CHAPTER IV

RELEVANT THEORIES

As previously mentioned, the main objective of this study is to test the applicability of the material balance equation (p/z plot) and the pseudo-steady state equation in estimation of GIIP for a volumetric gas reservoir. It is then appropriate to discuss about these two equations in detail.

4.1 Material Balance Equation

The material balance equation for volumetric gas reservoir seems to be the simplest material balance equation used in petroleum engineering. This equation can be developed from balance of all gas involved at standard conditions,

Total gas produced at standard conditions = GIIP - remaining gas in the reservoir at standard conditions, (4-1)

where GIIP is gas initially in place reported as volume at standard conditions. The above equation can be written in symbolized form as follows:

$$G_p = G - \frac{GB_{gi}}{B_g}, \qquad (4\text{-}2)$$

where G = gas initially in place, scf

G_P = produced gas, scf

B_{gi} = initial gas formation volume factor, ft^3/scf

B_g = gas formation volume factor, ft^3/scf

Equation 4-2 is written with the assumption that expansions of connate water and rock are neglected. This is reasonable because the connate water and rock expansions are usually very small compared to expansion of gas.

Substituting for B_g and B_{gi} in Equation 4-2 and rearranging, the following equation is obtained,

$$\frac{p}{z} = \frac{p_i}{z_i}\left(1 - \frac{G_P}{G}\right) \quad . \tag{4-3}$$

Here it is assumed that the temperature of the reservoir is constant throughout the life of the reservoir. This is a plausible assumption because the reservoir temperature generally does not change. In the above equation (Equation 4-3), p/z and G_P are variables while the other terms are constant. Therefore, a plot of p/z vs. G_P or G_P/G should give a straight line. From the plot, then, G or GIIP is obtained at pressure equal to zero while the ultimate gas reserves are obtained at pressure equal to abandonment pressure. This is a method of obtaining GIIP and ultimate gas reserves which is widely used. However, as it is also well known that Equation 4-3 is obtained by use of several assumptions as mentioned above. In addition, all material balance equations have two important assumptions in common. These assumptions are as follows:

1. The tank model is applicable. That is, it is assumed that the reservoir is not made up of porous medium, and in the system there is only fluid.

2. Changes of pressure and fluid properties are instantaneous and uniform throughout the system. Equivalently, it is assumed that rate of fluid movement in the system is approximately approaching infinity. Alternatively, it may be said that the system has zero dimension.

These two assumptions are quite unrealistic because there is a porous medium in the system, and changes of pressure and fluid properties are not instantaneous. Therefore, one of the objectives of this study is to investigate whether these assumptions will have significant effect such that the gas material balance equation cannot be applied for gas reservoirs. This investigation will be very useful for engineers who use or intend to use material balance equation for gas reservoirs.

4.2 Pseudo-Steady State Equation

In using the material balance equation, it is necessary to have an average reservoir pressure ($\bar{p}$). The average reservoir pressure will be calculated using the pseudo-steady state equation as follows:

$$m(\bar{p}) - m(p_{wf}) = 50{,}300\frac{P_{sc}q_gT}{T_{sc}kh}\left(\frac{1}{2}\ln\left(\frac{10.06A}{C_Ar_w^2}\right) - \frac{3}{4} + s + D'|q_g|\right), \quad (4\text{-}4)$$

where $m(\bar{p})$ = pseudo-pressure at average pressure at any time t (psia^2/ cp)

$m(p_{wf})$ = pseudo-pressure at well flowing pressure at time t (psia^2/ cp)

p_{sc} = pressure at standard condition = 14.7 psia

T_{sc} = temperature at standard condition = 520 °R (60 °F)

q_g = gas flow rate at time t (Mscf/d)

T = average reservoir temperature (°R)

k = reservoir permeability (md)

h = reservoir thickness (ft)

A = drainage area (ft^2)

C_A = shape factor for specific drainage-area shape and well location, dimensionless

r_w = well radius (ft)

s = skin factor (dimensionless)

D' = modified turbulence factor ((Mscf/d)$^{-1}$)

Equation 4-4 is applicable when the flow of gas in a reservoir is in a pseudo-steady state regime. The pseudo-steady state flow regime is prevailing in a reservoir when all boundaries have influence on the flow of gas in the reservoir. This is, in fact, a condition occurring during production period which is the period that most of the data can be obtained.

Equation 4-4 is used for evaluation of the average reservoir (or drainage area) pressure, $\bar{p}$. The average reservoir pressure is very important for material balance calculation. However, it should be noticed that for Equation 4-4 to accurately estimate average reservoir pressure, all parameters in Equation 4-4 must be equal or close to their actual values. Most of these parameters are reservoir properties or reservoir conditions. Therefore, it is quite difficult to obtain the values which are equal or close to their actual values. This is one of the limitation in obtaining the accurate value of $\bar{p}$. This, in turn, should affect the results obtained when using the material balance equation.

In addition, flowing bottom-hole pressure (p_{wf}) is also required in using Equation 4-4 for calculation of $\bar{p}$. The accurate value of p_{wf} is also difficult to be obtained because there is usually no bottom-hole gauges in the well during production. It is possible that p_{wf} is calculated by using the flowing surface pressure. However, here an equation or correlation to calculate pressure loss in the tubing is also needed. It is again very unfortunate that there is no equation or correlation that can give exact pressure loss for flow of well fluid in the tubing. All equations or correlations available can give only approximated value of pressure loss. It is, then, clear that the best we can do is to use the best estimation of p_{wf}, and hence the best estimation of $\bar{p}$.

Another point needed to be considered is that Equation 4-4 can be used only when the well produces at constant rate. To use the equation for variable rate case, one needs to use superposition concept. This concept is well known and several text books in well testing or reservoir engineering can be consulted if necessary.

4.3 Compressibility Factor Calculation

Because the reservoir simulation used in this study does not provide a correlation for calculation of compressibility factor (z), the equation proposed by Dranchuk and Abou-Kassem[23] is used here. This equation is as follows:

$$Z = 1+\left(A_1+\frac{A_2}{T_R}+\frac{A_3}{T_R^3}+\frac{A_4}{T_R^4}+\frac{A_5}{T_R^5}\right)\rho_R+\left(A_6+\frac{A_7}{T_R}+\frac{A_8}{T_R^2}\right)\rho_R - A_9\left(\frac{A_7}{T_R}+\frac{A_8}{T_R^2}\right)\rho_R^5 + A_{10}\left(1+A_{44}\rho_R^2\right)\frac{\rho_R^2}{T_R^3}e^{-A_{11}\rho_R^2} \qquad (4\text{-}5)$$

$$\rho_R = \frac{Z_C P_R}{Z T_R} \qquad (4\text{-}6)$$

where,

Z_c = 0.270

$$P_R = \frac{P}{P_C} \qquad (4\text{-}7)$$

$$T_R = \frac{T}{T_C} \qquad (4\text{-}8)$$

A_1 = 0.3265

A_2 = -1.0700

A_3 = -0.5339

A_4 = 0.01569

A_5 = -0.05165

A_6 = 0.5475

A_7 = -0.7361

A_8 = 0.1844

A_9 = 0.1056

A_{10} = 0.6134

and A_{11} = 0.7210