ปัญหาการเลือกพอร์ตการลงทุนบนพื้นฐานของฟังก์ชันการสูญเสียแบบเอกซ์โพเนนเชียล ภายใต้ตัวแบบทวินามที่เป็นอิสระต่อกันในเซต 50

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PORTFOLIO SELECTION PROBLEM BASED ON EXPONENTIAL LOSS FUNCTION UNDER INDEPENDENT BINOMIAL MODEL IN SET50

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ในวิทยานิพนธ์ฉบับนี้ เราศึกษาปัญหาการจัดสรรพอร์ตการลงทุนในตลาดหลักทรัพย์แห่งประเทศไทย โดยพิจารณาจากหุ้นของบริษัทที่ใหญ่ที่สุด 48 แห่งในแง่ของมูลค่าหลักทรัพย์ตามราคาตลาด โดยมีข้อกำหนดว่านักลงทุนจะต้องลงทุนภายใต้ 48 บริษัทนี้เท่านั้น และความเสี่ยงจะวัดจากฟังก์ชันการสูญเสียแบบเอกซ์โพเนนเชียล หุ้นที่พิจารณาจะต้องเป็นอิสระต่อกันและราคาของหุ้นเมื่อครบกำหนดจะถือว่าเป็นไปตามแบบจำลองทวินาม การทดสอบสหสัมพันธ์แบบเพียร์สันถูกใช้สำหรับการเลือกหุ้นที่มีคุณสมบัติเป็นอิสระต่อกัน ในการศึกษานี้ เราสนใจนักลงทุนที่ลงทุนในหุ้นที่แตกต่างกัน สองหุ้น สามหุ้น และสี่หุ้น อย่างไรก็ตาม เราสมมติว่านักลงทุนซื้อหุ้นครั้งแรกด้วยทรัพย์สินที่มีและถือไว้จนกว่าจะครบกำหนดโดยไม่มีการปรับเปลี่ยนการลงทุน สุดท้าย เราจะตรวจสอบประสิทธิภาพของพอร์ตการลงทุนที่เหมาะสมที่สุดที่ได้จากฟังก์ชันการสูญเสียแบบเอกซ์โพเนนเชียลด้วยวิธีการทดสอบย้อนหลัง

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In this thesis, we investigate the portfolio allocation problem in the Stock Exchange of Thailand by considering the stocks of the 48 biggest companies in terms of market capitalization, with the requirement that an investor wants to invest under these 48 assets only and their risk is measured by the exponential loss function. The stocks under consideration must be independent and the prices of the stocks at the maturity time are assumed to follow the well-known binomial model. The Pearson correlation test is used for selecting stocks with independence property. In this study, we are interested in an investor investing in two, three, and four distinct stocks. Anywise, we suppose that an investor buys assets at an initial time with an initial wealth and holds them until maturity time without rebalancing. Finally, we examine the efficiency of the obtained optimal portfolio with the backtesting method.

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CHAPTER I

INTRODUCTION

1.1 Motivation and Literature Surveys

One of the most important, probably the hardest, tasks in investing is choosing the best portfolio out of many assets available in financial markets. However, the word "the best" is subjective. One may view the best portfolio as the one with minimal risk. However, risk can be viewed differently from one investor to another. Moreover, the views on the future prices of the assets are also not certain, thus subjective.

The most famous risk measure was proposed by Markowitz [17] in 1959 which won him a noble price in 1990. The Markowitz framework measured risk as the variance of the terminal wealth. Later on, there were more works on some other risk measures such as Value-at-risk (VaR) [10] and Conditional Value-at-Risk (CVaR) [16]. Both VaR and CVaR, stated in [2, 4, 13, 14], measure the loss occurring at the tail. However, CVaR is more popular in practice as VaR is not sub-additive and convex. In 2018, Armstrong et al. [3] viewed the exponential loss function as a risk measure. This loss function is more sensible than the ones mentioned above as it does not penalize both upside and downside like that of Markowitz and does not ignore the risk everywhere except the tail like VaR and CVaR.

1.2 Research Objectives

In this work, we apply the portfolio optimization model based on the exponential loss function [3] to the Stock Exchange of Thailand (SET). As there are many stocks in the exchange, the questions are that if an investor would like to invest in only n stocks out of all tradable assets, which stocks should he/she invest in and how much should the fund be allocated to each asset? These questions lead us to a portfolio optimization model which helps the investor to select his/her own portfolio out of the stocks in SET50.

1.3 Scope and Assumptions

In this thesis, we consider the stocks in SET50 using the data from 3 January 2018 to 24 January 2020 (506 trading days). However, since the data for two stocks, AWC and OSP, are not available, only 48 stocks are considered in this research study. Additionally, this work follows the assumptions described in Assumptions 1.1.

Assumptions 1.1.

- The market is assumed to be perfectly liquid, meaning that one can buy or sell any amount of a particular asset and there are no transaction costs.
- The asset prices at maturity time are independent and follow the well-known binomial distribution explained in details in Chapter 2.

1.4 Thesis Structure

This thesis is divided into five chapters which are organized as follows. First, Chapter 1 is an introduction of this work, including motivation and literature reviews, research objectives and thesis overview. Chapter 2 presents the background knowledge on the models for asset prices, conditional expectation and a portfolio optimization model. In Chapter 3, we apply the exponential loss function to the portfolio selection problem which is given the prices of the 48 biggest stocks in terms of market capitalization. Then, we estimate some parameters and find an optimal portfolio based on the exponential loss function. Next, numerical results and discussion regarding the research are demonstrated in Chapter 4. Finally, Chapter 5 provides the conclusion and possible future works.

CHAPTER II

BACKGROUND KNOWLEDGE

In this chapter, the background knowledge about asset pricing models, conditional expectation and the well-known portfolio optimization model are introduced. Let us first assume that an investor has an initial wealth w to buy assets from the financial market which is a set J consisting of all tradeable assets. The investor buys the assets at time $t = 0$ and holds them until maturity time T without rebalancing. Moreover, the costs of buying assets denoted by the vector $\mathbf{S}_0 = (S_0^i)_{i\in J}$ are assumed to be known at time $t = 0$. The values of the assets at time $t = T$ denoted by the random vector $\mathbf{S}_T = (S_T^i)_{i\in J}$ are on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Also, the portfolio $\mathbf{x} = (x^i)_{i\in J}$ is a vector of units bought or sold in assets $i \in J$ at time $t = 0$. Note that these units can be both positive or negative values. Particularly, the negative unit means the short selling.

2.1 Binomial Models for Pricing Asset

One of the widely used models for asset prices is a geometric Brownian motion which can be expressed in the following stochastic differential equation (SDE)

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \tag{2.1}$$

where W_t is the standard Brownian motion called a Wiener process, S_t is the asset price at time t, μ is the growth rate, and σ is the volatility. Although this model is popular, solving for the asset prices S_t in (2.1) is not suitable for the data used in this work because the logarithm of its return is not normally distributed. Thus, a numerical method plays an important role to approximate S_t. However, the consuming time to simulate the asset prices S_t obtained from several numerical methods is actually very expensive.

Therefore, in this work, we propose another model that is relatively easy to seek for the asset prices at the maturity time $\mathbf{S}_T$ by applying the well-known binomial distribution given in [6]. It can be expressed as follows:

$$S_T^i = \begin{cases} S_0^i(1+u_i) & \text{with probability } p_i, \\ S_0^i(1+d_i) & \text{with probability } 1-p_i, \end{cases} \tag{2.2}$$

which is called a "binomial model for asset price", where S_0^i and S_T^i are the asset prices at the initial and the maturity times, respectively, of the i^{th} stock, $-1 < d_i < 0 < u_i$ and $0 < p_i < 1$ for $i = 1, 2, \ldots, n(J)$. However, the required parameters of this model are u_i, d_i and p_i. We can estimate them by using historical data (k trading days) with the adjusted formula of (2.2) that employs the stock prices of two consecutive trading days as follows:

$$S_t^i = \begin{cases} S_{t-1}^i(1+u_i) & \text{with probability } p_i, \\ S_{t-1}^i(1+d_i) & \text{with probability } 1-p_i, \end{cases} \tag{2.3}$$

where S_{t-1}^i and S_t^i are the i^{th} stock prices at consecutively previous and current days for $t > 0$. Furthermore, we can compute the relative return at time t of the i^{th} stock by

$$r_t^i = \frac{S_t^i - S_{t-1}^i}{S_{t-1}^i}, \tag{2.4}$$

which can be used to estimate u_i and d_i by the following formulas

$$u_i = \frac{1}{n} \sum_{t:\ r_t^i > 0} r_t^i, \tag{2.5}$$

$$d_i = \frac{1}{m} \sum_{t:\ r_t^i \leq 0} r_t^i, \tag{2.6}$$

where u_i is an average of the n positive relative returns and d_i is an average of the m non-positive relative returns. In fact, we have $m + n = k - 1$ and $p_i = \frac{n}{k-1}$ which means the probability of positive relative returns. To make it clearer, we will illustrate the process for estimating the parameters u_i, d_i and p_i via Example 2.1.

Example 2.1. Assume that stock prices for 10 consecutive trading days are as follows:

$$79.5,\quad 78.25,\quad 79,\quad 80,\quad 80.75,\quad 81,\quad 80.25,\quad 80,\quad 80.25,\quad 80.5$$

down   up   up   up   up   down   down   up   up

First, we can find the relative return r_t^i of each trading day $t \in \{1, 2, 3, \dots, 9\}$ by using (2.4) and their obtained values are shown in Table 2.1. Then, these values are utilized to compute u_i and d_i by (2.5) and (2.6), respectively, as also shown in Table 2.1. Obviously, from Table 2.1, there are 6 positive relative returns out of all (9) relative returns. Thus, the probability $p_i = \frac{6}{9} = \frac{2}{3}$.

Table 2.1: The values of the relative return r_t^i of each trading day

$r_t^i > 0$	$r_t^i \le 0$
$r_2^i = \frac{79-78.25}{78.25} = 0.00958$	$r_1^i = \frac{78.25-79.5}{79.5} = -0.0157$
$r_3^i = \frac{80-79}{79} = 0.0127$	$r_6^i = \frac{80.25-81}{81} = -0.00926$
$r_4^i = \frac{80.75-80}{80} = 0.00934$	$r_7^i = \frac{80-80.25}{80.25} = -0.00311$
$r_5^i = \frac{81-80.75}{80.75} = 0.0031$	
$r_8^i = \frac{80.25-80}{80} = 0.0031$	
$r_9^i = \frac{80.5-80.25}{80.25} = 0.0031$	
$\therefore u_i = \frac{r_2^i+r_3^i+r_4^i+r_5^i+r_8^i+r_9^i}{6} = 0.00682$	$\therefore d_i = \frac{r_1^i+r_6^i+r_7^i}{3} = -0.00936$

2.2 Expectation and Variance

The expected value or expectation of a random variable X is denoted by $\mathbb{E}[X]$ or μ_X. If we observe random values X, then their mean will be approximately equal to $\mathbb{E}[X]$. For continuous and discrete random variables, the expectation is defined in different ways.

Definition 2.1. Let X be a continuous random variable with probability density function $f_X(x)$. The expected value of X is

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx.$$

Definition 2.2. Let X be a discrete random variable with probability mass function $f_X(x)$. The expected value of X is

$$\mathbb{E}[X] = \sum_x x f_X(x) = \sum_x x P(X = x).$$

The variance of a random variable X is a measure of how dispersed its value is. In other words, the variance measures how far the values of X are from their mean. The exact definition of the variance is given below.

