combining Figures 7.10 and 7.11 to form Figure 13, the Bangkok and Si-racha data tend to lie in one group, while the Singapore and Bandung data tend to lie in a second and tighter group. The data for the two groups were averaged separately, resulting in the upper set of white circles for Bangkok and Si-racha, and the lower white circles for Singapore and Bundung. Again the vertical bars indicate standard deviations. The equations of the fitted lines are:

For ITU-R zone-N : $1/b = 2364 - 113\alpha$ ....(7.4)

and

For ITU-R zone-P: $1/b = 1261 - 77.7\alpha$ ....(7.5)

3) Modeling "a" parameter

An attempt was made to determine the value of 'a' by fitting the 12 annual data using values of "b" and "d" from Equations (7.3), (7.4), and (7.5), with 'a' as the single parameter, but it was not possible to determine a useful relation. Values of "a" were found to vary from about 0 to

0.5. Simple sensitivity tests showed that making predictions from values of 0.3, 0.4 and *0.5* did not create severe errors. It was decided to adopt a value of *0.5* for 'a'.

Results of this procedure are given in Figure 7.14, which shows the year 1993 distributions from Si-racha and Singapore at the 4 dB threshold. The predictions, based on the above equations with 'a' set equals to *0.5* are seen to be very good, even to small probability values. This Figure illustrates quite clearly the difference in the probability distributions between the two sites, with the Si-racha and Bangkok distributions having a significantly greater conditional probability of long duration fades than the Singapore and Bandung distributions.

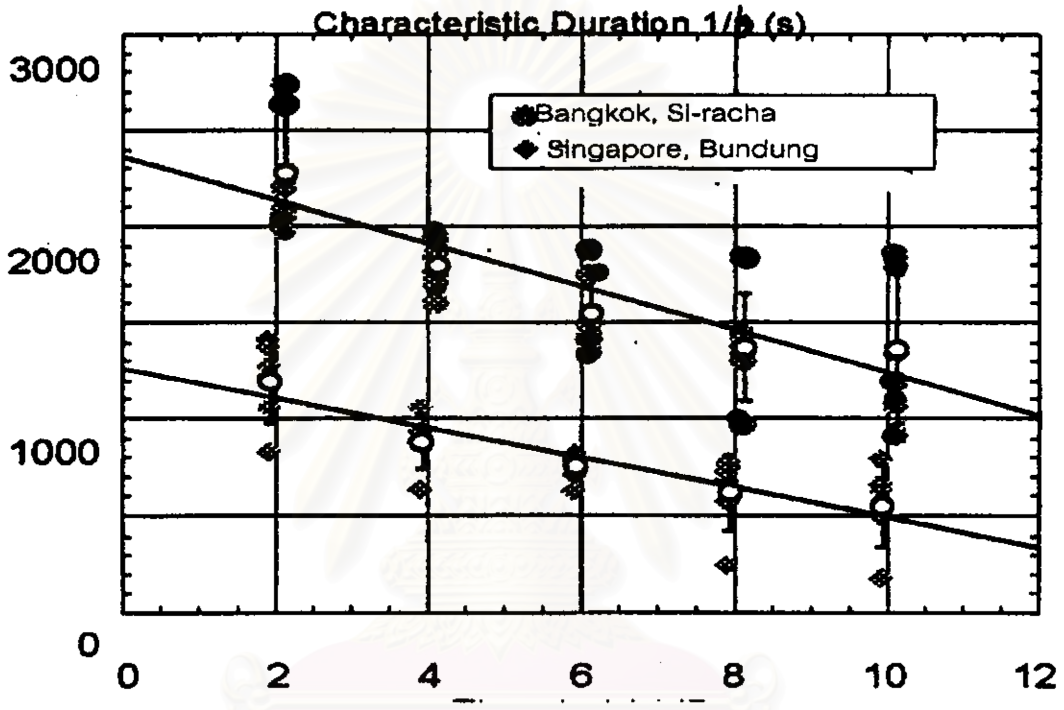


Figure 7.13 Value of characteristics duration "1/d" derived from all twelve station-years of data.

As a matter of interest, the Si-racha data contain 193 fades that exceed 4 dB for at least 2 seconds, while the Singapore data contain 272 fades with these characteristics. This is another indication that the fading regime for Bangkok and Si-racha (ITU-R zone N) tends to have very significantly more fades of long duration than Singapore and Bundung (ITU-R zone P).

7.5 Interpretation of Results

As evident from Figure 7.14, the measured probability density for the duration of rain fades exceeding specified time intervals is fitted very well by a double exponential distribution. These exponential components divide the population of durations into short - duration and long-duration regimes, which can be related to the physical processes responsible for the rain fading.