Definition 2.3. Let X be a random variable. The variance of X is

$$\mathrm{Var}[X] = \mathbb{E}[(X - \mu_X)^2] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2.$$

Theorem 2.1. *If random variables $X_1, X_2, \ldots, X_n$ are independent, then for any Borel measurable functions $f_1, f_2, \ldots, f_n$, we have*

$$\mathbb{E}[f_1(X_1)f_2(X_2)\ldots f_n(X_n)] = \mathbb{E}\left[f_1(X_1)\right]\mathbb{E}\left[f_2(X_2)\right]\ldots\mathbb{E}\left[f_n(X_n)\right].$$

2.3 Independence of Random Variables

In this section, we demonstrate the well-known models for testing the independence of random variables, consisting of Pearson correlation, Spearman's rank correlation and Kendall's τ. Moreover, their advantages and limitations are also addressed here.

2.3.1 Pearson Correlation Model

This section addresses the most popularly applied correlation concept in observed data, namely, the Pearson correlation model. Although, there are severe limitations when applied in finance, the Pearson correlation model is still the most widely applied in this field, which we will mention later. Let's take a closer look at the Pearson correlation model.

Let $S_x = \{x_1, x_2, ..., x_n\}$ and $S_y = \{y_1, y_2, ..., y_n\}$ be two samples from the continuous random variables X and Y, respectively. The Pearson correlation coefficient r_{xy} for the samples is defined as follows:

$$r_{xy} = \frac{s_{xy}}{s_x s_y} \tag{2.7}$$

where s_{xy} is the sample covariance while s_x and s_y are the sample standard deviations of S_x and S_y, respectively. The sample covariance s_{xy} is defined below.

$$s_{xy} = \frac{1}{n-1}\sum_{t=1}^{n}(x_t - \bar{x})(y_t - \bar{y}), \tag{2.8}$$

where $\bar{x}$ and $\bar{y}$ are the sample means of S_x and S_y, respectively, i.e,

$$\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t, \tag{2.9}$$

$$\bar{y} = \frac{1}{n}\sum_{t=1}^{n} y_t. \tag{2.10}$$

The sample standard deviations s_x and s_y are defined as follows:

$$s_x = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(x_t - \bar{x})^2}, \tag{2.11}$$

$$s_y = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(y_t - \bar{y})^2}. \tag{2.12}$$

Therefore, from (2.7), the Pearson correlation coefficient r_{xy} of the samples S_x and S_y can be written as the following formula.

$$r_{xy} = \frac{\sum_{t=1}^{n}(x_t - \bar{x})(y_t - \bar{y})}{\sqrt{\sum_{t=1}^{n}(x_t - \bar{x})^2}\sqrt{\sum_{t=1}^{n}(y_t - \bar{y})^2}} \tag{2.13}$$

The population Pearson correlation between X and Y is defined by

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sigma(X)\sigma(Y)}. \tag{2.14}$$

In finance, the properties of the Pearson correlation coefficient are standard statistical tools to analyze the behavior of the data. For example, in 2005, the Pearson correlation coefficient was applied by Altman et al. [1] to verify the negative correlation between the default rates and recovery rates; in 2006, the correlations between the returns of the asset with sector specific regional factor loadings was studied by Fitch [15]; and Das et al. [8] regressed the mean of the default with market volatility and also debt to asset ratios.

2.3.2 Spearman's Rank Correlation

Spearman's rank correlation is a measure of ordinal correlation given in [11]. This indicates that the order of the items in a set is more important than their numerical values for determining the connection. For ranking variables, the Spearman's correlation coefficient is also known as the Pearson correlation coefficient. A perfect correlation coefficient of 1 will arise if an increase in the variables x_i is always accompanied by an increase in y_i, regardless of the magnitude of the increase, and vice versa. In the sense that it may be used without knowing the joint distribution of the variables, the Spearman correlation technique is nonparametric. ρ_S representing the Spearman rank correlation coefficient is defined by

$$\rho_S = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}, \tag{2.15}$$

where d_i is the difference in rank for x and y.

2.3.3 Kendall's τ

Kendall's τ is an additional, widely used ordinal correlation metric in finance. The Kendall's τ is nonparametric, like the Spearman's correlation coefficient. If a rise of the variables x and y, regardless of the numerical increase, then the correlation coefficient perfectly yield the value at 1. The two rank correlation measurements are not equivalent in most other circumstances. Kendall's τ is given in [11] and defined as follows:

$$\tau = \frac{n_c - n_d}{n(n-1)/2}, \tag{2.16}$$

where n_c is the number of concordant data pairs and n_d is the number of discordant pairs. Rigorously, a concordant pair denotes mathematically as the observed pair for all $t \neq t^*$ where $x_t > y_t$ and $x_{t^*} > y_{t^*}$ or $x_t < y_t$ and $x_{t^*} < y_{t^*}$. On the other hand, the observed discordant data pair is defined as the cases for all $t \neq t^*$ where $x_t > y_t$ and $x_{t^*} < y_{t^*}$ or $x_t < y_t$ and $x_{t^*} > y_{t^*}$. In this respect, for the case of $x_t = y_t$ or $x_{t^*} = y_{t^*}$, the pair is defined neither concordant nor discordant.

2.3.4 Should We Apply Spearman's Rank Correlation and Kendall's τ?

Meissner [11] concludes that the properties and applications of the statistical correlation measures in finance are limited in various situations. One primary concern with the Pearson correlation coefficients is that the coefficients are evaluated for the linear relationships. However, real-world samples in finance are always nonlinear. In contrast, Spearman's rank and Kendall's τ are statistical rank correlation measures that should be only applied with the financial variables which are ordinal such as the rating categories. From the above reason, this work will focus on the Pearson correlation model.

2.3.5 Significance of Pearson Correlation Coefficient Test

In order to describe the independence of the stock pairs, this subsection addresses a statistical correlation approach, which is the testing for the significance of the Pearson correlation coefficient. Because of its mathematical simplicity of the Pearson correlation model, it becomes one of the most popularity applied correlation and independence concepts in finance. However, as mentioned in the previous subsection, there are some major problems for the Pearson correlation model. For example, it measures only linear relationships but the most of financial correlations are always nonlinear. Most importantly, it should be noted that the zero correlation calculated by the Pearson correlation approach does not necessarily imply that the pair of financial data are independent. For this very reason, the Pearson correlation model may lead us misunderstanding. However, the outcome performed by the Pearson correlation model can give a good approximation for nonlinear financial correlations found in real-world problems [11]. In this work, the testing for the significance of the Pearson correlation coefficient is applied inevitably. We just need to be aware of its limitations.

Briefly, the Pearson correlation coefficient of the sample, r, with the sample size, n, can be applied to test whether the relationship between the two variables is significant. Technically speaking, r is an estimate of the Pearson correlation of the population, ρ, which can infer whether ρ is significantly different from 0 as described in the following hypothesis

$$\begin{array}{ll} \text{Null hypothesis} & H_0 : \rho = 0, \\ \text{Alternative hypothesis} & H_a : \rho \neq 0. \end{array}$$

To decide whether to reject the null hypothesis or not, one can check by comparing the t- and t^*-values which are the test statistic and the critical value of t, respectively. Given the significance level α, if the t-value is greater than the t^*-value (or p-value $< \alpha$), then the relationship is statistically significant. This implies that the sample allows us to reject the null hypothesis, H_0, and vice versa for the case that the t-value is less than the t^*-value.

Anywise, for our convenience, this work applies the basic package in Mathematica which is `IndependenceTest` with its option, namely, `PearsonCorrelation` and $\alpha = 0.05$ that will perform the Pearson correlation coefficient test described above with significant level $\alpha = 0.05$.

2.4 Portfolio Optimization Model

The portfolio optimization model is specified as a constrained disutility-minimization problem. The disutility function is used to measure the unhappiness of investors. Many works of literature use this function to describe risk. Due to subjectivity, this function is severally proposed in different forms.

The simplest disutility is to use the expectation, $v(x) = \mathbb{E}[x]$ for $x \in L^{\infty}(\Omega, \mathcal{F}, \mathcal{P})$, see [5] for more details. The portfolio selection problem using the expectation as a disutility function can be written as,

$$\left.\begin{array}{ll}\text{Minimize} & \mathbb{E}[C - \mathbf{S}_T \cdot \mathbf{x}], \\ \text{subject to} & \mathbf{S}_0 \cdot \mathbf{x} \leq w,\end{array}\right\} \tag{2.17}$$

where $\mathbf{x} \in \mathbb{R}^n$ is a portfolio consisting of n assets, $\mathbf{S}_0 \in \mathbb{R}^n$ is a vector of initial costs of the assets, $\mathbf{S}_T$ is a vector of random prices of the assets and C is a liability claim which is a future expense.

Since the expectation of a claim is a constant, the problem is just to minimize the negative value of the portfolio at time $t = T$. Obviously, the portfolio which yields the minimized value is the one which invests all initial money in the asset which has the highest return.

One of the drawbacks of this disutility is that it will not take the distribution of the data into account. Although the portfolio yields the minimum expected value of $C - \mathbf{S}_T \cdot \mathbf{x}$, the biggest loss could be unexceptionably high.

2.5 Mean-Variance Criterion

In 1959, Markowitz [17] proposed a mathematical formulation for a portfolio selection problem by taking a variance of the terminal wealth as a risk measure. Given a required expected return r, the optimal portfolio is a combination of assets yielding the minimal variance. The formulation can be expressed as

$$\left.\begin{array}{ll}\text{Minimize} & \mathrm{Var}\left[\mathbf{S}_T \cdot \mathbf{x}\right], \\ \text{subject to} & \mathbf{S}_0 \cdot \mathbf{x} \leq w, \\ & \mathbb{E}\left[\mathbf{S}_T \cdot \mathbf{x}\right] \geq r,\end{array}\right\} \tag{2.18}$$

where $\mathbf{S}_T \cdot \mathbf{x}$ denotes value of the portfolio at the maturity time T which $\mathbf{x} \in \mathbb{R}^n$ is a portfolio consisting of n assets, $\mathbf{S}_0 \in \mathbb{R}^n$ is a vector of initial costs of the assets and $\mathbf{S}_T$ is a vector of random prices of the assets.

One of the benefits of this disutility does not only provide the low expectation of losses, but the portfolio also gives the low variance. However, it also eliminates the opportunity to take advantage of the possibility that the net investment is greater than the expectation, since it minimizes the fluctuation for both sides of the distributions.

CHAPTER III

PORTFOLIO OPTIMIZATION BASED ON EXPONENTIAL LOSS FUNCTION UNDER BINOMIAL MODEL

In this chapter, we will show the exponential loss function which is the main model used in this work. Moreover, we apply the binomial model with the exponential loss function in order to describe the prices at maturity time.

3.1 Exponential Loss Function

In 2018, Armstrong et al. [3] measured the risk by the exponential loss function. The scheme was illustrated by implementing the model for a portfolio selection in S&P 500 options markets. The model can be written as follows

$$\left.\begin{aligned} &\text{Minimize} \quad \mathbb{E}[v(C - \mathbf{S}_T \cdot \mathbf{x})], \\ &\text{subject to} \quad \mathbf{S}_0 \cdot \mathbf{x} \leq w, \end{aligned}\right\} \tag{3.1}$$

where $v(x) = e^{\frac{\lambda x}{w}}$ is the exponential loss function, $\lambda > 0$ is a risk aversion factor, C is a liability, $\mathbf{S}_T \cdot \mathbf{x}$ denotes value of the portfolio at the maturity time T, $\mathbf{x} \in \mathbb{R}^n$ is a portfolio consisting of n assets, $\mathbf{S}_0 \in \mathbb{R}^n$ is a vector of initial costs of the assets and $\mathbf{S}_T$ is a vector of random prices of the assets. The exponential loss function has an advantage over the mean-variance criterion because, unlike the mean-variance criterion which penalizes both profit and loss, it only sees the loss as risk.

If we apply the properties of expectation and the binomial model (2.3) into the model (3.1), we obtain the following proposition,

Proposition 3.1. Suppose that $\mathbf{S}_T$ follows the binomial model as described by (2.2). The model (3.1) can be expressed as

$$\left.\begin{array}{ll} \text{Minimize} & e^{\frac{\lambda C}{w}} \displaystyle\prod_{i=1}^{n} \left(p_i \, e^{-\frac{\lambda}{w}(1+u_i)S_0^i x_i} + (1-p_i) \, e^{-\frac{\lambda}{w}(1+d_i)S_0^i x_i} \right), \\ \text{subject to} & \mathbf{S}_0 \cdot \mathbf{x} \leq w, \end{array}\right\} \tag{3.2}$$

under the assumption that S_T^i's for $i = 1, 2, 3, \ldots, n$ are mutually independent.

Proof. By considering (3.1) applied with the binomial model, we have

$$\begin{aligned} \mathbb{E}\left[e^{\frac{\lambda}{w}(C - \mathbf{S}_T \cdot \mathbf{x})} \right] &= e^{\frac{\lambda C}{w}} \, \mathbb{E}\left[e^{-\frac{\lambda}{w} \mathbf{S}_T \cdot \mathbf{x}} \right] \\ &= e^{\frac{\lambda C}{w}} \, \mathbb{E}\left[e^{-\frac{\lambda}{w} \sum_{i=1}^{n} S_T^i x_i} \right] \\ &= e^{\frac{\lambda C}{w}} \, \mathbb{E}\left[e^{-\frac{\lambda}{w}(S_T^1 x_1)} \, e^{-\frac{\lambda}{w}(S_T^2 x_2)} \cdots e^{-\frac{\lambda}{w}(S_T^n x_n)} \right] \\ &= e^{\frac{\lambda C}{w}} \, \mathbb{E}\left[e^{-\frac{\lambda}{w}(S_T^1 x_1)} \right] \mathbb{E}\left[e^{-\frac{\lambda}{w}(S_T^2 x_2)} \right] \cdots \mathbb{E}\left[e^{-\frac{\lambda}{w}(S_T^n x_n)} \right] \\ &= e^{\frac{\lambda C}{w}} \prod_{i=1}^{n} \mathbb{E}\left[e^{-\frac{\lambda}{w}(S_T^i x_i)} \right] \\ &= e^{\frac{\lambda C}{w}} \prod_{i=1}^{n} \left(p_i \, e^{-\frac{\lambda}{w}(1+u_i)S_0^i x_i} + (1-p_i) \, e^{-\frac{\lambda}{w}(1+d_i)S_0^i x_i} \right). \end{aligned}$$

Note that by applying the properties of mutual independence into the third line of this proof, it can be separated into the product of expectation as shown in the fourth line. □

Next, we will show that the objective function in (3.2) is a convex function. Let us provide some important properties of convex function as the following lemmas, given in [7, 12].

Lemma 3.1 ([12]). *Let $f_i : \mathbb{R}^n \mapsto \mathbb{R}$ be convex functions for all $i = 1, 2, \ldots, m$. Then, the following functions are convex as well:*

(i) *The multiplication by scalars λf_i for any $\lambda > 0$.*

(ii) *The sum function $\sum_{i=1}^{m} f_i$.*

Proof. The proof of this lemma can be found in the reference [12]. □

Lemma 3.2 ([7]). *For any $\mathbf{b} \in \mathbb{R}^n$, the function $f : \mathbb{R}^n \mapsto \mathbb{R}^+$ defined as $f(\mathbf{x}) = e^{\mathbf{b}^\top \mathbf{x}}$ is convex.*

Proof. It is easy to see that the Hessian matrix of $f(\mathbf{x})$ is $e^{\mathbf{b}^\top \mathbf{x}}\mathbf{b}\mathbf{b}^\top = f(\mathbf{x})\mathbf{b}\mathbf{b}^\top$. Next, we will show that the Hessian matrix of $f(\mathbf{x})$ is semi-positive definite. Let $\mathbf{v} \in \mathbb{R}^n$ be a nonzero vector, then we have

$$\mathbf{v}^\top \left(f(\mathbf{x})\mathbf{b}\mathbf{b}^\top \right) \mathbf{v} = f(\mathbf{x}) \left(\mathbf{v}^\top \mathbf{b} \right) \left(\mathbf{b}^\top \mathbf{v} \right) = f(\mathbf{x}) \left(\mathbf{v}^\top \mathbf{b} \right)^2 \geq 0,$$

because $f(\mathbf{x}) \in \mathbb{R}^+$ and $\left(\mathbf{v}^\top \mathbf{b}\right)^2 \geq 0$. Therefore, the Hessian matrix of $f(\mathbf{x})$ is the semi-positive definite. It is a result that $f(\mathbf{x})$ is a convex function. □

Proposition 3.2. The objective function in (3.2) is convex.

Proof. Let us first define the universal set $\mathcal{U} = \{1, 2, 3, \ldots, n\}$. Then, we rewrite the product term of the objective function into the summation form as follows

$$\begin{aligned}
&e^{\frac{\lambda C}{w}} \prod_{i=1}^{n} \left(p_i e^{-\frac{\lambda}{w}(1+u_i)S_0^i x_i} + (1-p_i) e^{-\frac{\lambda}{w}(1+d_i)S_0^i x_i} \right) \\
&\quad = e^{\frac{\lambda C}{w}} \sum_{Y \subseteq \mathcal{U}} \left(\prod_{i \in Y} p_i e^{-\frac{\lambda}{w}(1+u_i)S_0^i x_i} \prod_{i \in Y^{\mathrm{C}}} (1-p_i) e^{-\frac{\lambda}{w}(1+d_i)S_0^i x_i} \right) \\
&\quad = e^{\frac{\lambda C}{w}} \sum_{Y \subseteq \mathcal{U}} \left(\left(\prod_{i \in Y} p_i \prod_{i \in Y^{\mathrm{C}}} (1-p_i) \right) e^{-\frac{\lambda}{w}\left(\sum\limits_{i \in Y} (1+u_i)S_0^i x_i + \sum\limits_{i \in Y^{\mathrm{C}}} (1+d_i)S_0^i x_i \right)} \right) \\
&\quad = e^{\frac{\lambda C}{w}} \sum_{Y \subseteq \mathcal{U}} \left(a(Y)\, e^{\mathbf{b}(Y)^\top \mathbf{x}} \right), \qquad (3.3)
\end{aligned}$$

where $a(Y) := \prod_{i \in Y} p_i \prod_{i \in Y^c} (1 - p_i)$ is always greater than zero, $\mathbf{b}(Y) \in \mathbb{R}^n$ is the coefficient vector of $\mathbf{x}$ which means that $\mathbf{b}(Y)^\top \mathbf{x} := -\frac{\lambda}{w} \left(\sum_{i \in Y} (1 + u_i) S_0^i x_i + \sum_{i \in Y^c} (1 + d_i) S_0^i x_i \right)$ is a linear combination of $x_1, x_2, \ldots, x_n$ and Y^c is a complement of the set Y. Note that an empty product, when $Y = \varnothing$, is defined to be 1.

From (3.3), we can see that $\mathbf{b}(Y) \in \mathbb{R}^n$ for all $Y \subseteq \mathcal{U}$; thus, $e^{\mathbf{b}(Y)^\top \mathbf{x}}$ is actually the convex function by Lemma 3.2. Also, since $e^{\frac{\lambda C}{w}} \geq 0$ and $a(Y) \geq 0$, by using Lemma 3.1, hence (3.3) is obviously convex function. □

Since the objective function and the constraint of (3.2) are both convex, (3.2) is a convex optimization problem. In fact, if we know that the considered problem is the convex optimization, then its local minimizer is also a global minimizer. Therefore, if we can find a local minimum of the convex optimization problem by using certain methods (e.g. `NMinimize` command in Mathematica), the found result indeed becomes the global minimum.

3.2 The Market, Views, and Preferences

In this work, we consider 48 biggest stocks, in terms of market capitalization, included in SET50 index computation in SET from 3 January 2018 to 24 January 2020 (506 trading days) excluding AWC and OSP as their data are not available. The quotes were obtained from SET website [18] on 25 January 2020. The market is assumed to be perfectly liquid, meaning that one can buy or sell any amount of a particular asset and there are no transaction costs. We assume that an investor, with an initial wealth w, buys assets at time $t = 0$ and holds them until time T without rebalancing.

In addition, we collect the combinations of two, three and four assets that are independent by using Pearson correlation test with 5% significant level as mentioned in Chapter 2. Remark that the Pearson correlation test measures only the linear uncorrelation, not the real independence.

Due to the assumption of Proposition 3.1, the stocks within the three- and four-asset combinations under consideration must be mutually independent. However, testing for mutually independence is complicated while pairwise independence can be tested more easily. Moreover, Feller [9, p. 126] mentioned that three-event instances with pairwise independent events but not mutually independent are scarce. Therefore, we will use Pearson correlation test for pairwise independence instead to approximate the mutually independence of the stocks within such combinations.

We found two- and three-combination of 48 stocks that are independent, including, 42 pairs out of $\binom{48}{2} = 1{,}128$ pairs and 3 three-combinations out of $\binom{48}{3} = 17{,}296$ combinations. Unfortunately, there are no independent four-combinations under 5% significant level. All independent combinations are shown in Tables 3.1 and 3.2. Thus, we apply the model (3.1) with the data in these tables to seek the optimal portfolio for different "risk aversion" of investors. Finally, we will investigate the efficiency of the optimal portfolio by using backtesting as provided in Chapter 4.

Table 3.1: The stock pairs that are considered independent by Pearson correlation test

Stock pairs		***p*-value**	**Stock pairs**		***p*-value**	**Stock pairs**		***p*-value**
RATCH	TU	0.051239	ADVANC	TU	0.092964	BDMS	TOP	0.239417
BBL	RATCH	0.056201	BTS	PTTEP	0.093095	BTS	TOA	0.262216
KBANK	TOA	0.057090	BTS	IRPC	0.095334	CBG	DELTA	0.278802
BBL	BDMS	0.061888	CPN	DELTA	0.110941	BEM	DELTA	0.283987
BH	RATCH	0.062640	GLOBAL	RATCH	0.112973	BBL	TOA	0.287464
EGCO	KTB	0.066369	KTB	RATCH	0.132111	LH	TOA	0.406997
BH	TOA	0.067611	DELTA	VGI	0.135416	DELTA	TISCO	0.539678
BTS	DTAC	0.068647	BEM	TOA	0.140315	ADVANC	DELTA	0.549352
BJC	DELTA	0.077417	DELTA	RATCH	0.144312	DELTA	TCAP	0.577700
TOA	TU	0.080730	RATCH	TMB	0.144746	DELTA	GULF	0.599336
CPF	VGI	0.081921	KTB	TOA	0.160038	DELTA	TOA	0.654170
TMB	VGI	0.083364	TOA	VGI	0.164075	BDMS	DELTA	0.926874
DELTA	PTTEP	0.087297	DELTA	INTUCH	0.182432	DELTA	GPSC	0.955458
DELTA	DTAC	0.091314	DELTA	LH	0.201683	DELTA	KTC	0.971446

Table 3.2: The three-stock combinations that are considered independent by Pearson correlation test

Three-stock combinations	**Pairwise p-value**								
	Stock pairs		**p-value**	**Stock pairs**		**p-value**	**Stock pairs**		**p-value**
(BEM, DELTA, TOA)	BEM	DELTA	0.283987	DELTA	TOA	0.654170	TOA	BEM	0.140315
(DELTA, LH, TOA)	DELTA	LH	0.201683	LH	TOA	0.406997	TOA	DELTA	0.654170
(DELTA, TOA, VGI)	DELTA	TOA	0.654170	TOA	VGI	0.164075	VGI	DELTA	0.135416

CHAPTER IV

NUMERICAL RESULTS AND DISCUSSION

This chapter shows numerical results of the optimal portfolios obtained from (3.2) under parameters estimated by historical data. Moreover, an important assumption such as divisibility, liquidity and short selling are also provided in this chapter.

4.1 The Experiment

Recall from the previous chapter that this work considers 48 biggest stocks in the terms of market capitalization that are in the SET50 index provided by the Stock Exchange of Thailand from 3 January 2018 to 21 November 2019, excluding AWC and OSP as their data are not available. The quotes were obtained from the SET website [18] on 25 January 2020. Actually, we use the data to estimate parameters u_i, d_i and p_i for the binomial model. In this experiment, we study portfolio selection based on the model (3.1) under Assumptions 1.1.