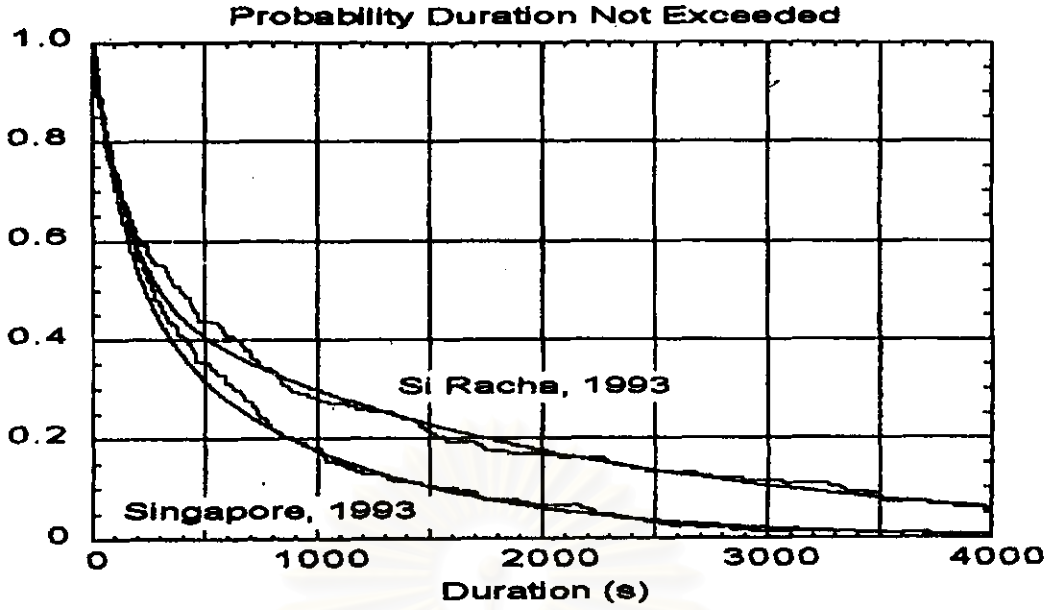


Figure 7.14 Comparison of measured data with prediction from equation (7.2), with parameters substituted from Equations (7.3), (7.4), (7.5). The value of the parameter "a" was assumed to be 0.5and the fade threshold to be 4 dB

A negative exponential distribution implies a physical process in which the probability of an event is a discrete random variable obeying Poisson statistics with each event being independent of previous events. The occurrence rate of events is parameterized by the average number of occurrences in a given time interval; the reciprocal of the average event occurrence rate is the average event duration. Here, an "event" refers to crossing a specified fade threshold, and the event duration is the time interval between two crossings.

The short and long-duration regimes can preliminarily be identified as belonging to fade events generated by convective and stratiform rainfall, respectively. Indeed, the double-exponential distributions have previously been applied to describe the overall rainfall rate distribution [6], again by dividing the rainfall rate statistics into stratiform and convective terms. It is interesting to interpret the measured duration statistics in the same fashion.

In the simplest case, presumably the ratio between the characteristics durations, equivalent to "a" in equation (7.3), should correspond approximately to the convectivity component, " β" of the Rice-Holmberg [1973] rain-rate model. Since values ranging from 0.3 - 0.5 are found to perform acceptable in the duration model, β values of this order for the locations in question would support such a presumption.

From the Rice-Holmberg maps for β the approximate convectivity components for the sites in question are found to be 0.5 for Bangkok and Si-racha and 0.7 for Singapore and Bandung These values are in nominal agreement with observed "a" values. While the relation between these parameters is conjectural at present, the approximate correlation between the “a” and β factors is interesting to say the least, and may offer possibilities for modeling of rain fade durations.

Daily attenuation and rainfall data of all four sites were observed. Another effect to the long fade duration may be due to low elevation angle operation (Bangkok, Si-racha and Bundung) which rainstorm may last longer time.

7.6 Comparison with Other Models

From the previous works, researchers found that a fade-duration longer than 1 minute shows a behavior of log-normal distribution. For example, fade duration data from 11 GHz and 14 GHz radiometric measurement of three locations in Australia show an approximate log-normal distribution.

Recently, an analysis of fade durations at 20 GHz using the Olympus satellite over a three year period at the Netherlands by G. Brussaard [1995] found that fade duration data can be modeled by the single and double exponential distributions. The Brussaard model, indicated similar results whether there was a different in frequencies. In addition, the Rice and Holmberg rain-intensity model using a double exponential distribution has the β parameter implied by our “a” parameter.

7.7 Concluding Remarks

We have presented the fade-duration analysis of three year data of rain attenuation by 12 GHz radiometric measurement at four locations in Southeast Asia covering two rain climates (ITU-R zone N and ITU-R zone-P). It is found that fade duration statistics (a conditional probability of event number of rain fade) can be well defined by a double exponential distributions over the range of 2 seconds to 4000 seconds.

This model is developed to obtain the most fitted parameters of “a”, “b”, and “d”. It is found that parameter “a” = 0.5 and this parameter implies by the β parameter of Rice & Holmberg model. Using the parameters a =0.5, and equations (7.3), (7.4) and (7.5) to obtain the parameters “b”, and “d” , it is possible to obtain a fade-duration distribution from equation (7.2).