In this case, the model for optimizing the portfolio is proposed in Proposition 3.1 based on the binomial model in which the condition for mutual independence of all stocks in each combination is required. In the context of independent property, Pearson correlation test for selecting the stocks is applied in this study. We apply the backtesting method to the data from 22 November 2019 to 24 January 2020 (34 trading days) in order to study the volatility of return of the optimal portfolio.

4.2 Parameter Estimation for Binomial Model

In this section, we can find the parameters u_i and d_i for each asset by employing (2.5) and (2.6), respectively. Moreover, the probability p_i of positive returns for each stock is also provided. Thus, these obtained parameters are displayed in Table 4.1.

Table 4.1: Estimated value of u_i, d_i and p_i for each stock i

Stocks	u_i	d_i	p_i	**Stocks**	u_i	d_i	p_i
AOT	0.01146	−0.00714	0.40000	IRPC	0.01874	−0.00859	0.37659
BBL	0.01706	−0.00725	0.42766	IVL	0.02681	−0.00648	0.44255
ADVANC	0.01883	−0.00570	0.42766	KBANK	0.01937	−0.00875	0.41063
BANPU	0.01163	−0.01023	0.45745	KTB	0.01516	−0.00966	0.38085
BDMS	0.01020	−0.01180	0.42766	KTC	0.01135	−0.01153	0.38510
BEM	0.01430	−0.01427	0.38085	LH	0.01025	−0.01314	0.45532
BGRIM	0.01130	−0.00965	0.41277	MINT	0.01727	−0.01168	0.45957
BH	0.01773	−0.00545	0.41277	MTC	0.01453	−0.01042	0.41276
BJC	0.01625	−0.00768	0.41277	PTT	0.01050	−0.00813	0.40638
BPP	0.01037	−0.00074	0.38511	PTTEP	0.01504	−0.00763	0.38510
BTS	0.01449	−0.00711	0.40000	PTTGC	0.01321	−0.00726	0.39574
CBG	0.01464	−0.01144	0.33192	RATCH	0.02337	−0.01605	0.36170
CPALL	0.02132	−0.01845	0.44255	SAWAD	0.01167	−0.01216	0.42767
CPF	0.01283	−0.01478	0.39149	SCB	0.02158	−0.00750	0.38936
CPN	0.01847	−0.01092	0.45106	SCC	0.01841	−0.01223	0.41489
DELTA	0.00138	−0.00967	0.40851	TCAP	0.01102	−0.00789	0.47446
DTAC	0.00957	−0.01324	0.44043	TISCO	0.01181	−0.00922	0.44468
EA	0.01380	−0.01641	0.40638	TMB	0.01374	−0.00973	0.39361
EGCO	0.01210	−0.00950	0.39362	TOA	0.01296	−0.01078	0.42760
GLOBAL	0.01256	−0.00914	0.44255	TOP	0.01050	−0.01193	0.35531
GPSC	0.01591	−0.00813	0.39575	TU	0.00968	−0.00691	0.40638
GULF	0.01982	−0.01006	0.37447	VGI	0.01517	−0.01167	0.36808
HMPRO	0.01736	−0.00755	0.37447	WHA	0.01411	−0.00792	0.40425
INTUCH	0.01451	−0.01800	0.45532	TRUE	0.01903	−0.01031	0.33617

4.3 Numerical Results

We examine the portfolio optimization models given in Chapter 3 by implementing the model (3.2) with $\lambda \in \{1, 2, \ldots, 10\}$, $C = 0$ and $w = 100{,}000$. In this experiment, we estimated the i^{th} asset price S_T^i by using the values p_i, u_i and d_i, where p_i is the probability that the i^{th} stock price goes up in a day estimated from the 482-day data, and u_i and d_i are the average of returns for each day that the i^{th} stock price goes up and down, respectively.

Our numerical results of (3.2) are solved based on the convex methods. It is implemented by applying the basic packages in Mathematica and the implemented code is provided in the appendix. In addition, all our calculations use the software of Wolfram Mathematica 9 that runs on a PC with the following configurations: Intel(R) Core(TM) i7-8750H, CPU @2.20GHz, 16.0GB RAM, Windows 10, 64-bit Operating System.

The obtained numerical results including the optimized portfolios for each $\lambda \in \{1, 2, \ldots, 10\}$ and objective values of model are demonstrated in Table 4.2. From the 42 stock pairs that are independent and the exponential loss function based framework (3.2), the pair (DELTA, GULF) is the best choice to invest in for risk aversions from 1 to 10 as they give the minimum exponential loss values. The optimal pairs are acquired by doing portfolio optimization for all possible pairs. The pair with the smallest objective value is the optimal pair. The result shows optimal portfolios; for example, in the case of $\lambda = 1$, an investor should buy DELTA for -16738.80327 and GULF for 5450.16883. Recall that the negative value means short selling.

Moreover, we apply the backtesting method for 34 trading days in order to observe the fluctuate of return of portfolio given by the model (3.2). The obtained results are shown in Figure 4.1 for risk-lover investors and Figure 4.2 for risk-averse investors. We also compute the mean and standard deviation of daily returns for $\lambda \in \{1, 2, \ldots, 10\}$ as shown in Table 4.3. From this table, we notice here that the mean of returns closes to zero and the standard deviation of returns ever decreases for increasing λ values, which correspond to the risk preference of investors.

For the combination of three stocks, we found that the best choice is (BEM, DELTA, TOA) as shown in Table 4.4. The backtesting results of (BEM, DELTA, TOA) are depicted in Figure 4.3 for $\lambda \in \{1, 2, \ldots, 5\}$ and Figure 4.4 for $\lambda \in \{6, 7, \ldots, 10\}$. Furthermore, their results provide the mean and standard deviation similar to the stock pairs where the mean of returns tends towards zero and the standard deviation of returns decreases for increasing λ values, as displayed in Table 4.5. However, we observe that all means obtained are negative values which mean that investors have an opportunity to lose the profit.

Table 4.2: Numerical results for stock pairs obtained from (3.2).

λ	Stock Pairs	Portfolios	Obj. Value
1	(DELTA, GULF)	$(-16738.80327, 5450.16883)$	3.63×10^{-1}
2	(DELTA, GULF)	$(-7772.13334, 2855.26889)$	1.34×10^{-1}
3	(DELTA, GULF)	$(-4783.36031, 1990.33609)$	4.91×10^{-2}
4	(DELTA, GULF)	$(-3289.05870, 1557.89426)$	1.81×10^{-2}
5	(DELTA, GULF)	$(-2392.54341, 1298.44817)$	6.64×10^{-3}
6	(DELTA, GULF)	$(-1794.91943, 1125.49941)$	2.44×10^{-3}
7	(DELTA, GULF)	$(-1368.08891, 1001.97724)$	8.99×10^{-4}
8	(DELTA, GULF)	$(-1048.00291, 909.34629)$	3.31×10^{-4}
9	(DELTA, GULF)	$(-799.07871, 837.30914)$	1.22×10^{-4}
10	(DELTA, GULF)	$(-599.96667, 779.68732)$	4.48×10^{-5}

Table 4.3: Backtesting of (DELTA, GULF) from model (3.2).

λ	Mean of Daily Returns	Standard Deviation of Daily Returns
1	12118.35	58242.31
2	9119.11	30936.00
3	8119.42	21981.42
4	7619.61	17595.67
5	7319.74	15024.41
6	7119.85	13351.56
7	6977.08	12185.82
8	6870.02	11332.73
9	6786.76	10684.99
10	6720.16	10178.41

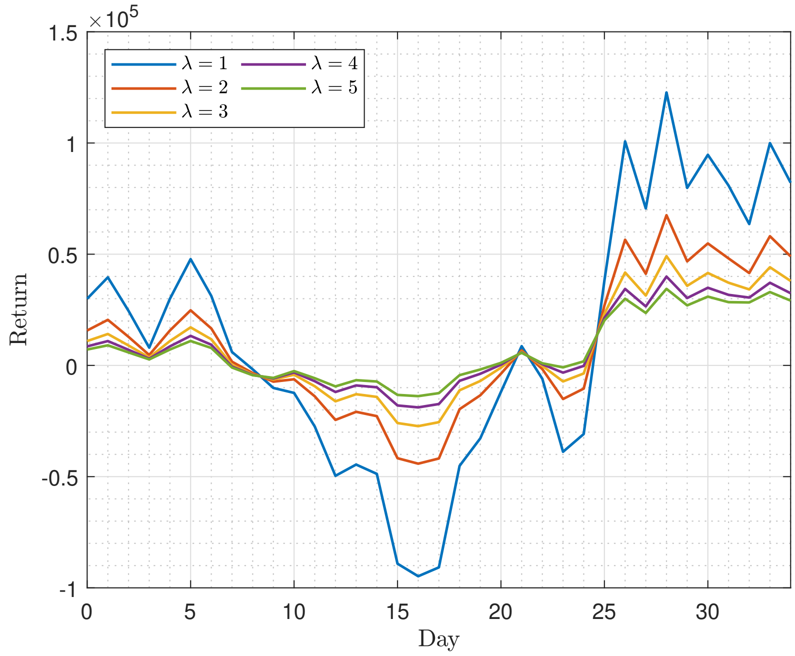

Figure 4.1: Backtesting of (DELTA, GULF) from (3.2) with $\lambda \in \{1, 2, \dots, 5\}$

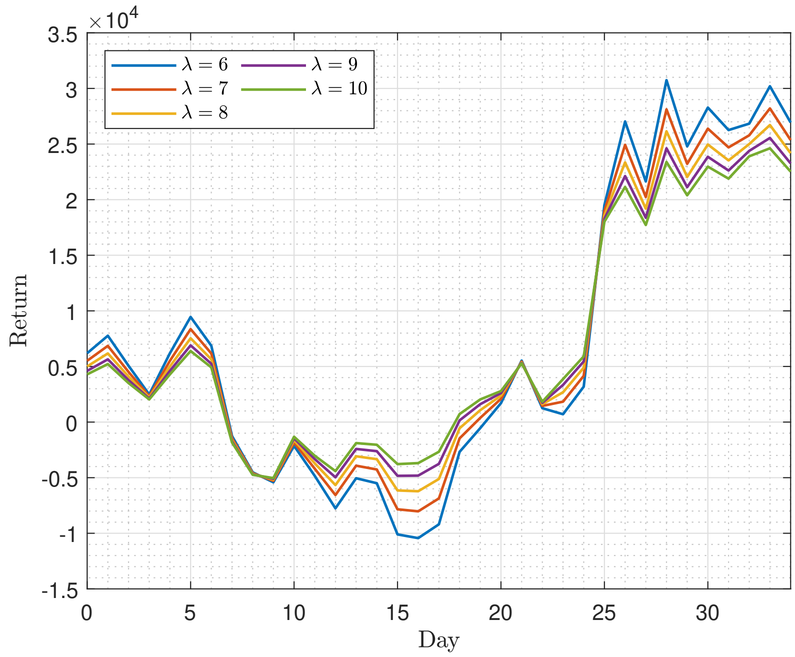

Figure 4.2: Backtesting of (DELTA, GULF) from (3.2) with $\lambda \in \{6, 7, \dots, 10\}$

Table 4.4: Numerical results for three-combination of stocks obtained from (3.2).

λ	Combinations	Portfolios	Obj. Value
1	(BEM, DELTA, TOA)	(48161.72264, −14666.21764, 6745.12331)	3.66×10^{-1}
2	(BEM, DELTA, TOA)	(26140.97411, −6985.91096, 3641.86569)	1.34×10^{-1}
3	(BEM, DELTA, TOA)	(18802.21462, −4426.03175, 2607.32118)	4.95×10^{-2}
4	(BEM, DELTA, TOA)	(15133.96391, −3146.26021, 2089.95293)	1.82×10^{-2}
5	(BEM, DELTA, TOA)	(12933.92600, −2378.53238, 1779.45356)	6.67×10^{-3}
6	(BEM, DELTA, TOA)	(11468.00231, −1866.82696, 1572.38728)	2.46×10^{-3}
7	(BEM, DELTA, TOA)	(10421.57921, −1501.42055, 1424.42446)	9.06×10^{-4}
8	(BEM, DELTA, TOA)	(9637.34995, −1227.45144, 1313.40029)	3.33×10^{-4}
9	(BEM, DELTA, TOA)	(9027.92191, −1014.44094, 1227.00096)	1.22×10^{-4}
10	(BEM, DELTA, TOA)	(8540.85959, −844.10182, 1157.83821)	4.51×10^{-5}

Table 4.5: Backtesting of (BEM, DELTA, TOA) from model (3.2).

λ	Mean Daily of Returns	Standard Deviation of Daily Returns
1	−37668.50	54170.10
2	−17686.50	26725.90
3	−11025.40	17589.00
4	−7694.64	13029.30
5	−5695.92	10300.60
6	−4363.25	8487.79
7	−3411.16	7198.39
8	−2696.94	6236.29
9	−2141.29	5492.53
10	−1696.63	4901.71

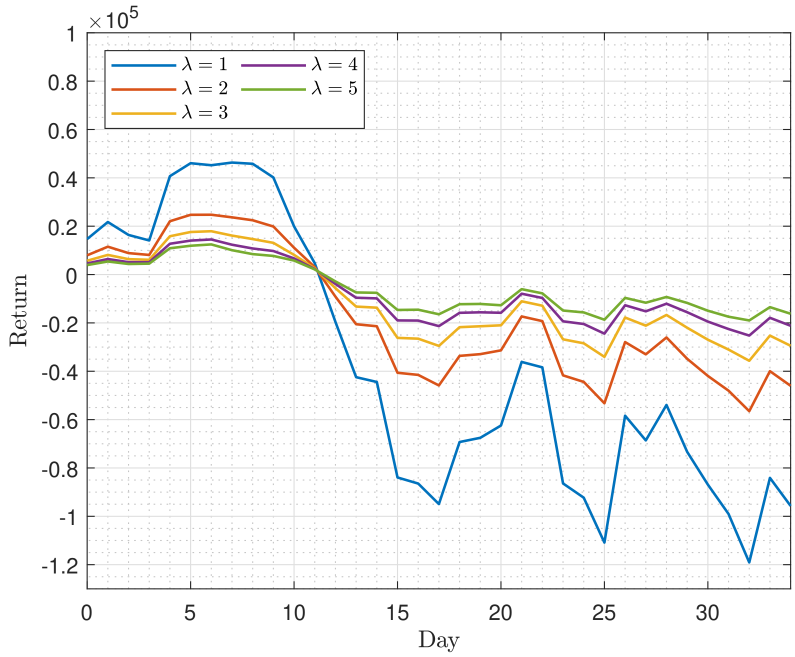

Figure 4.3: Backtesting of (BEM, DELTA, TOA) from (3.2) with $\lambda \in \{1, 2, \ldots, 5\}$

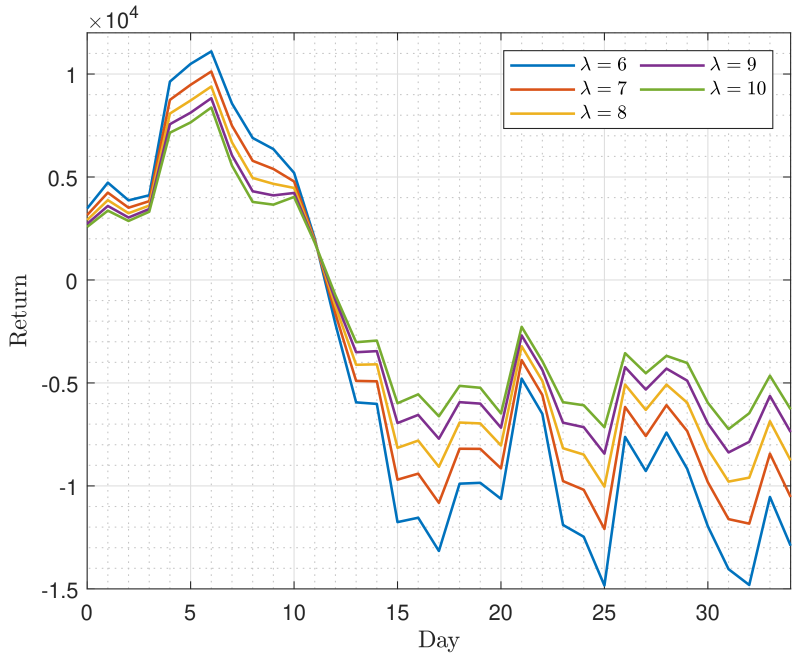

Figure 4.4: Backtesting of (BEM, DELTA, TOA) from (3.2) with $\lambda \in \{6, 7, \ldots, 10\}$

CHAPTER V

CONCLUSION AND FUTURE WORK

5.1 Conclusion

In this work, we have studied the portfolio allocation of the 48 biggest stocks, in terms of market capitalization, included in SET50 index in the Stock Exchange of Thailand using the data from 3 January 2018 to 21 November 2020 (482 trading days) to construct portfolio optimization model based on exponential loss function and applied the results with the data from 22 November 2019 to 24 January 2020 to examine the volatility of the daily returns by the backtesting method, excluding AWC and OSP as their data are not available. By using independent properties of the stock prices at time T and following the well-known binomial distribution, we have purposed the method to choose the optimal combination of two and three stocks for investor under the Armstrong's frameworks.

In this experiment, we found that the portfolio allocations under the Armstrong's framework for two and three combinations of stocks are (DELTA, GULF) and (BEM, DELTA, TOA), respectively. These combinations yield the smallest exponential loss values for all $\lambda \in \{1, 2, \ldots, 10\}$ as can be seen from each table in Appendix.

After that, we apply the backtesting method in order to observe the volatility of the obtained returns via mean and standard deviation for each of the optimal portfolios. As a result of (DELTA, GULF) and (BEM, DELTA, TOA), their standard deviations of the daily returns usually decrease and their means of the daily returns tend to zero when λ increases. A small value of λ represents a behavior of an investor who is a risk lover. In contrast, a behavior of an investor who is a risk aversion is represented by a large value of λ.

For the example of (DELTA, GULF), we can obviously see in Table 4.3 that from $\lambda = 1$ to $\lambda = 10$, the means of the daily returns change from 12118.35 to 6720.16 which mean that the obtained returns are decreasing. Moreover, the standard deviations of the daily returns decrease from 58242.31 to 10178.41 indicating that their volatility is also decreasing. Similarly, for the example of (BEM, DELTA, TOA) in Table 4.5 from $\lambda = 1$ to $\lambda = 10$, the means of the daily returns change from -37668.50 to -1696.63. We notice here that they are negative values which mean that an investor has an opportunity to lose their profit. The standard deviation of the daily return for (BEM, DELTA, TOA) provides the same behavior of the stock pair (DELTA, GULF). However, this thesis focuses on one of the models proposed for both risk-lover and risk-averse investors to choose a suitable portfolio. Therefore, the investors and practitioners who want to invest or hedge in the markets should realize and understand the fund features, conditions of the returns and also the risk before making an investment decision.

5.2 Future work

For the future work, we will investigate the portfolio allocation problem in combination of different markets including SET50, S&P500, and especially, cryptocurrencies which are poorly studied. We can also change the binomial model to other models in order to estimate the prices at maturity time using the Monte Carlo simulations. In the context of utility function, we can change the exponential utility function to others such as VaR and CVaR.

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APPENDIX

Table 1: Portfolios of stock pairs obtained from (3.2) with $\lambda = 1$

Stock Pairs	Portfolios	Obj. Value	Stock Pairs	Portfolios	Obj. Value
(DELTA,GULF)	(−16738.8,5450.17)	0.363090	(KTB,TOA)	(−19791.82,10049.37)	0.367063
(BTS,IRPC)	(73259.04,−255458.93)	0.363602	(BDMS,TOP)	(17165.59,−4721.96)	0.367121
(RATCH,TMB)	(14277.74,−562489.72)	0.364770	(BBL,TOA)	(−1750.2,9574.78)	0.367125
(DELTA,KTC)	(−9392.21,13542.67)	0.364987	(ADVANC,TU)	(2292.9,−28597.77)	0.367191
(EGCO,KTB)	(2666.68,−49512.46)	0.365625	(DELTA,GPSC)	(−6032.21,4703.49)	0.367195
(TMB,VGI)	(−570326.51,102154.68)	0.365727	(DELTA,INTUCH)	(−8156.5,8625.07)	0.367206
(BEM,DELTA)	(63380.82,−12108.37)	0.366028	(BTS,PTTEP)	(21927.69,−1647.26)	0.367295
(DELTA,RATCH)	(−13799.42,10959.17)	0.366035	(BTS,DTAC)	(19293.14,−3100.63)	0.367299
(BH,RATCH)	(−4709.78,10625.56)	0.366324	(LH,TOA)	(−22645.73,7646.4)	0.367322
(RATCH,TU)	(10016.43,−45315.87)	0.366343	(TOA,VGI)	(8541.06,−26618.35)	0.367336
(KBANK,TOA)	(−2808.13,12436.26)	0.366523	(BBL,BDMS)	(−1861.61,17232.49)	0.367388
(DELTA,TOA)	(−8268.88,11712.16)	0.366582	(DELTA,DTAC)	(−4538.11,6119.7)	0.367389
(KTB,RATCH)	(−40753.22,11095.35)	0.366629	(DELTA,TCAP)	(−6427.2,7170.03)	0.367448
(BBL,RATCH)	(−3570.59,10415.64)	0.366762	(BTS,TOA)	(12537.69,−1639.26)	0.367450
(TOA,TU)	(10820.86,−27265.74)	0.366768	(CBG,DELTA)	(3199.97,−3417.76)	0.367462
(BH,TOA)	(−2689.45,10960.38)	0.366796	(GLOBAL,RATCH)	(−9341.71,3696.85)	0.367550
(BDMS,DELTA)	(22160.06,−9322.25)	0.366807	(BEM,TOA)	(5136.05,1066.14)	0.367587
(DELTA,VGI)	(−12208.99,69691.78)	0.366940	(DELTA,LH)	(−5548.57,37050.17)	0.367613
(DELTA,PTTEP)	(−7508.92,3853.37)	0.366948	(CPF,VGI)	(9623.94,−15819.83)	0.367696
(DELTA,TISCO)	(−9305.67,5583.03)	0.366977	(CPN,DELTA)	(2195.26,−756.14)	0.368106
(ADVANC,DELTA)	(2574.04,−9064.44)	0.366991	(BJC,DELTA)	(2114.66,145.65)	0.368148

Table 2: Portfolios of stock pairs obtained from (3.2) with $\lambda = 2$

Stock Pairs	Portfolios	Obj. Value	Stock Pairs	Portfolios	Obj. Value
(DELTA,GULF)	(−7772.13,2855.27)	0.133528	(ADVANC,DELTA)	(1439.27,−4145.11)	0.135033
(BTS,IRPC)	(39316.15,−123783.92)	0.133714	(DELTA,PTTEP)	(−3178.4,2115.7)	0.135033
(RATCH,TMB)	(7655.54,−272244.48)	0.134189	(BTS,TOA)	(9004.85,−510.43)	0.135045
(DELTA,KTC)	(−3914.43,7084.3)	0.134296	(KTB,TOA)	(−7577.27,5308.1)	0.135047
(EGCO,KTB)	(1397.51,−23045.62)	0.134473	(BDMS,TOP)	(9924.28,−2111.9)	0.135061
(TMB,VGI)	(−284934.32,56142.47)	0.134535	(BBL,TOA)	(−657.38,5074.19)	0.135064
(DELTA,RATCH)	(−6603.47,5997.34)	0.134636	(ADVANC,TU)	(1298.39,−12883)	0.135100
(BEM,DELTA)	(34366.52,−5606.74)	0.134651	(LH,TOA)	(−7818.43,4189.62)	0.135114
(BH,RATCH)	(−2235.59,5802.23)	0.134740	(DELTA,INTUCH)	(−3709.27,4883.13)	0.135121
(RATCH,TU)	(5524.79,−21571.91)	0.134744	(TOA,VGI)	(4276.37,−8232.33)	0.135125
(KTB,RATCH)	(−18971.36,5936.9)	0.134854	(BEM,TOA)	(5632.35,940.45)	0.135131
(KBANK,TOA)	(−1189.55,6632.34)	0.134878	(DELTA,GPSC)	(−2390.54,2595.74)	0.135139
(DELTA,TOA)	(−3514.03,6338.34)	0.134892	(GLOBAL,RATCH)	(−3958.82,2398.73)	0.135153
(BBL,RATCH)	(−1653.48,5598.63)	0.134900	(BBL,BDMS)	(−754.03,9398.44)	0.135158
(TOA,TU)	(5891.1,−11366.32)	0.134950	(DELTA,DTAC)	(−1569.16,3380.24)	0.135225
(BH,TOA)	(−1109.36,5911.57)	0.134961	(DELTA,TCAP)	(−2786.43,4106.64)	0.135226
(BDMS,DELTA)	(12247.26,−4215.34)	0.134963	(CPF,VGI)	(4827.21,−2849.09)	0.135258
(DELTA,VGI)	(−6097.02,39911.49)	0.134981	(CBG,DELTA)	(1752.6,−924.64)	0.135263
(BTS,DTAC)	(12587.06,−1351.21)	0.135000	(DELTA,LH)	(−2366.85,21626.09)	0.135295
(BTS,PTTEP)	(13556.98,−697.64)	0.135002	(CPN,DELTA)	(1550.31,81.28)	0.135521
(DELTA,TISCO)	(−4286.64,3124.99)	0.135025	(BJC,DELTA)	(1641.11,582.01)	0.135546

Table 3: Portfolios of stock pairs obtained from (3.2) with $\lambda = 3$

Stock Pairs	Portfolios	Obj. Value	Stock Pairs	Portfolios	Obj. Value
(DELTA,GULF)	$(-4783.36,1990.34)$	0.04911	(BEM,TOA)	$(5799.4,898.14)$	0.049680
(BTS,IRPC)	$(28003.57,-79898.92)$	0.049176	(DELTA,TISCO)	$(-2613.68,2305.68)$	0.049684
(RATCH,TMB)	$(5448.34,-175504.63)$	0.049366	(ADVANC,DELTA)	$(1061.02,-2505.35)$	0.049688
(DELTA,KTC)	$(-2088.44,4931.43)$	0.049419	(KTB,TOA)	$(-3505.24,3727.48)$	0.049688
(EGCO,KTB)	$(974.44,-14223.08)$	0.049459	(BDMS,TOP)	$(7511.18,-1242.13)$	0.049692
(TMB,VGI)	$(-189803.66,40805.08)$	0.049490	(BBL,TOA)	$(-293.1,3573.96)$	0.049692
(DELTA,RATCH)	$(-4205.15,4343.62)$	0.049524	(DELTA,PTTEP)	$(-1734.68,1536.4)$	0.049695
(BEM,DELTA)	$(24697.17,-3439.99)$	0.049537	(GLOBAL,RATCH)	$(-2165.19,1966.19)$	0.049700
(BH,RATCH)	$(-1411,4194.73)$	0.049562	(LH,TOA)	$(-2874.19,3036.94)$	0.049703
(RATCH,TU)	$(4027.81,-13658.48)$	0.049562	(TOA,VGI)	$(2854.81,-2103.65)$	0.049706
(KTB,RATCH)	$(-11711.2,4217.53)$	0.049604	(ADVANC,TU)	$(966.9,-7644.86)$	0.049710
(BBL,RATCH)	$(-1014.52,3993.15)$	0.049619	(DELTA,INTUCH)	$(-2227.05,3635.98)$	0.049723
(BTS,DTAC)	$(10353.07,-768.43)$	0.049622	(BBL,BDMS)	$(-384.88,6787.33)$	0.049725
(BTS,PTTEP)	$(10768.49,-381.3)$	0.049623	(DELTA,GPSC)	$(-1176.65,1893.15)$	0.049739
(BTS,TOA)	$(7828.36,-134.5)$	0.049635	(CPF,VGI)	$(3228.3,1474.5)$	0.049755
(KBANK,TOA)	$(-650.06,4697.85)$	0.049638	(DELTA,TCAP)	$(-1573.07,3085.71)$	0.049768
(DELTA,TOA)	$(-1929.13,4547.12)$	0.049641	(DELTA,DTAC)	$(-579.29,2466.88)$	0.049777
(DELTA,VGI)	$(-4059.7,29984.74)$	0.049654	(CBG,DELTA)	$(1269.95,-93.27)$	0.049796
(TOA,TU)	$(4247.93,-6066.81)$	0.049658	(DELTA,LH)	$(-1306.74,16486.97)$	0.049796
(BDMS,DELTA)	$(8943.56,-2513.33)$	0.049662	(CPN,DELTA)	$(1335.26,360.5)$	0.049896
(BH,TOA)	$(-582.73,4228.84)$	0.049662	(BJC,DELTA)	$(1483.17,727.55)$	0.049909

Table 4: Portfolios of stock pairs obtained from (3.2) with $\lambda = 4$

Stock Pairs	Portfolios	Obj. Value	Stock Pairs	Portfolios	Obj. Value
(DELTA,GULF)	$(-3289.06,1557.89)$	0.018063	(BH,TOA)	$(-319.46,3387.62)$	0.018276
(BTS,IRPC)	$(22348.59,-57961.49)$	0.018086	(GLOBAL,RATCH)	$(-1268.88,1750.04)$	0.018277
(RATCH,TMB)	$(4344.88,-127141.27)$	0.018162	(DELTA,TISCO)	$(-1777.25,1896.04)$	0.018282
(DELTA,KTC)	$(-1175.37,3854.91)$	0.018188	(KTB,TOA)	$(-1468.81,2937)$	0.018283
(EGCO,KTB)	$(762.9,-9811.62)$	0.018192	(BBL,TOA)	$(-110.95,2823.81)$	0.018283
(TMB,VGI)	$(-142238.38,33136.39)$	0.018205	(BDMS,TOP)	$(6305.16,-807.43)$	0.018284
(DELTA,RATCH)	$(-3006.24,3516.94)$	0.018218	(ADVANC,DELTA)	$(871.89,-1685.49)$	0.018284
(BEM,DELTA)	$(19864.07,-2356.98)$	0.018225	(TOA,VGI)	$(2144.03,960.7)$	0.018285
(RATCH,TU)	$(3279.5,-9702.71)$	0.018231	(LH,TOA)	$(-400.68,2460.28)$	0.018285
(BH,RATCH)	$(-998.82,3391.2)$	0.018231	(DELTA,PTTEP)	$(-1012.67,1246.68)$	0.018290
(BTS,DTAC)	$(9237.12,-477.32)$	0.018241	(ADVANC,TU)	$(801.16,-5025.9)$	0.018292
(BTS,PTTEP)	$(9375.58,-223.28)$	0.018241	(BBL,BDMS)	$(-200.32,5481.95)$	0.018295
(BTS,TOA)	$(7240.97,53.18)$	0.018244	(DELTA,INTUCH)	$(-1486.08,3012.52)$	0.018298
(KTB,RATCH)	$(-8081.46,3357.92)$	0.018247	(CPF,VGI)	$(2428.84,3636.3)$	0.018303
(BBL,RATCH)	$(-695.09,3190.55)$	0.018252	(DELTA,GPSC)	$(-569.71,1541.87)$	0.018309
(DELTA,VGI)	$(-3041.04,25021.38)$	0.018265	(DELTA,TCAP)	$(-966.57,2575.4)$	0.018318
(BEM,TOA)	$(5884.17,876.67)$	0.018266	(DELTA,DTAC)	$(-84.18,2010.04)$	0.018325
(KBANK,TOA)	$(-380.35,3730.71)$	0.018269	(DELTA,LH)	$(-777.03,13919.12)$	0.018329
(DELTA,TOA)	$(-1136.71,3651.55)$	0.018270	(CBG,DELTA)	$(1028.48,322.66)$	0.018334
(TOA,TU)	$(3426.42,-3417.26)$	0.018275	(CPN,DELTA)	$(1227.68,500.18)$	0.018372
(BDMS,DELTA)	$(7292.14,-1662.55)$	0.018275	(BJC,DELTA)	$(1404.12,800.39)$	0.018378

Table 5: Portfolios of stock pairs obtained from (3.2) with $\lambda = 5$

Stock Pairs	Portfolios	Obj. Value	Stock Pairs	Portfolios	Obj. Value
(DELTA,GULF)	(−3289.06,1557.89)	0.018063	(BH,TOA)	(−319.46,3387.62)	0.018276
(BTS,IRPC)	(22348.59,−57961.49)	0.018086	(GLOBAL,RATCH)	(−1268.88,1750.04)	0.018277
(RATCH,TMB)	(4344.88,−127141.27)	0.018162	(DELTA,TISCO)	(−1777.25,1896.04)	0.018282
(DELTA,KTC)	(−1175.37,3854.91)	0.018188	(KTB,TOA)	(−1468.81,2937)	0.018283
(EGCO,KTB)	(762.9,−9811.62)	0.018192	(BBL,TOA)	(−110.95,2823.81)	0.018283
(TMB,VGI)	(−142238.38,33136.39)	0.018205	(BDMS,TOP)	(6305.16,−807.43)	0.018284
(DELTA,RATCH)	(−3006.24,3516.94)	0.018218	(ADVANC,DELTA)	(871.89,−1685.49)	0.018284
(BEM,DELTA)	(19864.07,−2356.98)	0.018225	(TOA,VGI)	(2144.03,960.7)	0.018285
(RATCH,TU)	(3279.5,−9702.71)	0.018231	(LH,TOA)	(−400.68,2460.28)	0.018285
(BH,RATCH)	(−998.82,3391.2)	0.018231	(DELTA,PTTEP)	(−1012.67,1246.68)	0.018290
(BTS,DTAC)	(9237.12,−477.32)	0.018241	(ADVANC,TU)	(801.16,−5025.9)	0.018292
(BTS,PTTEP)	(9375.58,−223.28)	0.018241	(BBL,BDMS)	(−200.32,5481.95)	0.018295
(BTS,TOA)	(7240.97,53.18)	0.018244	(DELTA,INTUCH)	(−1486.08,3012.52)	0.018298
(KTB,RATCH)	(−8081.46,3357.92)	0.018247	(CPF,VGI)	(2428.84,3636.3)	0.018303
(BBL,RATCH)	(−695.09,3190.55)	0.018252	(DELTA,GPSC)	(−569.71,1541.87)	0.018309
(DELTA,VGI)	(−3041.04,25021.38)	0.018265	(DELTA,TCAP)	(−966.57,2575.4)	0.018318
(BEM,TOA)	(5884.17,876.67)	0.018266	(DELTA,DTAC)	(−84.18,2010.04)	0.018325
(KBANK,TOA)	(−380.35,3730.71)	0.018269	(DELTA,LH)	(−777.03,13919.12)	0.018329
(DELTA,TOA)	(−1136.71,3651.55)	0.018270	(CBG,DELTA)	(1028.48,322.66)	0.018334
(TOA,TU)	(3426.42,−3417.26)	0.018275	(CPN,DELTA)	(1227.68,500.18)	0.018372
(BDMS,DELTA)	(7292.14,−1662.55)	0.018275	(BJC,DELTA)	(1404.12,800.39)	0.018378

Table 6: Portfolios of stock pairs obtained from (3.2) with $\lambda = 6$

Stock Pairs	Portfolios	Obj. Value	Stock Pairs	Portfolios	Obj. Value
(DELTA,GULF)	(−1794.92,1125.5)	0.002444	(DELTA,TOA)	(−344.36,2756.05)	0.002475
(BTS,IRPC)	(16696.27,−36034.38)	0.002447	(KBANK,TOA)	(−110.69,2763.76)	0.002475
(RATCH,TMB)	(3241.74,−78791.33)	0.002459	(BBL,TOA)	(71.21,2073.58)	0.002475
(EGCO,KTB)	(551.33,−5399.75)	0.002461	(TOA,TU)	(2605.02,−768.09)	0.002476
(TMB,VGI)	(−94673.22,25467.72)	0.002464	(KTB,TOA)	(568.48,2146.2)	0.002476
(DELTA,KTC)	(−262.15,2778.21)	0.002464	(BH,TOA)	(−56.28,2546.68)	0.002476
(BTS,DTAC)	(8123.31,−186.76)	0.002465	(BDMS,TOP)	(5100.2,−373.11)	0.002476
(BTS,TOA)	(6655.35,240.3)	0.002465	(DELTA,TISCO)	(−940.93,1486.45)	0.002476
(BTS,PTTEP)	(7985.36,−65.57)	0.002465	(ADVANC,DELTA)	(682.78,−865.68)	0.002476
(DELTA,RATCH)	(−1807.86,2690.62)	0.002465	(CPF,VGI)	(1629.37,5798.12)	0.002477
(RATCH,TU)	(2531.55,−5748.86)	0.002467	(BBL,BDMS)	(−15.81,4176.9)	0.002477
(BH,RATCH)	(−586.85,2588.09)	0.002467	(ADVANC,TU)	(635.43,−2407.17)	0.002477
(BEM,DELTA)	(15034.16,−1274.67)	0.002467	(DELTA,PTTEP)	(−290.33,956.83)	0.002478
(KTB,RATCH)	(−4452.43,2498.48)	0.002469	(DELTA,INTUCH)	(−745.41,2389.31)	0.002479
(BBL,RATCH)	(−375.78,2388.24)	0.002470	(DELTA,GPSC)	(37.23,1190.57)	0.002481
(BEM,TOA)	(5971.5,854.56)	0.002470	(DELTA,TCAP)	(−360.44,2065.39)	0.002482
(DELTA,VGI)	(−2022.38,20058.03)	0.002472	(DELTA,LH)	(−248.04,11354.72)	0.002484
(GLOBAL,RATCH)	(−373.64,1534.15)	0.002472	(DELTA,DTAC)	(411.29,1552.86)	0.002484
(TOA,VGI)	(1433.24,4025.06)	0.002474	(CBG,DELTA)	(786.7,739.14)	0.002486
(LH,TOA)	(2075.68,1882.95)	0.002475	(CPN,DELTA)	(1120.02,639.98)	0.002491
(BDMS,DELTA)	(5641.61,−812.22)	0.002475	(BJC,DELTA)	(1324.94,873.36)	0.002493

Table 7: Portfolios of stock pairs obtained from (3.2) with $\lambda = 7$

Stock Pairs	Portfolios	Obj. Value	Stock Pairs	Portfolios	Obj. Value
(DELTA,GULF)	(−1368.09,1001.98)	0.000899	(BDMS,DELTA)	(5170.41,−569.47)	0.000911
(BTS,IRPC)	(15082.48,−29774)	0.0009	(KTB,TOA)	(1150.94,1920.11)	0.000911
(RATCH,TMB)	(2926.69,−64982.96)	0.000905	(CPF,VGI)	(1400.95,6415.8)	0.000911
(EGCO,KTB)	(490.88,−4139.04)	0.000905	(BDMS,TOP)	(4756.4,−249.19)	0.000911
(TMB,VGI)	(−81083.22,23276.68)	0.000906	(DELTA,TOA)	(−117.99,2500.21)	0.000911
(BTS,TOA)	(6488.82,293.51)	0.000906	(KBANK,TOA)	(−33.66,2487.57)	0.000911
(BTS,DTAC)	(7806.02,−103.99)	0.000906	(DELTA,TISCO)	(−702.02,1369.45)	0.000911
(BTS,PTTEP)	(7589.34,−20.64)	0.000906	(BH,TOA)	(18.88,2306.53)	0.000911
(DELTA,RATCH)	(−1465.69,2454.68)	0.000907	(TOA,TU)	(2370.37,−11.32)	0.000911
(DELTA,KTC)	(−1.15,2470.49)	0.000907	(BBL,BDMS)	(36.88,3804.17)	0.000911
(RATCH,TU)	(2318.01,−4620.02)	0.000908	(ADVANC,DELTA)	(628.76,−631.47)	0.000911
(BH,RATCH)	(−469.25,2358.82)	0.000908	(ADVANC,TU)	(588.09,−1659.08)	0.000912
(BEM,DELTA)	(13655.57,−965.75)	0.000908	(DELTA,INTUCH)	(−533.92,2211.36)	0.000912
(BEM,TOA)	(5997.58,847.95)	0.000908	(DELTA,PTTEP)	(−83.81,873.97)	0.000912
(KTB,RATCH)	(−3415.86,2253)	0.000908	(DELTA,GPSC)	(210.65,1090.2)	0.000914
(BBL,RATCH)	(−284.6,2159.13)	0.000909	(DELTA,TCAP)	(−187.42,1919.81)	0.000914
(DELTA,VGI)	(−1731.34,18639.94)	0.000909	(DELTA,LH)	(−97.21,10623.53)	0.000914
(GLOBAL,RATCH)	(−118.32,1472.58)	0.000909	(DELTA,DTAC)	(553.03,1422.09)	0.000915
(TOA,VGI)	(1230.16,4900.59)	0.000910	(CBG,DELTA)	(717.48,858.37)	0.000916
(LH,TOA)	(2784.47,1717.7)	0.000911	(CPN,DELTA)	(1089.21,679.97)	0.000917
(BBL,TOA)	(123.27,1859.19)	0.000911	(BJC,DELTA)	(1302.25,894.26)	0.000918

Table 8: Portfolios of stock pairs obtained from (3.2) with $\lambda = 8$

Stock Pairs	Portfolios	Obj. Value	Stock Pairs	Portfolios	Obj. Value
(DELTA,GULF)	(−1048.00291,909.3463)	0.000331	(BBL,TOA)	(162.32275,1698.3631)	0.000335
(BTS,IRPC)	(13872.83161,−25081.38701)	0.000332	(KTB,TOA)	(1588.02548,1750.44691)	0.000335
(RATCH,TMB)	(2690.47965,−54630.1998)	0.000333	(BDMS,DELTA)	(4817.23721,−387.51907)	0.000335
(EGCO,KTB)	(445.53122,−3193.39492)	0.000333	(BDMS,TOP)	(4498.83692,−156.35734)	0.000335
(BTS,TOA)	(6364.38762,333.2726)	0.000333	(BBL,BDMS)	(76.39301,3524.69984)	0.000335
(BTS,DTAC)	(7568.61949,−42.05533)	0.000333	(DELTA,TISCO)	(−522.87646,1281.71642)	0.000335
(BTS,PTTEP)	(7293.0142,12.97738)	0.000333	(BH,TOA)	(75.23288,2126.47482)	0.000335
(TMB,VGI)	(−70890.74985,21633.40661)	0.000333	(ADVANC,DELTA)	(588.24265,−455.83725)	0.000335
(DELTA,RATCH)	(−1209.19239,2277.81858)	0.000334	(TOA,TU)	(2194.41746,556.17271)	0.000335
(RATCH,TU)	(2157.95035,−3773.89784)	0.000334	(KBANK,TOA)	(24.09398,2280.46775)	0.000335
(BH,RATCH)	(−381.09585,2186.97386)	0.000334	(DELTA,TOA)	(51.77571,2308.34817)	0.000335
(DELTA,KTC)	(194.66076,2239.62836)	0.000334	(ADVANC,TU)	(552.5842,−1098.08618)	0.000336
(BEM,TOA)	(6017.8129,842.82608)	0.000334	(DELTA,INTUCH)	(−375.38183,2077.96445)	0.000336
(BEM,DELTA)	(12622.44393,−734.24398)	0.000334	(DELTA,PTTEP)	(71.1668,811.77971)	0.000336
(KTB,RATCH)	(−2638.6168,2068.92874)	0.000334	(DELTA,TCAP)	(−57.75122,1810.70697)	0.000336
(BBL,RATCH)	(−216.23878,1987.37254)	0.000334	(DELTA,GPSC)	(340.72637,1014.91292)	0.000336
(GLOBAL,RATCH)	(72.88263,1426.46729)	0.000334	(DELTA,LH)	(15.7286,10076.0365)	0.000337
(DELTA,VGI)	(−1513.05683,17576.37386)	0.000334	(DELTA,DTAC)	(659.43118,1323.90649)	0.000337
(TOA,VGI)	(1077.84508,5557.24951)	0.000335	(CBG,DELTA)	(665.47812,947.94606)	0.000337
(LH,TOA)	(3316.81354,1593.59495)	0.000335	(CPN,DELTA)	(1066.09262,709.99493)	0.000338
(CPF,VGI)	(1229.63292,6879.05384)	0.000335	(BJC,DELTA)	(1285.20708,909.96626)	0.000338

Table 9: Portfolios of stock pairs obtained from (3.2) with $\lambda = 9$

Stock Pairs	Portfolios	Obj. Value	Stock Pairs	Portfolios	Obj. Value
(DELTA,GULF)	(−799.08,837.31)	0.0001218	(BBL,TOA)	(192.7,1573.25)	0.0001234
(BTS,IRPC)	(12932.61,−21433.99)	0.0001219	(KTB,TOA)	(1928.19,1618.41)	0.0001234
(RATCH,TMB)	(2506.84,−46581.24)	0.0001225	(BBL,BDMS)	(107.11,3307.4)	0.0001234
(BTS,TOA)	(6268.03,364.06)	0.0001225	(BDMS,DELTA)	(4542.75,−246.11)	0.0001234
(BTS,PTTEP)	(7063.17,39.05)	0.0001225	(BDMS,TOP)	(4298.76,−84.24)	0.0001234
(BTS,DTAC)	(7384.48,5.98)	0.0001225	(DELTA,TISCO)	(−383.57,1213.49)	0.0001234
(EGCO,KTB)	(410.26,−2457.8)	0.0001225	(ADVANC,DELTA)	(556.73,−319.25)	0.0001235
(TMB,VGI)	(−62963.3,20355.31)	0.0001226	(BH,TOA)	(119.04,1986.49)	0.0001235
(DELTA,RATCH)	(−1009.82,2140.34)	0.0001228	(ADVANC,TU)	(524.98,−661.83)	0.0001235
(RATCH,TU)	(2033.54,−3116.25)	0.0001228	(TOA,TU)	(2057.58,997.5)	0.0001235
(BH,RATCH)	(−312.59,2053.42)	0.0001229	(KBANK,TOA)	(69.01,2119.42)	0.0001235
(BEM,TOA)	(6034.16,838.68)	0.0001229	(DELTA,TOA)	(183.81,2159.13)	0.0001235
(KTB,RATCH)	(−2034.25,1925.8)	0.0001229	(DELTA,INTUCH)	(−252.15,1974.27)	0.0001236
(BBL,RATCH)	(−163.1,1853.85)	0.000123	(DELTA,PTTEP)	(191.78,763.38)	0.0001237
(BEM,DELTA)	(11819.64,−554.35)	0.000123	(DELTA,TCAP)	(43.02,1725.92)	0.0001238
(DELTA,KTC)	(347.01,2060.01)	0.0001230	(DELTA,GPSC)	(441.9,956.35)	0.0001239
(GLOBAL,RATCH)	(221.34,1390.67)	0.0001230	(DELTA,LH)	(103.41,9651.01)	0.0001239
(DELTA,VGI)	(−1343.28,16749.16)	0.0001230	(DELTA,DTAC)	(742.28,1247.46)	0.0001241
(TOA,VGI)	(959.38,6067.98)	0.0001232	(CBG,DELTA)	(624.95,1017.75)	0.0001242
(CPF,VGI)	(1096.38,7239.37)	0.0001233	(CPN,DELTA)	(1048.09,733.37)	0.0001245
(LH,TOA)	(3731.54,1496.91)	0.0001233	(BJC,DELTA)	(1271.92,922.21)	0.0001246

Table 10: Portfolios of stock pairs obtained from (3.2) with $\lambda = 10$

Stock Pairs	Portfolios	Obj. Value	Stock Pairs	Portfolios	Obj. Value
(DELTA,GULF)	(−799.08,837.31)	0.0001218	(BBL,TOA)	(192.7,1573.25)	0.0001234
(BTS,IRPC)	(12932.61,−21433.99)	0.0001219	(KTB,TOA)	(1928.19,1618.41)	0.0001234
(RATCH,TMB)	(2506.84,−46581.24)	0.0001225	(BBL,BDMS)	(107.11,3307.4)	0.0001234
(BTS,TOA)	(6268.03,364.06)	0.0001225	(BDMS,DELTA)	(4542.75,−246.11)	0.0001234
(BTS,PTTEP)	(7063.17,39.05)	0.0001225	(BDMS,TOP)	(4298.76,−84.24)	0.0001234
(BTS,DTAC)	(7384.48,5.98)	0.0001225	(DELTA,TISCO)	(−383.57,1213.49)	0.0001234
(EGCO,KTB)	(410.26,−2457.8)	0.0001225	(ADVANC,DELTA)	(556.73,−319.25)	0.0001235
(TMB,VGI)	(−62963.3,20355.31)	0.0001226	(BH,TOA)	(119.04,1986.49)	0.0001235
(DELTA,RATCH)	(−1009.82,2140.34)	0.0001228	(ADVANC,TU)	(524.98,−661.83)	0.0001235
(RATCH,TU)	(2033.54,−3116.25)	0.0001228	(TOA,TU)	(2057.58,997.5)	0.0001235
(BH,RATCH)	(−312.59,2053.42)	0.0001229	(KBANK,TOA)	(69.01,2119.42)	0.0001235
(BEM,TOA)	(6034.16,838.68)	0.0001229	(DELTA,TOA)	(183.81,2159.13)	0.0001235
(KTB,RATCH)	(−2034.25,1925.8)	0.0001229	(DELTA,INTUCH)	(−252.15,1974.27)	0.0001236
(BBL,RATCH)	(−163.1,1853.85)	0.000123	(DELTA,PTTEP)	(191.78,763.38)	0.0001237
(BEM,DELTA)	(11819.64,−554.35)	0.000123	(DELTA,TCAP)	(43.02,1725.92)	0.0001238
(DELTA,KTC)	(347.01,2060.01)	0.0001230	(DELTA,GPSC)	(441.9,956.35)	0.0001239
(GLOBAL,RATCH)	(221.34,1390.67)	0.0001230	(DELTA,LH)	(103.41,9651.01)	0.0001239
(DELTA,VGI)	(−1343.28,16749.16)	0.0001230	(DELTA,DTAC)	(742.28,1247.46)	0.0001241
(TOA,VGI)	(959.38,6067.98)	0.0001232	(CBG,DELTA)	(624.95,1017.75)	0.0001242
(CPF,VGI)	(1096.38,7239.37)	0.0001233	(CPN,DELTA)	(1048.09,733.37)	0.0001245
(LH,TOA)	(3731.54,1496.91)	0.0001233	(BJC,DELTA)	(1271.92,922.21)	0.0001246

Table 11: Portfolios for three combinations obtained from (3.2) with $\lambda = 1$

Combinations	Portfolios	Obj. Value
(BEM, DELTA, TOA)	(48161.72,−14666.22,6745.12)	0.365614
(DELTA, TOA, VGI)	(517724894.30,1755658.96,−234364262.50)	3.727424
(DELTA, LH, TOA)	(4271110847.00,−1205490259.00,−2132215884.00)	9.065911

Table 12: Portfolios for three combinations obtained from (3.2) with $\lambda = 2$

Combinations	Portfolios	Obj. Value
(BEM, DELTA, TOA)	(26140.97,−6985.91,3641.87)	0.134473
(DELTA, TOA, VGI)	(70670364.64,−19939597.70,−35278867.34)	1.266002
(DELTA, LH, TOA)	(49387872.88,31564.60,−4249872.18)	2.256337

Table 13: Portfolios for three combinations obtained from (3.2) with $\lambda = 3$

Combinations	Portfolios	Obj. Value
(BEM, DELTA, TOA)	(18802.21,−4426.03,2607.32)	0.049462
(DELTA, LH, TOA)	(3984017.12,−1124359.62,−1988896.84)	1.062857
(DELTA, TOA, VGI)	(370420809.80,1201491.09,−167693936.00)	2.016559

Table 14: Portfolios for three combinations obtained from (3.2) with $\lambda = 4$

Combinations	Portfolios	Obj. Value
(BEM, DELTA, TOA)	(15133.96,−3146.26,2089.95)	0.018194
(DELTA, LH, TOA)	(2165460.87,−610378.74,−1080888.32)	0.793073
(DELTA, TOA, VGI)	(193185947.8,674669.82,−87458730.01)	0.850695

Table 15: Portfolios for three combinations obtained from (3.2) with $\lambda = 5$

Combinations	Portfolios	Obj. Value
(BEM, DELTA, TOA)	(12933.93,−2378.53,1779.45)	0.006693
(DELTA, TOA, VGI)	(24413867.96,85177.38,−11052576.74)	0.674440
(DELTA, LH, TOA)	(199384909.1,−56164517.23,−99516031.67)	0.723128

Table 16: Portfolios for three combinations obtained from (3.2) with $\lambda = 6$

Combinations	Portfolios	Obj. Value
(BEM, DELTA, TOA)	(11468.00,−1866.83,1572.39)	0.002462
(DELTA, LH, TOA)	(2082847.94,−587494.11,−1039726.79)	0.324044
(DELTA, TOA, VGI)	(274438.51,913.54,−124241.93)	0.456907

Table 17: Portfolios for three combinations obtained from (3.2) with $\lambda = 7$

Combinations	Portfolios	Obj. Value
(BEM, DELTA, TOA)	(10421.58,−1501.42,1424.42)	0.000906
(DELTA, LH, TOA)	(2082847.94,−587494.11,−1039726.79)	0.059457
(DELTA, TOA, VGI)	(274438.51,913.54,−124241.93)	0.062168

Table 18: Portfolios for three combinations obtained from (3.2) with $\lambda = 8$

Combinations	Portfolios	Obj. Value
(BEM, DELTA, TOA)	(9637.35,−1227.45,1313.40)	0.000333
(DELTA, LH, TOA)	(2082847.94,−587494.11,−1039726.79)	0.010918
(DELTA, TOA, VGI)	(274438.51,913.54,−124241.93)	0.028132

Table 19: Portfolios for three combinations obtained from (3.2) with $\lambda = 9$

Combinations	Portfolios	Obj. Value
(BEM, DELTA, TOA)	(9027.92,−1014.44,1227.00)	0.000123
(DELTA, LH, TOA)	(186549.15,−52655.44,−93130.36)	0.002261
(DELTA, TOA, VGI)	(180397.61,585.07,−81667.92)	0.002954

Table 20: Portfolios for three combinations obtained from (3.2) with $\lambda = 10$

Combinations	Portfolios	Obj. Value
(BEM, DELTA, TOA)	(8540.86,−844.1,1157.84)	0.000045
(DELTA, LH, TOA)	(186549.15,−52655.44,−93130.36)	0.000651
(DELTA, TOA, VGI)	(180397.61,585.07,−81667.92)	0.000687

Example of Mathematica Code

```
V0 = 100000;
num = 48;
Lcompany = {AOT, BBL, ADVANC, BANPU, BDMS, BEM, BGRIM, BH, BJC, BPP,
   BTS, CBG, CPALL, CPF, CPN, DELTA, DTAC, EA, EGCO, GLOBAL, GPSC,
   GULF, HMPRO, INTUCH, IRPC, IVL, KBANK, KTB, KTC, LH, MINT, MTC,
   PTT, PTTEP, PTTGC, RATCH, SAWAD, SCB, SCC, TCAP, TISCO, TMB, TOA,
   TOP, TU, VGI, WHA, TRUE, AWC, OSP};
Table[S0[i] =
   Import["TESTIMPORT.xlsx", {"Data", i, Range[37, 37], 3}], {i, num}];
Table[Data[i] =
   Import["TESTIMPORT.xlsx", {"Data", i, Range[38, 507], 2}], {i,
   num}];
Table[u[i] = Mean[Select[(Data[i])/100, # > 0 &]], {i, 1, num}];
Table[d[i] = Mean[Select[(Data[i]/100), # <= 0 &]], {i, 1, num}];
Table[p[i] = (Count[Data[i], u_ /; u > 0])/(Length[Data[i]]), {i, 1,
   num}];
Table[EK[i] = 100*((1 + u[i])*p[i] + (1 + d[i]) (1 - p[i]) - 1), {i,
   1, num}];
Table[VarData[i] = (100/S0[i])^2*p[i]*(1 - p[i]), {i, 1, num}];
Table[Var[x1_, x2_][i,
    j] = (x1*S0[i]/V0)^2*VarData[i] + (x2*S0[j]/V0)^2*VarData[j], {i,
   1, num}, {j, i + 1, num}];
Table[Expec[x1_, x2_][i,
    j] = (x1*S0[i]/V0)*EK[i] + (x2*S0[j]/V0)*EK[j], {i, 1, num}, {j,
   i + 1, num}];
com = Table[{Lcompany[[i]], Lcompany[[j]]}, {i, 1, num}, {j, i + 1,
    num}];
inde2 = Subsets[Range[1, 48], {2}];
listinde =
  Table[{IndependenceTest[Data[inde2[[i]][[1]]],
     Data[inde2[[i]][[2]]],"PearsonCorrelation"]}, {i, 1, Length[inde2
        ]}];
```

```
companyinde =
  Table[If[listinde[[i]][[1]] > 0.05, {inde2[[i]]}, {}], {i, 1,
    Length[listinde]}];
abc = Flatten[companyinde, 1]
lambda = 1;
claim = 0;
ansxx = Table[{Lcompany[[abc[[i]][[1]]]], Lcompany[[abc[[i]][[2]]]],
   Flatten[OPx[i] =
     NMinimize[{Exp[
         lambda*claim/V0]*(p[abc[[i]][[1]]]*
           Exp[-(lambda/V0)*(1 + u[abc[[i]][[1]]])*S0[abc[[i]][[1]]]*
             x1] + (1 - p[abc[[i]][[1]]])*
           Exp[-(lambda/V0)*(1 + d[abc[[i]][[1]]])*S0[abc[[i]][[1]]]*
             x1])*(p[abc[[i]][[2]]]*
           Exp[-(lambda/V0)*(1 + u[abc[[i]][[2]]])*S0[abc[[i]][[2]]]*
             x2] + (1 - p[abc[[i]][[2]]])*
           Exp[-(lambda/V0)*(1 + d[abc[[i]][[2]]])*S0[abc[[i]][[2]]]*
             x2]),
       x1*S0[abc[[i]][[1]]] + x2*S0[abc[[i]][[2]]] <= V0}, {x1, x2}]]
   }, {i, 1, Length[abc]}]
```

BIOGRAPHY

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Scholarships	Development and Promotion of Science and Technology Talents Project (DPST)

Publications

Krerkkiat Charoenying, Boonyarit Intiyot and Sanae Rujivan, Portfolio selection problem in SET50 based on binomial model, *Proceeding of Annual Pure and Applied Mathematics Conference 2021*, pp. 128–136, 2021.